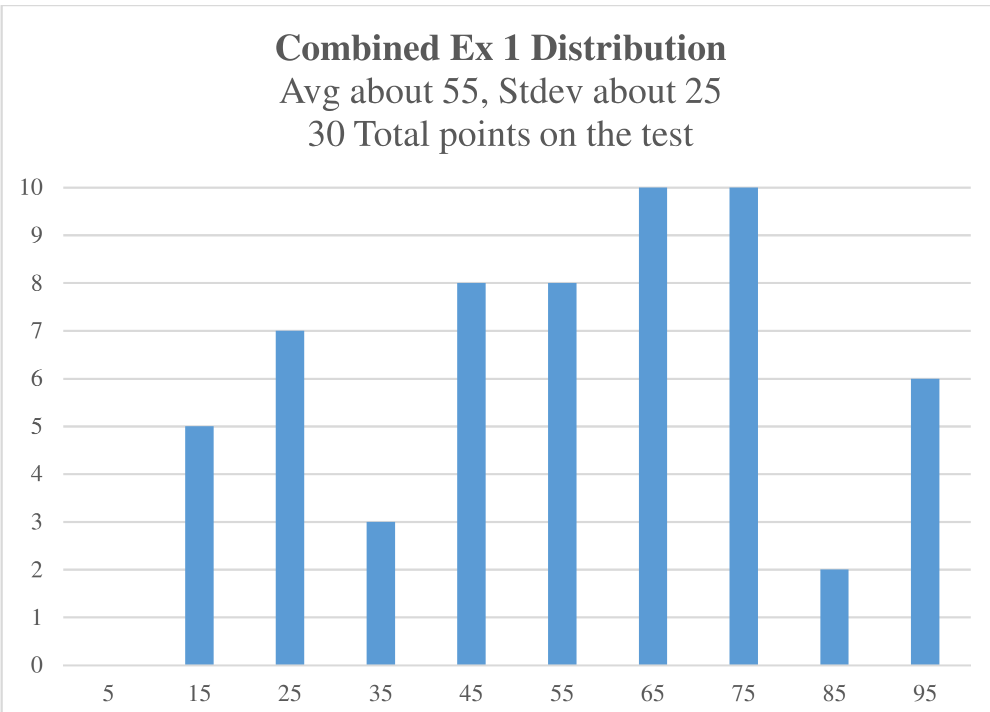
110Fa19t1aSoln

Distribution shown below.
Solutions begin on the next page.

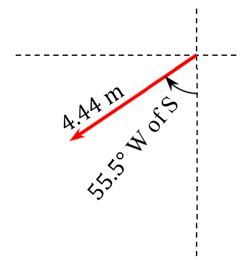
I grade super stingy on these basics since we need to use them for the next year and a half together!
If you scored below 40%, the odds are not in your favor to pass this course this go-around.

If you scored in the 40-60% range, you are on the bubble.
You still have a realistic shot…but only if you do significantly more practice per week outside of class.
If you are unwilling or unable to work more outside of class…be honest with yourself about your chances.

If you scored above 60%, you will probably make it if you continue to work.



1a) I sketched the vector at right.



$$\vec{z} = \left(4.44\sin 55.5°\,(-\hat{\imath}) + 4.44\cos 55.5°\,(-\hat{\jmath})\right)\text{m}$$

$$\vec{z} = \left(-3.6\underline{5}9\hat{\imath} - 2.5\underline{1}5\hat{\jmath}\right)\text{m}$$

$$\boldsymbol{\vec{z} = -(3.66\hat{\imath} + 2.52\hat{\jmath})\text{m}}$$

Either of the last two answer is fine.
Note: if you need to use one of these answers *in a subsequent part*, it is important to use the *first* form (unrounded) to reduce intermediate rounding error.

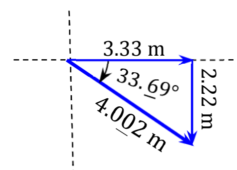
1b) The *unit* vector $\hat{w}$ is given by dividing the vector by the magnitude.
The magnitude is $w = \sqrt{w_x^2 + w_y^2} = \sqrt{(3.33)^2 + (-2.22)^2} = 4.0\underline{0}2$ m.

$$\hat{w} = \frac{\vec{w}}{w} = \frac{(3.33\hat{\imath} - 2.22\hat{\jmath})\text{m}}{4.0\underline{0}2\text{ m}} = \frac{3.33}{4.0\underline{0}2}\hat{\imath} - \frac{2.22}{4.0\underline{0}2}\hat{\jmath} = 0.83\underline{2}0\hat{\imath} - 0.55\underline{4}7\hat{\jmath}$$

$$\boldsymbol{\hat{w} = 0.832\hat{\imath} - 0.555\hat{\jmath}}$$

WATCH OUT! Notice the units of m cancel when writing the unit vector!!!

