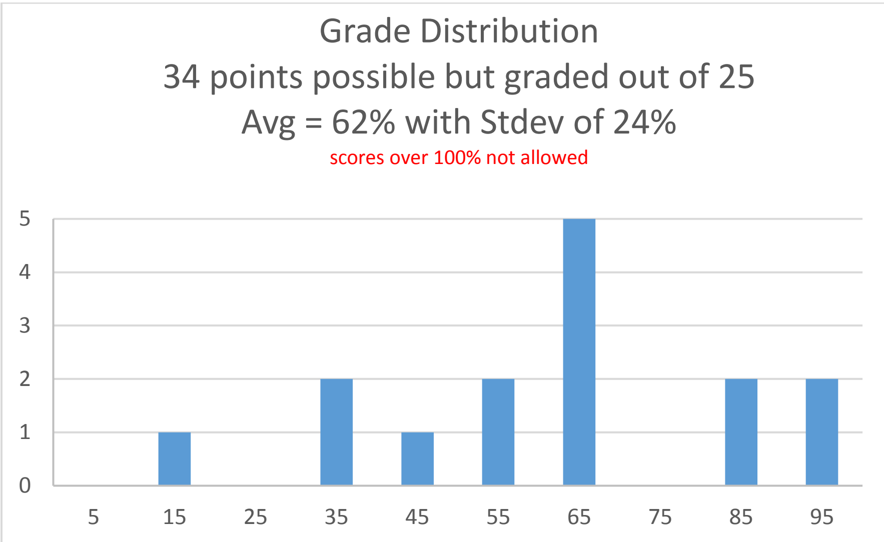
110fa22t2aSoln

Distribution here.



1a) Since starting from rest in stage 1 $v_{ix1} = 0$.

$$\Delta x_1 = \frac{1}{2} a_{1x} t_1^2 \quad \rightarrow \quad t_1 = \sqrt{\frac{2\Delta x_1}{a_{1x}}}$$

$$\boldsymbol{t_1 \approx 2.8\underline{2}8\ \mathrm{s}}$$

WATCH OUT! The entire race is 40.0 m, but the runner is only accelerating for $\Delta x_1 = 15.00$ m.

1b)

You could use either $v_{fx1}^2 = v_{ix1}^2 + 2a_{x1}\Delta x_1$ or use the time you just found in $v_{fx1} = v_{ix1} + a_{x1}t_1$.

Note: if you use the time found in part 1a), be sure to use the UNROUNDED result to avoid intermediate rounding error.

$$\boldsymbol{v_{fx_1} = 10.6\underline{0}7\,\frac{\mathrm{m}}{\mathrm{s}}}$$

When the first digit is a 1, we often keep an extra sig fig (engineering style).

I kept the unrounded solution so I can use that in part 1c…

1c) Displacement during the second stage is

$$\Delta x_2 = 40.0\ \mathrm{m} - \Delta x_1 = 25.0\ \mathrm{m}$$

Speed in the second stage is the final speed from stage 1!

We also know $a_{2x} = 0$ since we were told constant speed during the second stage of the race

$$\Delta x_2 = v_{2ix} t_2$$

I found $t_2 = 2.3\underline{5}7$ s

Total time is thus

$$t_{total} = t_1 + t_2$$

$$\boldsymbol{t_{total} \approx 5.19\ \mathrm{s}}$$

2a) On an xt-plot, slope corresponds to velocity.
Zero slope at times 0.40 ms and 1.60 ms.

2b) Moving left on xt-plot means negative slope.
Speeding up implies acceleration is same sign as velocity.
Concavity corresponds to acceleration on an xt-plot.
Putting it altogether, we want regions with negative slope that are also concave down.
This occurs from about 1.60 ms to about 3.00 ms.

