110Fa19t3aSoln

1a) We are told the system is moving down ($v < 0$) while slowing down (a & v have opposite signs).
This tells us the system is **accelerating *upwards***!

1b) Since block two is accelerating upwards, we know the *upwards* normal force must be larger than the net downwards force ($\Sigma\vec{F} = m\vec{a}$). The force equation in the vertical direction is

$$n_{1on2} - n_{3on2} - m_2 g = m_2 a$$

$$n_{1on2} = n_{3on2} + m_2 g + m_2 a$$

In this equation, all terms on the right side are positive.

$$\boldsymbol{n_{1on2} > n_{3on2}}$$

1c) By Newton's THIRD law, two objects exert forces of equal magnitude (but opposite direction) on each other *regardless* of their masses or acceleration.

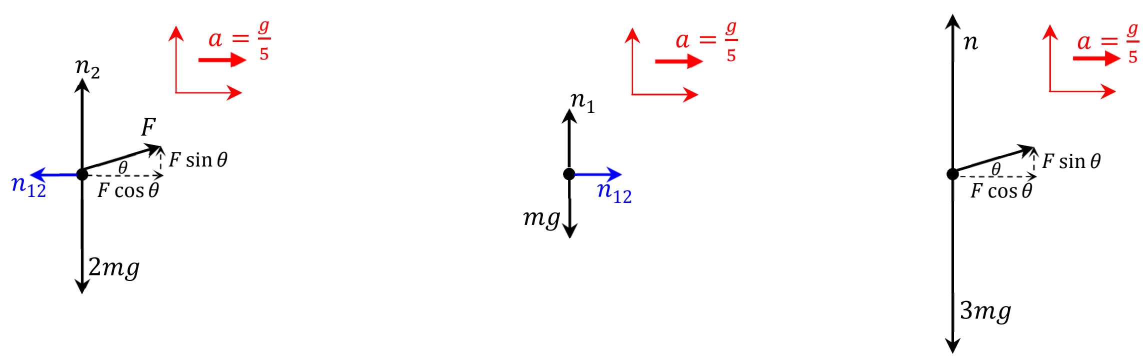
$$\boldsymbol{n_{1on2} = n_{2on1}}$$

2a) We expect the normal force upwards & weight downwards balance.
There is a frictional force to the left on this block.
Since no other force exists in the horizontal direction, we know acceleration points to the *left* ($-\hat{\imath}$).

2b) There is a frictional force to the left on this block ($-\hat{\imath}$).

2c) Once the block comes to rest, no friction is present!

3a)



ΣF_x: $F\cos\theta - n_{12} = (2m)\dfrac{g}{5}$	ΣF_x: $n_{12} = (m)\dfrac{g}{5}$	ΣF_x: $F\cos\theta = (3m)\dfrac{g}{5}$
ΣF_y: $F\sin\theta + n_2 = 2mg$	ΣF_y: $n_1 = mg$	ΣF_y: $F\sin\theta + n = 3mg$

3b) The horizontal system force equation gives

$$F = \frac{3mg}{5\cos\theta}$$

$$F = \frac{3mg}{5\cos 15.0°}$$

$$\boldsymbol{F \approx 0.621mg}$$

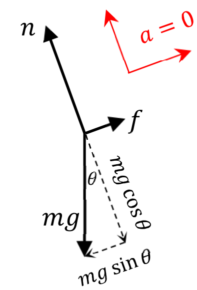
3c) Consider the horizontal force equation for m_1…the equation is already solved for n_{12}!

$$\boldsymbol{n_{12} = 0.200mg}$$

4a) We are asked for the *largest* angle for which the block remains at rest.
This implies two things:

1. Acceleration is zero.
2. The block is on the verge of slipping; use $f = \mu_s n$.

The FBD appropriate is shown at right.



$\Sigma F_x = ma_x$	$\Sigma F_y = ma_y$
$mg\sin\theta - f = 0$	$n - mg\cos\theta = 0$
$f = mg\sin\theta$	$n = mg\cos\theta$

An interesting way to solve this problem is to take a ratio!

$$\frac{f}{n} = \frac{mg\sin\theta}{mg\cos\theta}$$

When we are at angle θ_{max} we can use $f = \mu_s n$.

$$\frac{\mu_s n}{n} = \frac{mg\sin\theta_{max}}{mg\cos\theta_{max}}$$

$$\mu_s = \tan\theta_{max}$$

$$\theta_{max} = \tan^{-1}\mu_s$$

$$\theta_{max} = \tan^{-1} 0.888$$

$$\boldsymbol{\theta_{max} = 41.6°}$$

4b) For angles *larger* than θ_{max}, the block should slide and one should use $f = \mu_k n$.
For angles *less* than θ_{max}, the block remains at rest *but is no longer on the verge of slipping* ($f \neq \mu_s n$)!!!!
We can still use the same FBD.

