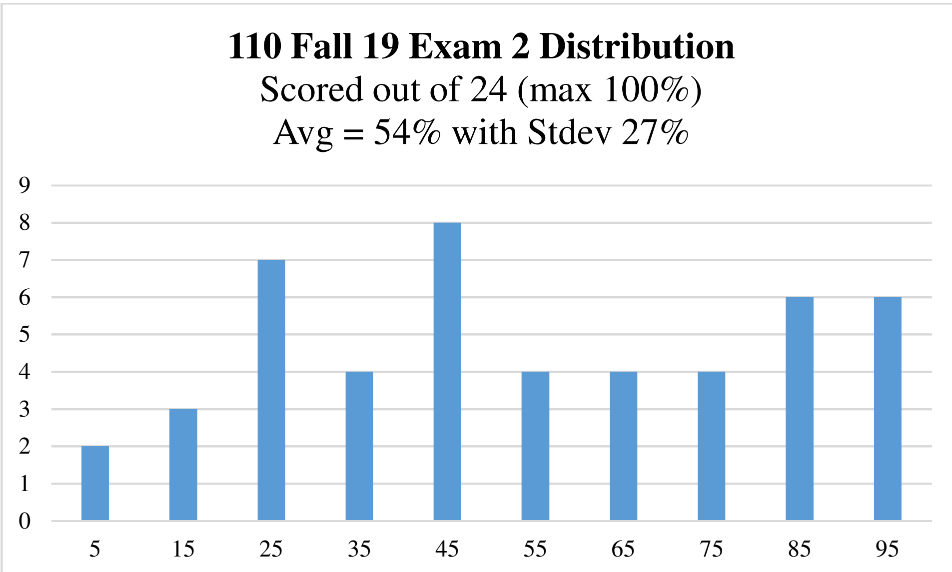
110Fa19t2aSoln

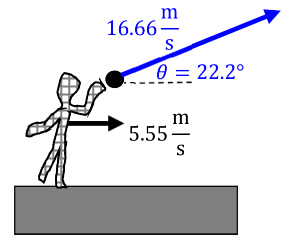
Distribution (for both sections combined) shown below.

More than 10% of the class is getting A's.

Solutions begin on the next page.



*****1) The athlete throws a ball while she is running to the right at a speed of $5.55\,\frac{\text{m}}{\text{s}}$.
The ball leaves her hand travelling at $16.66\,\frac{\text{m}}{\text{s}}$ (relative to the earth) angle at $\theta = 22.2°$.
*****1) What direction did she *aim* (relative to her body) while throwing the ball?
Sketch the direction and include a numerical value for the angle.
The figure is shown at right (approximately to scale)
Please pay close attention to the *ordering* of the subscripts in this solution.



According to the wording of the problem statement:

$$\vec{v}_{ae} = 5.55\hat{\imath}\,\frac{\text{m}}{\text{s}}$$

$$\vec{v}_{be} = (16.66\cos 22.2°\,\hat{\imath} + 16.66\sin 22.2°\,\hat{\jmath})\,\frac{\text{m}}{\text{s}}$$

$$\vec{v}_{be} = (15.425\hat{\imath} + 6.295\hat{\jmath})\,\frac{\text{m}}{\text{s}}$$

We are asked about the angle of the throw (basically, which way did she aim)?
Think: she had to aim at some angle greater than $22.2°$.
Why? Her horizontal velocity (relative to earth) as she throws the ball increases the horizontal component of the ball's velocity (relative to the earth) which causes the angle of the throw (relative to the earth) to become closer to the horizontal axis…

The math says

$$\vec{v}_{ba} = \vec{v}_{be} + \boldsymbol{\vec{v}_{ea}}$$

$$\vec{v}_{ba} = \vec{v}_{be} - \boldsymbol{\vec{v}_{ae}}$$

$$\vec{v}_{ba} = (15.425\hat{\imath} + 6.295\hat{\jmath})\,\frac{\text{m}}{\text{s}} - \left(5.55\hat{\imath}\,\frac{\text{m}}{\text{s}}\right)$$

$$\vec{v}_{ba} = (9.875\hat{\imath} + 6.295\hat{\jmath})\,\frac{\text{m}}{\text{s}}$$

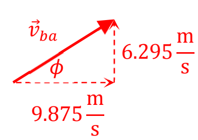
This vector describes the ball's velocity *relative to the athlete*…
The direction of this vector tells us about the direction she aims!