1c) First I make the sketch shown at right.



The magnitude was found in the previous part.
The angle was found using

$$\phi = \tan^{-1}\left|\frac{w_y}{w_x}\right| = \tan^{-1}\left|-\frac{2.22\text{ m}}{3.33\text{ m}}\right| = 33.\underline{6}9°$$

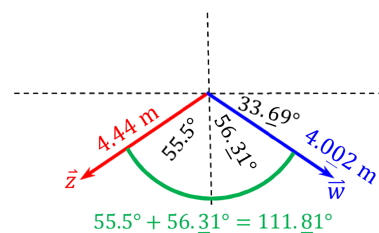
The three most common ways to correctly write the answer are:

$$\boldsymbol{\vec{w} = 4.0\underline{0}2\text{ m @ }33.\underline{6}9°\text{ S of E}}$$

$$\boldsymbol{\vec{w} = 4.0\underline{0}2\text{ m @ }56.\underline{3}1°\text{ E of S}}$$

$$\boldsymbol{\vec{w} = 4.0\underline{0}2\text{ m @ }326.\underline{3}1°\text{ from the positive }x\text{ axis}}$$

1d) It is easiest to get the *magnitude* of the cross-product using



$$\|\vec{z}\times\vec{w}\| = zw\sin\theta_{zw}$$

$$\|\vec{z}\times\vec{w}\| = (4.44\text{ m})(4.0\underline{0}2\text{ m})\sin(111.\underline{8}1°)$$

$$\|\vec{z}\times\vec{w}\| = \boldsymbol{16.\underline{4}97\text{ m}^2}$$

Don't forget the units!!!

You could do the full wheel of pain method (req'd for 3D vectors), then remember to take the magnitude.

$$\vec{z}\times\vec{w} = \left(-3.6\underline{5}9\hat{\imath} - 2.5\underline{1}5\hat{\jmath}\right)\text{m} \times (3.33\hat{\imath} - 2.22\hat{\jmath})\text{m}$$

Ignore $\hat{\imath}\times\hat{\imath} = 0$ & $\hat{\jmath}\times\hat{\jmath} = 0$ terms.

$$\vec{z}\times\vec{w} = \left(-3.6\underline{5}9\hat{\imath}\right)\text{m} \times (-2.22\hat{\jmath})\text{m} + \left(-2.5\underline{1}5\hat{\jmath}\right)\text{m} \times (3.33\hat{\imath})\text{m}$$

$$\vec{z}\times\vec{w} = 8.1\underline{2}3\text{m}^2\hat{k} + 8.3\underline{7}5\text{m}^2\hat{k} = 16.\underline{4}98\text{m}^2\hat{k}$$

Notice the magnitude is again $16.\underline{4}97$ m^2.

1e) I asked for the angle between & the NEGATIVE x-axis.
You could use the figure.
The standard method is to use

$$\theta_{-x\,axis} = \cos^{-1}\left(-\frac{w_x}{w}\right) = 14\underline{6}.3°$$

1f) The sketch is shown above in part 1d.
Sometimes on exams, it is easier to do the parts of the question out of order.
For me it was easiest to do part 1f) before part 1d)!

2a) We are asked for the *mass* of the pipe.
We know the *volume* of the pipe is

$$V_{total} = V_{outer} - V_{inner}$$

Notice, the following does NOT compute the total volume:

$$V_{total} \neq \pi L\left(\frac{D}{2} - \frac{d}{2}\right)^2$$

This was a common mistake.
I don't really understand why students try this but, if *you* did, I guess take comfort in the fact you were not alone in thinking it would work. The units check out in this mistake…but that doesn't mean it is correct.

Proceeding in the correct way gives

$$V_{total} = V_{outer} - V_{inner}$$
$$V_{total} = \pi R_{outer}^2 L - \pi R_{inner}^2 L$$
$$V_{total} = \pi L(R_{outer}^2 - R_{inner}^2)$$
$$V_{total} = \pi L\left(\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right)$$

If you did any homework at all, you know you are expected to simplify this ugly looking result.
Students lost points for not simplifying their work to a reasonable level.

$$V_{total} = \frac{\pi L}{4}(D^2 - d^2)$$

To get mass from volume, use the density equation.

$$\rho = \frac{M}{V} \quad \rightarrow \quad M = \rho V$$

Therefore the total *mass* is

$$\boldsymbol{M = \frac{\pi}{4}\rho L(D^2 - d^2)}$$

I check the units to feel more confident.
The output units of this equation are units of mass…CHECK!

2b) The given parameters of this problem are shown in the table at right.
Notice: while using the units at right, the output units are grams = g.
This means I can leave off units during my calculation to reduce clutter.
I do need to remember to write the correct units on the final answer.