$$f = mg\sin\theta$$

$$f = (0.300\text{ kg})\left(9.8\frac{\text{m}}{\text{s}^2}\right)\sin 32.1°$$

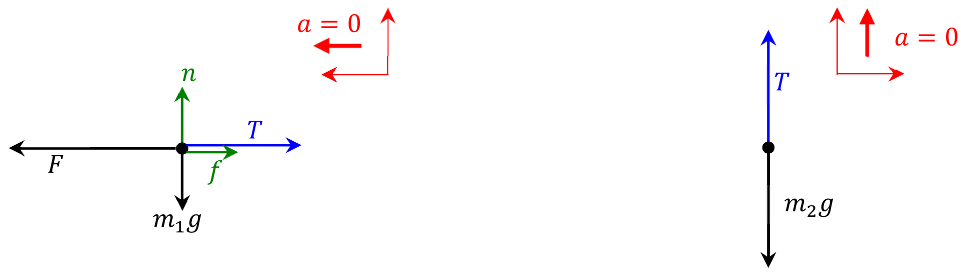
$$\boldsymbol{f = 1.562\text{ N}}$$

Note: when the first digit of a numerical result is 1, we typically include an extra sig fig.

4c) Notice, in the derivation shown in part a, that θ_{max} does *not* depend on mass.
We expect $\boldsymbol{\theta_{max}}$ remains the same.

4d) Notice, in the derivation shown in part b, that f *does* depend on mass.
We expect $\boldsymbol{f}$ increases.

5a) FBDs and force equations are shown below.
Note: in part a we are asked for the largest possible applied force (magnitude) that does *not* cause motion.
Once again, the problem is on the verge of slipping.
We can set $a = 0$ and use $f = \mu_s n$ for part a…



ΣF_x: $F - T - f = m_1 a$	ΣF_x: $T - m_2 g = m_2 a$
ΣF_y: $n = m_1 g$	

Adding the ΣF_x: equations together gives

$$\begin{aligned} F - T - f &= m_1 a \\ + \quad T - m_2 g &= m_2 a \\ \hline F - m_2 g - f &= (m_1 + m_2) a \end{aligned}$$

Now sub in $f = \mu_s n = \mu_s m_1 g$ and $a = 0$.

$$F - m_2 g - \mu_s m_1 g = 0$$

Rearrange and solve for F.

$$F = m_2 g + \mu_s m_1 g$$

Finally, use $m_1 = m, m_2 = 2m, \& \mu_s = 0.888$.

$$F = (2m)g + (0.888)(m)g$$

$$F = 2.888mg$$

$$\boldsymbol{F \approx 2.89mg}$$

5b) In part b we are told the applied force (magnitude) is $F = 5mg$.
Because this is larger than the previous result, we know the blocks move!
This implies $a \neq 0$ and $f = \mu_k n$.
Notice the FBDs are identical!
First I will use the FBDs to solve for a, then I will use that a to solve for T.

$$F - m_2 g - \mu_k m_1 g = (m_1 + m_2) a$$

$$a = \frac{F - m_2 g - \mu_k m_1 g}{m_1 + m_2}$$

$$a = \frac{5mg - 2mg - 0.777mg}{3m}$$

$$a = 0.741g$$

Now use the force equation from m_2.

$$T - m_2 g = m_2 a$$

$$T = m_2 g + m_2 a$$

$$T = 2mg + 2m(0.741g)$$

$$T = 3.482mg$$

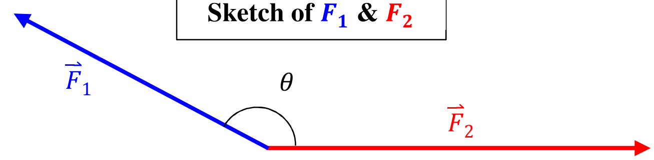
$$\boldsymbol{T \approx 3.48mg}$$

Extra Credit:

Without loss of generality, we can align one force along the horizontal axis.

I choose to align the horizontal axis with the larger force ($\vec{F}_2$).

Sketch of $\vec{F}_1$ & $\vec{F}_2$



In this case, I chose to define the angle from the positive x-axis.
This differs from how we typically do problems with free body diagrams.
If you are clever, you could use the law of cosines.
If you are not clever, it is straightforward to derive it.

$$\vec{F}_{Net} = \vec{F}_2 + \vec{F}_1$$

$$\vec{F}_{Net} = (F_2 + F_1 \cos\theta)\hat{\imath} + (F_1 \sin\theta)\hat{\jmath}$$

$$F_{Net}^2 = (F_2 + F_1 \cos\theta)^2 + (F_1 \sin\theta)^2$$

$$F_{Net}^2 = F_2^2 + F_1^2 \cos^2\theta + 2F_2F_1\cos\theta + F_1^2 \sin^2\theta$$

$$F_{Net}^2 = F_2^2 + F_1^2 + 2F_2F_1\cos\theta$$

$$\cos\theta = \frac{F_{Net}^2 - F_2^2 - F_1^2}{2F_2F_1}$$

$$\theta = \cos^{-1}\left(\frac{F_{Net}^2 - F_2^2 - F_1^2}{2F_2F_1}\right)$$

Notice the units of the term inside the parentheses will all cancel!

$$\theta = \cos^{-1}\left(\frac{2.00^2 - 4.00^2 - 3.00^2}{2(4.00)(3.00)}\right)$$

$$\theta = \cos^{-1}\left(\frac{2.00^2 - 4.00^2 - 3.00^2}{2(4.00)(3.00)}\right)$$

$$\boldsymbol{\theta \approx 151.0°}$$

Graphical Vector Addition