Sketching it out (see figures at right) one finds

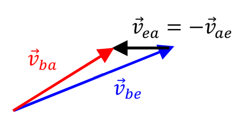
$$\phi = \tan^{-1}\left(\frac{6.295}{9.875}\right)$$

$$\boldsymbol{\phi = 32.5°}$$

Sketch of $\vec{v}_{ba}$



Graphical vector addition of
$\vec{v}_{ba} = \vec{v}_{be} + \boldsymbol{\vec{v}_{ea}}$
$\vec{v}_{ba} = \vec{v}_{be} - \boldsymbol{\vec{v}_{ae}}$



2a) We are looking at a plot of x versus t.
The question asks about "moving right"…which is the same as asking about when $v > 0$.
On an xt-plot velocity corresponds to the slope of the curve.
All said, the question is simply asking about which curves initially have positive slope.
None of them have initial positive velocity.
Many indicated object 1 initially had postivite velocity.
In my opinion it is *clearly/unamiguously* zero slope on the large scale graph included on your exam.
Since I ended up grading out of 24 (instead of the true total of 26), you get no leeway on this one.

2b) We are looking at a plot of x versus t.
We are asked about acceleration.
On a plot of x versus t, acceleration corresponds to concavity.
Non-zero, positive acceleration corresponds to graphs which are concave up.
Objects 1 & 3 have non-zero, positive acceleration.

2c) *Distance* traveled is trickier than *displacement*.
To get *displacement*, on an xt-plot, simply use $\Delta x = x_f - x_i$.
To get *distance*, first split up each curve whenever the object reverses direction (when slope flips sign).
Determine each positive and negative displacement separately.
Get each distance traveled by taking the absolute value of each displacement.
Sum up the terms to get total distance traveled.
Note: if the object never reverses direction, total distance is the absolute value of total displacement.
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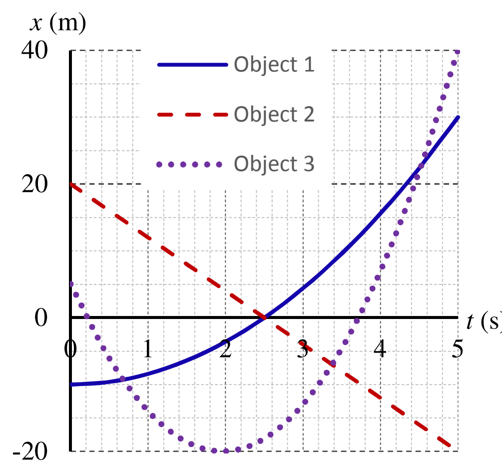
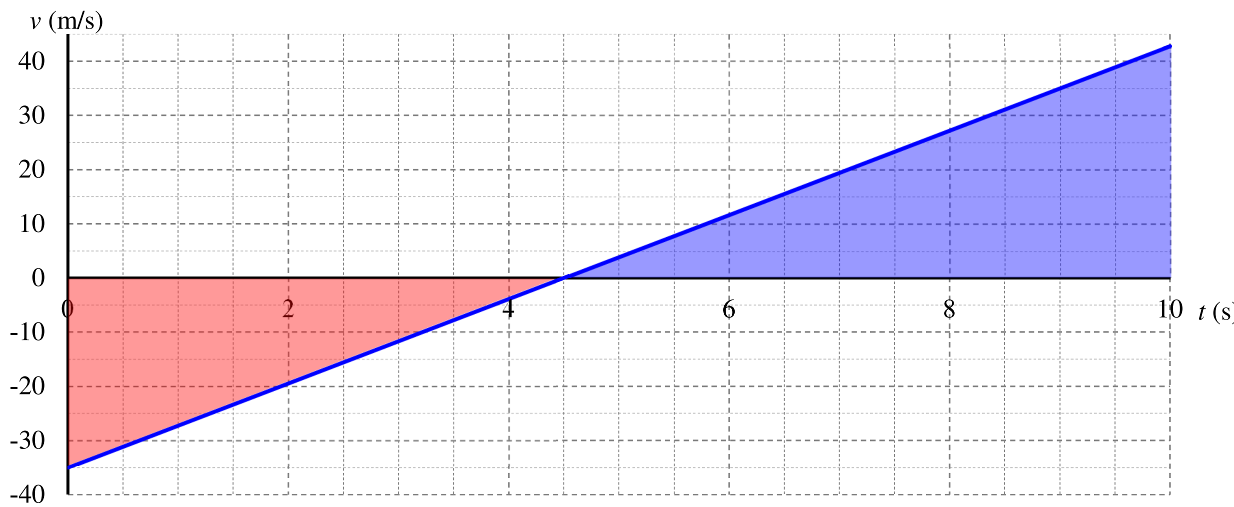


