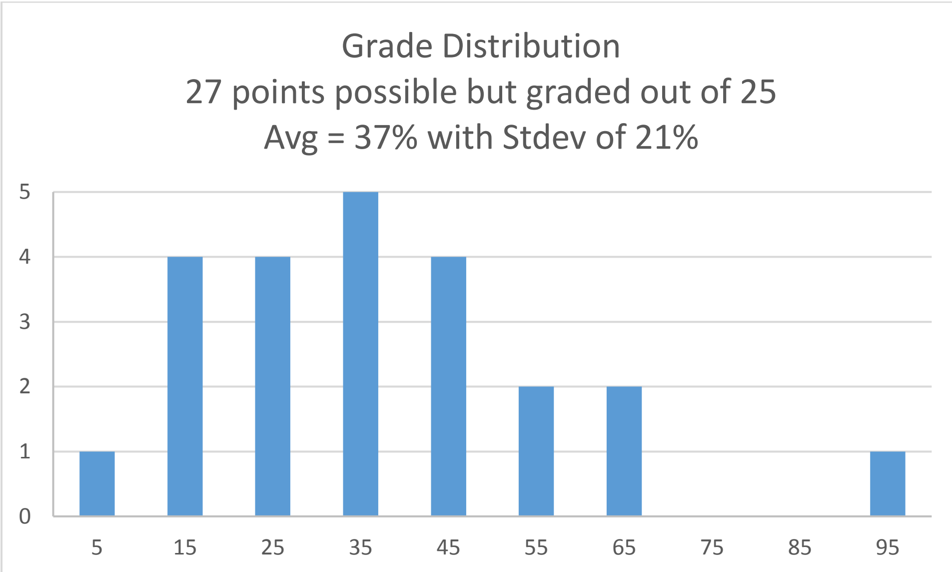
110Fa22t1aSoln

Distribution on this page, solutions begin on next page.



1)

Number of sig figs implied on A	A in engineering notation with best choice of prefix	Number of sig figs implied on B	B in scientific notation
4	203.0 μs	2	2.4×10^{-7} s *standard scientific notation uses no prefix on the units

2)

$$8.15 \times 10^{-2} \frac{\mathrm{g}}{\mathrm{cm}^3} \times \frac{1\ \mathrm{kg}}{1000\ \mathrm{g}} \times \frac{1\ \mathrm{slug}}{14.5939\ \mathrm{kg}} \times \frac{2.54^3\ \mathrm{cm}^3}{1^3\ \mathrm{in}^3} = \mathbf{9.15 \times 10^{-5} \frac{slug}{in^3}}$$

3a) One finds

$$x = \sqrt{\frac{m}{k}\left(v^2 - gh\right)}$$

Before moving on, I check the units.
The term $\frac{m}{k}$ has units of s^2.
Both v^2 & gh have units of $\frac{\mathrm{m}^2}{\mathrm{s}^2}$.
Final units on the right side of the equation will turn out to be m as expected.

3b) First I choose to convert everything to units that match. I converted grams to kg but you could have instead converted kg to grams. Similar with mm to meters.

$v\ \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$	$k\ \left(\frac{\mathrm{kg}}{\mathrm{s}^2}\right)$	m (kg)	$g\ \left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)$	h (m)
7.77×10^{-2}	-800.0	9.99×10^{-3}	9.80	6.78×10^{-4}

I know the final units must be in meters so I choose to leave units off in the calculation.
Plugging in and tracking sig figs gives

$$x = \sqrt{\frac{9.9\underline{9} \times 10^{-3}}{-800.\underline{0}}\left[\left(7.7\underline{7} \times 10^{-2}\right)^2 - \left(9.8\underline{0}\right)\left(6.7\underline{8} \times 10^{-4}\right)\right]}$$

$$x = \sqrt{-1.2\underline{4}88 \times 10^{-5}\left[6.0\underline{3}7 \times 10^{-3} - 6.6\underline{4}4 \times 10^{-3}\right]}$$

$$x = \sqrt{-1.2\underline{4}88 \times 10^{-5}\left[-0.6\underline{0}7 \times 10^{-3}\right]}$$

Notice the minus signs do cancel under the square root.
Notice the subtraction drops us down to just two sig figs!!!

$$x = 8.71 \times 10^{-5}\ \mathrm{m}$$

$$\boldsymbol{x = 87\ \mu\mathrm{m}}$$

4) The standard trick is to know the units of any two terms in an expression must match. This avoids having to solve algebraically for some nasty expression.