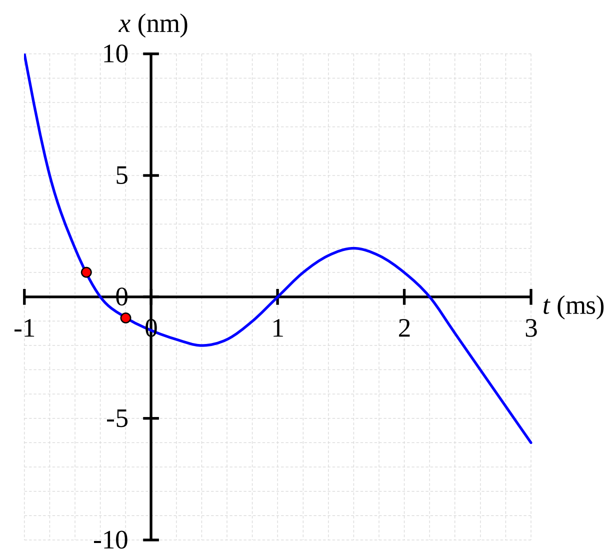
2c) Positions on an xt-plot are found by reading the x values off the vertical axis.
Displacement is then found using

$$\Delta x = x_f - x_i$$

I found **<u>displacement</u> is -16.0 nm**…WATCH OUT FOR THOSE UNITS!

2d) First split the displacement into chunks.
The splits should occur wherever the object reverse direction.
The object reverses direction whenever v= 0 (points where slope is zero).
Between –1.0 to 0.4 ms displacement was –12 nm.
From 0.4 to 1.6 ms displacement was +4 nm.
From 1.6 to 3.0 ms displacement was –8 nm.
Distance is found by summing the absolute value of these numbers.
I found **<u>distance</u> is 24 nm**.

2e) Get velocity using the slope of an xt-plot.
Usually the best estimate is obtained by using comparing a point just before to a point just after the time of interest.
I chose the points shown on the plot at right.



I found

$$v_x \approx \frac{rise}{run}$$

$$v_x \approx \frac{x_f - x_i}{t_f - t_i}$$

$$v_x \approx \frac{-1.9\ \mathrm{nm}}{0.30\ \mathrm{ms}}$$

$$v_x \approx -6.3 \frac{\mathrm{nm}}{\mathrm{ms}}$$

Standard practice:

- Convert the prefix to powers of 10.
- Simplify the powers of 10.
- Convert this new power of 10 back to a single prefix in the numerator.

$$v_x \approx -6.3 \frac{10^{-9}\mathrm{m}}{10^{-3}\mathrm{s}}$$

$$v_x \approx -6.3 \frac{10^{-6}\mathrm{m}}{\mathrm{s}}$$

$$\boldsymbol{v_x \approx -6.3 \frac{\mu\mathrm{m}}{\mathrm{s}}}$$

3a) Moving forwards corresponds to positive velocity.

On a vt-plot, positive velocity means a positive value on the vertical axis.

Objects 2 & 3 are initially moving forwards.

3b) Displacement is given by the area under the curve of a vt-plot.

Be careful: convention tells us you get *negative* areas if the velocity value is *negative*!

Only object 3 has more area below the time axis than above the time axis.

Only object 3 experiences negative displacement over the entire time interval.

3c) Speed is the magnitude of velocity.

For object in 1D motion, one can get speed by taking the absolute vlue of velocity (vertical axis).

Notice both objects 1 & 2 have an initial speed of 10 m/s!!!!

Object 1 & 2 tie for largest initial speed.

4a) **Zero**

At the start we see $x_i = 0$. Upon crossing vertical axis we once again have $x_f = 0$. One finds $\Delta x = 0$.

4b) **Positive**

The object is initially moving left (and up) but accelerates to the right.

To return to the vertical axis the object must have reversed direction horizontally.

4c) **Positive**

No acceleration in the vertical direction.

Vertical velocity component (which is initially positive) should remain unchanged throughout the trajectory.

4d) $\boldsymbol{v_0 = v_f}$

We know the $v_{fy} = v_{iy}$ from part 4c.

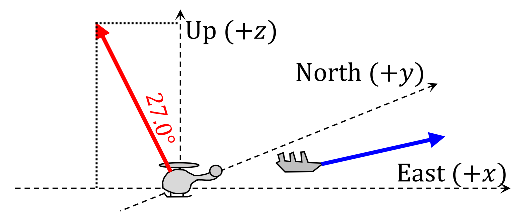
We also know $v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$. Since $\Delta x = 0$ from part 4a, we know $v_{fx} = \pm v_{ix}$.