Object 1 never reverses direction	Object 2 never reverses direction	Object 3 reverses direction at $t = 2.0$ s
$d_1 = \lvert 40.0\text{ m}\rvert$ $d_1 = 40.0\text{ m}$	$d_2 = \lvert -40.0\text{ m}\rvert$ $d_2 = 40.0\text{ m}$	$d_3 = \lvert -25.0\text{ m}\rvert + \lvert 60.0\text{ m}\rvert$ $d_3 = 85.0\text{ m}$

After all this one finds

$$\boldsymbol{d_3 > d_1 = d_2}$$

2d) For an xt-plot, asking which object moves fastest at $t = 4.0$ s implies which object has the speed at $t = 4.0$ s.
Speed is not the slope…it is the absolute value of the slope.
Object 3 is moving fastest at $t = 4.0$ s.



3a) Acceleration is slope of a vt-plot.
Notice this particular vt-plot has constant slope so we can choose any points we want to do rise over run.
For non-constant slopes, one must choose points close to the time of interest to ensure a good estimate of slope.
At $t = 4.5$ s the slope gives

$$a = \frac{rise}{run}$$

$$a = \frac{42.5\,\frac{\mathrm{m}}{\mathrm{s}} - \left(-35.0\,\frac{\mathrm{m}}{\mathrm{s}}\right)}{10.00\ \mathrm{s}}$$

$$\boldsymbol{a = 7.75\,\frac{\mathbf{m}}{\mathbf{s^2}}}$$

I accepted various answers as long as you were within about $\pm 10\%$ of mine.

3b) The displacement relates to are under a vt-curve

$$\Delta x = A_1 + A_2$$

$$\Delta x = \frac{1}{2}\left(-35.0\,\frac{\mathrm{m}}{\mathrm{s}}\right)(4.50\ \mathrm{s}) + \frac{1}{2}\left(42.5\,\frac{\mathrm{m}}{\mathrm{s}}\right)(5.50\ \mathrm{s})$$

$$\Delta x = -78.\underline{7}5\ \mathrm{m} + 11\underline{6}.875\ \mathrm{m}$$

$$\boldsymbol{\Delta x \approx 38.1\ \mathbf{m}}$$

4a) The table of knowns shown at right applies.
The tricky part was a speed of "11.2% less than initial".
The number 11.2% is equivalent to 0.112.
This means $v_{fx} = v - 0.112v = 0.888v$.