Also, numerical constants like 3, $\frac{1}{2}$, or π have no units.

Also, the minus sign will not affect the units.

In this particular case we know:

$$[v^2] = \frac{[k][a^2]}{[m][x]}$$

$$[k] = \frac{[v^2][m][x]}{[a^2]}$$

$$[k] = \frac{\frac{\mathrm{m}^2}{\mathrm{s}^2} \cdot \mathrm{kg} \cdot \mathrm{m}}{\frac{\mathrm{m}^2}{\mathrm{s}^4}}$$

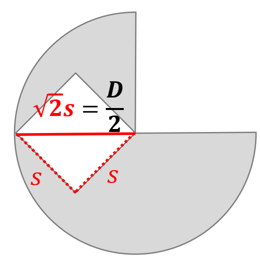
$$[k] = \frac{\mathrm{m}^2}{\mathrm{s}^2} \cdot \mathrm{kg} \cdot \mathrm{m} \cdot \frac{\mathrm{s}^4}{\mathrm{m}^2}$$

$$\boldsymbol{[k] = \mathbf{kg} \cdot \mathbf{m} \cdot \mathbf{s}^2}$$

5) The trick on this one is to recognize the main diagonal of the square has length $R = \frac{D}{2}$.

This helps us realize the relationship $\sqrt{2}s = \frac{D}{2} \;\rightarrow\; s = \frac{D}{2\sqrt{2}}$

The area shown is given by:



$$A_{total} = \frac{3}{4}A_{circle} - A_{square}$$

$$A_{total} = \frac{3}{4}\pi R^2 - s^2$$

$$A_{total} = \frac{3}{4}\pi\left(\frac{D}{2}\right)^2 - \left(\frac{D}{2\sqrt{2}}\right)^2$$

$$A_{total} = \frac{3}{16}\pi D^2 - \frac{D^2}{8}$$

$$\boldsymbol{A_{total} = 0.464D^2}$$

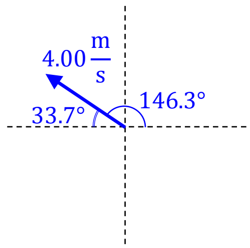
6a) $v = 4.0\underline{0}2\,\frac{\text{m}}{\text{s}}$. The angle should be described in one of the following ways

$33.7°$ **above** the *negative* $x -$ axis

$33.7°$ north of west

$56.3°$ west of north

$146.3°$ **from** the *positive* $x -$ axis



6b) Use $\hat{v} = \frac{\vec{v}}{v}$ where v is the magnitude of vector $\vec{v}$.

$$\hat{v} = -0.832\hat{\imath} + 0.555\hat{\jmath}$$

6c) Use

$$\theta = \cos^{-1}\left(\frac{\vec{r}\cdot\vec{v}}{rv}\right)$$

Notice the only term which survives the dot product comes from the $\hat{\jmath}\cdot\hat{\jmath}$ term since $\hat{k}\cdot\hat{\imath} = 0$ & $\hat{\jmath}\cdot\hat{\imath} = 0$.
Notice the units will cancel out inside the inverse trig function (so I will ignore units for the rest of this part).
Notice we require the magnitude of $\vec{r}$ which is $r = 7.0\underline{7}1$ m.

$$\theta = \cos^{-1}\left(\frac{(4.44)(2.22)}{(7.0\underline{7}1)(4.0\underline{0}2)}\right)$$

$$\boldsymbol{\theta = 69.6°}$$

Note: standard practice dictates we keep an extra sig fig on the final answer when the first digit is a 1.