From part b we know we must use the negative root.

Final speed is $v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(-v_{ix})^2 + v_{iy}^2} = v_0$

CRAZY!!!!

5a) & 5b)



$$\vec{v}_{HB} = \vec{v}_{HE} + \vec{v}_{EB}$$

$$\vec{v}_{HB} = \vec{v}_{HE} - \vec{v}_{BE}$$

Most common mistake was thinking both vectors were in the xy-plane.

Read the problem statement…it is *painfully* clear this is NOT true.

$$\vec{v}_{HE} = \left(-9.25 \sin 27.0° \,\hat{\imath} + 9.25 \cos 27.0° \,\hat{k}\right)\frac{\text{m}}{\text{s}}$$

$$\vec{v}_{HE} = \left(-4.199\hat{\imath} + 8.242\hat{k}\right)\frac{\text{m}}{\text{s}}$$

$$\vec{v}_{BE} = (13.00 \cos 36.5° \,\hat{\imath} + 13.00 \sin 36.5° \,\hat{\jmath})\frac{\text{m}}{\text{s}}$$

$$\vec{v}_{BE} = (10.450\hat{\imath} + 7.733\hat{\jmath})\frac{\text{m}}{\text{s}}$$

Now one finds

$$\vec{v}_{HB} = \left(-4.199\hat{\imath} + 8.242\hat{k}\right)\frac{\text{m}}{\text{s}} - (10.450\hat{\imath} + 7.733\hat{\jmath})\frac{\text{m}}{\text{s}}$$

Notice I am keeping an extra sig fig or two to avoid intermediate rounding…

$$\vec{v}_{HB} = \left(-14.650\hat{\imath} - 7.733\hat{\jmath} + 8.242\hat{k}\right)\frac{\text{m}}{\text{s}}$$

Take the magnitude of velocity to get speed.

Divide by any vector by its magnitude to get the unit vector.

$$\boldsymbol{v_{HB} = 18.5\underline{0}2\,\frac{\text{m}}{\text{s}}}$$

$$\boldsymbol{\hat{v}_{HB} = -0.792\hat{\imath} - 0.418\hat{\jmath} + 0.445\hat{k}}$$

6a) **Non-zero**
For a standard projectile in flight we assume air resistance is negligible and

$$\vec{a} = 0\hat{\imath} - g\hat{\jmath}$$

6b) **Non-zero**
Think…it is drawn in the frickin picture as non-zero!!!
If a ball is thrown straight up…then velocity is zero at max height.
In general, for an angled throw, at max height $v_y = 0$ while $v_x \neq 0$.

6c) **Same**
For a standard projectile, where $a_x = 0$, we know $v_{fx} = v_{ix}$.

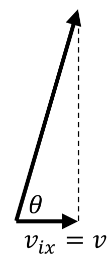
6d) I was hoping you would realize the velocity at max height gives you $v_{ix} = v_{fx} = v$.
We know this because $a_x = 0$ for a standard projectile problem.
Now use

$$\Delta x = v_{ix} t \quad \rightarrow \quad t = \frac{d}{v}$$

6e) I drew the initial velocity vector.
We know the angle and the horizontal component.
Use SOH CAH TOA to get the speed.

I found

$$v_i = \frac{v}{\cos\theta} \quad \& \quad v_{iy} = v\tan\theta$$



6f) A number of you found clever ways to get there…I was impressed!

One way is to use the above value of v_{iy} in the $v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$ equation to get $\Delta y = \frac{v^2\tan^2\theta}{2g}$.

Another way is to use the time from part a in the $v_{fy} = v_{iy} + a_y t$ equation to get $v_{iy} = \frac{gd}{v}$.
From there you can once again use the $v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$ equation to get $\Delta y = \frac{gd^2}{2v^2}$.

Alternatively, use time from part a in the $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ equation to get $\Delta y = d\tan\theta - \frac{gd^2}{2v^2}$.