$D = 7.77 \text{ nm} = 7.77 \times 10^{-9} \text{ m}$
$d = 7.43 \text{ nm} = 7.43 \times 10^{-9} \text{ m}$
$L = 44.4 \text{ cm} = 44.4 \times 10^{-2} \text{ m}$
$\rho = 2.267 \times 10^6 \dfrac{\text{g}}{\text{m}^3}$

$$M = \frac{\pi}{4}(2.267 \times 10^6)(44.4 \times 10^{-2})((7.77 \times 10^{-9})^2 - (7.43 \times 10^{-9})^2)$$

Watch out! Correct ways to write L are $0.444 \text{ m} = 4.44 \times 10^{-1} \text{ m} = 44.4 \times 10^{-2} \text{ m}$.

$$M = 7.9\underline{0}5 \times 10^5\left(6.0\underline{3}7 \times 10^{-17} - 5.5\underline{2}0 \times 10^{-17}\right)$$

Notice when we do the subtraction we must keep the column of sig figs…

$$M = 7.9\underline{0}5 \times 10^5\left(0.5\underline{1}7 \times 10^{-17}\right)$$
$$M = 4.\underline{0}85 \times 10^{-12} \text{ g}$$

If you are uncertain if this in engineering notation, use the engr mode on your calculator to check…
Finally, I used the prefix chart on the equation sheet and finished rounding my answer.

$$M = 4.1 \text{ pg}$$

3) I found $2.61 \times 10^{-6} \, \frac{\text{lbs}}{\text{yr}^2}$

For some reason, when people start doing conversions, they sometimes write way too many sig figs.
It is ok to include an extra digit IF you clearly indicate the rounding digit.
Generally speaking, we expect most numbers to have the same number of sig figs before & after conversion.

The nanograms seemed to throw people off.
When I need to convert a prefix I simply replace the letter with $\times 10^{appropriate\ power}$.
Alternatively, write out the prefix on relationship on a separate line to ensure you don't mess it up.

$$1 \text{ ng} = 1 \times 10^{-9} \text{ g}$$

4a) 2

4b) 8.8×10^{-1} m. WATCH OUT! Read the problem statement carefully.
I asked for the number written <u>in units of m</u> with scientific notation…<u>not units of mm</u> in scientific notation.

4c) 3

4d) 444 mm

5) The UNITS of $k = [k] = \frac{\text{s}^2}{\text{m}^2}$.
Remember: $k \neq \frac{\text{s}^2}{\text{m}^2}$…the UNITS of k are $\frac{\text{s}^2}{\text{m}^2}$.
Gotta have those brackets on k…if you didn't, you lost points.

6) The key sentence in the problem statement is this one:

"After the 3rd displacement, the drone is located 11.11 m above of its starting point."

Upon close inspection, this sentence tells us about the *final result* after an *unknown 3rd displacement*.

The vectors we are given are as follows (in Cartesian form):

$$\vec{A} = -22.22\hat{\jmath}\text{ m}$$
$$\vec{B} = \left(27.8\underline{4}8\hat{\imath} + 18.3\underline{1}4\hat{\jmath}\right)\text{ m}$$
$$\vec{R} = 11.11\hat{k}\text{ m}$$

The wording of the problem statement tells us

$$\vec{A} + \vec{B} + \vec{C} = \vec{R}$$

Since the third displacement is unknown I solve this equation for $\vec{C}$ before plugging in numbers.

$$\vec{C} = \vec{R} - \left(\vec{A} + \vec{B}\right)$$
$$\vec{C} = 11.11\hat{k}\text{ m} - \left(-22.22\hat{\jmath}\text{ m} + \left(27.8\underline{4}8\hat{\imath} + 18.3\underline{1}4\hat{\jmath}\right)\text{ m}\right)$$

I now group up like terms then reorder to match standard form ($\hat{\imath}$ term 1st, $\hat{\jmath}$ term 2nd)

$$\vec{C} = \left(-27.8\underline{4}8\hat{\imath} + 3.9\underline{0}6\hat{\jmath} + 11.11\hat{k}\right)\text{ m}$$

This is the required 3rd displacement vector.

The question asked for the *magnitude* of the 3rd displacement vector (and to write the final answer with 3 sig figs).

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2}$$
$$\boldsymbol{C = 30.2\text{ m}}$$

Don't forget to include the units!!!!

Extra credit: I found $d = 1.4\underline{8}9$ m.

I am tempted to tell you, but I think you can figure it out on your own if you work at it.

Challenge: figure out the required individual displacement magnitude for a regular n-gon (arbitrary value of n).

Your answer should be a formula with x_{max} and n in it.

Verify your formula works for both even and odd values of n.

It may be practical to write your solution with series notation!

Then think about the shape as $n \to \infty$…