6d) I like to move all the units to the right end right away so I don't forget to include them on the final answer. When doing the cross-product, I recommend using the wheel of pain. It is usually faster than the determinant method if at least one component in either vector is zero…

$$\vec{L} = m(\vec{r}\times\vec{v})$$

$$\vec{L} = 1.00\left[\left(4.44\hat{\jmath} - 5.55\hat{k}\right)\times(-3.33\hat{\imath} + 2.22\hat{\jmath})\right]\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

We know the cross-product of any vector with itself is zero (including a unit vector with itself).
Also, the order of the cross-product matters!

$$\vec{L} = \left[(4.44)(-3.33)(\hat{\jmath}\times\hat{\imath}) + (-5.55)(-3.33)\left(\hat{k}\times\hat{\imath}\right) + (-5.55)(2.22)\left(\hat{k}\times\hat{\jmath}\right)\right]\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

$$\vec{L} = \left[-14.785\left(-\hat{k}\right) + 18.482(\hat{\jmath}) + (-12.321)(-\hat{\imath})\right]\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$$

It is standard practice to put the vectors in traditional form ($\hat{\imath}$-term before $\hat{\jmath}$-term before $\hat{k}$-term).
Also, we can now round our intermediate results to 4 sig figs (since each term has first digit of 1).

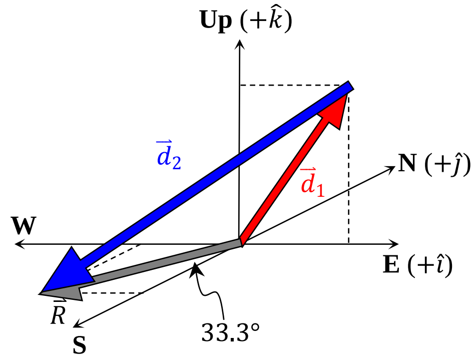
$$\boldsymbol{\vec{L} = \left[12.32\hat{\imath} + 18.48\hat{\jmath} + 14.79\hat{k}\right]\frac{\text{kg}\cdot\text{m}^2}{\text{s}}}$$

7) We know

$$\vec{d}_1 = \left(8.88\hat{\imath} + 5.55\hat{k}\right)\text{m}$$

$$\vec{R} = 7.77\text{ m @ }33.3°\text{ west of south}$$

We are looking for the vector which connects the tip of $\vec{d}_1$ to the tip of $\vec{R}$!



In equation form, we know

$$\vec{d}_1 + \vec{d}_2 = \vec{R}$$

$$\vec{d}_2 = \vec{R} - \vec{d}_1$$

Strategy is to convert all vectors to Cartesian, do the math, then compute the magnitude.

We notice the angle of 33.3° is adjacent to the negative y-axis.
Also both x- and y-components will be negative.
Stick the units at the right end so you don't forget them!

$$\vec{R} = (-7.77\sin 33.3°\,\hat{\imath} - 7.77\cos 33.3°\,\hat{\jmath}\,)\text{m}$$

$$\vec{R} = \left(-4.2\underline{6}6\hat{\imath} - 6.494\hat{\jmath}\,\right)\text{m}$$

Before moving on, THINK.
This vector is angled closer to the y-axis than it is to the x-axis.
We expect the y-component should be the larger number.
We already discussed the signs (both should be negative from the image).

$$\vec{d}_2 = \left(-4.2\underline{6}6\hat{\imath} - 6.4\underline{9}4\hat{\jmath}\,\right)\text{m} - \left(8.88\hat{\imath} + 5.55\hat{k}\right)\text{m}$$

$$\vec{d}_2 = \left(-13.146\hat{\imath} - 6.494\hat{\jmath} - 5.55\hat{k}\right)\text{m}$$

THINK: Do these signs agree with the signs we expect from the picture?
YES; in the figure $\vec{d}_2$ point left, out of the page, and down…
Move on to get the magnitude.

$$d_2 = \left\|\vec{d}_2\right\| = \sqrt{(-13.146)^2 + (-6.494)^2 + (5.55)^2}$$

$$\boldsymbol{d_2 = 15.68\text{ m}}$$

Notice I waited until the last step to round.
Here I rounded to 4 sig figs since the first digit was a 1…