Δx	d
v_{ix}	v
v_{fx}	$0.888v$
a_x	?
t	?

$$v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x$$

$$a_x = \frac{v_{fx}^2 - v_{ix}^2}{2\Delta x}$$

$$a_x = \frac{(0.888v)^2 - (v)^2}{2(d)}$$

$$\boldsymbol{a_x = -0.1057\frac{v^2}{d}}$$

4b) The time to travel distance d can be found in various ways.
I suppose the easiest might be to use

$$v_{fx} = v_{ix} + a_x t$$

$$t = \frac{v_{fx} - v_{ix}}{a_x}$$

$$t = \frac{0.888v - v}{\left(-0.1057\frac{v^2}{d}\right)}$$

$$t = \frac{-0.112v}{-0.1057\frac{v^2}{d}}$$

$$t = 0.112v \cdot \frac{d}{0.1057v^2}$$

$$\boldsymbol{t = 1.060\frac{d}{v}}$$

5a) At max height, velocity is given by $\vec{v} = v_{ix}\hat{\imath} + 0\hat{\jmath}$.
For a purely vertical throw, $v_{ix} = 0$.
For this special case, velocity at max height is zero.

5b) The change in vertical position is negative (displacement is negative).

5c) Just before impact the *velocity* is negative…but *speed* is positive.

6a) The list of knowns and unknowns is shown at right.
We are asked to find time to reach the vertical axis.
Reaching the vertical axis corresponds to reaching $x_f = 0$.
Because $a_x = 0$ we know $v_{fx} = v_{ix}$ and can also write

$$\Delta x = v_{ix} t$$

$$t = \frac{\Delta x}{v_{ix}}$$

$$\boldsymbol{t = 1.192\ \mathrm{s}}$$

x_i	$3.33\cos 66.6°$ $\mathbf{1.323\ m}$	y_i	$3.33\sin 66.6°$ $\mathbf{3.056\ m}$
x_f	0	y_f	?
Δx	$-1.323\ \mathrm{m}$	Δy	?
v_{ix}	$-1.11\frac{\mathrm{m}}{\mathrm{s}}$	v_{iy}	0
v_{fx}	$-1.11\frac{\mathrm{m}}{\mathrm{s}}$	v_{fy}	?
a_x	0	a_y	$4.44\frac{\mathrm{m}}{\mathrm{s}^2}$
t	?		

6b) When the astronaut crosses the vertical axis, the distance from the origin corresponds to the astronaut's vertical position.

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$\Delta y = \frac{1}{2} a_y t^2$$

$$y_f - y_i = \frac{1}{2} a_y t^2$$

$$y_f = y_i + \frac{1}{2} a_y t^2$$

$$y_f = y_i + \frac{1}{2} a_y \left(\frac{\Delta x}{v_{ix}}\right)^2$$

$$y_f = 3.056\ \mathrm{m} + \frac{1}{2}\left(4.44\frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(\frac{-1.323\ \mathrm{m}}{-1.11\frac{\mathrm{m}}{\mathrm{s}}}\right)^2$$

$$\boldsymbol{y_f = 6.21\ \mathrm{m}}$$

6c) & 6d) We are essentially asked to find the final velocity in polar form (speed and direction).
In particular, the question asking "how fast" is essentially asking for *speed* (not the velocity vector).

$$v_{fy} = v_{iy} + a_y t$$

$$v_{fy} = a_y t$$

$$v_{fy} = a_y \left(\frac{\Delta x}{v_{ix}}\right)$$

$$v_{fy} = 5.2\underline{9}2\frac{\mathrm{m}}{\mathrm{s}}$$

A sketch of the astronaut's *velocity* (as she crosses the vertical axis) is shown at right.

Notice one finds *speed* $v_f = 5.41\frac{\mathrm{m}}{\mathrm{s}}$ with *heading* $\phi = 78.2°$ or $\psi = 11.8°$.