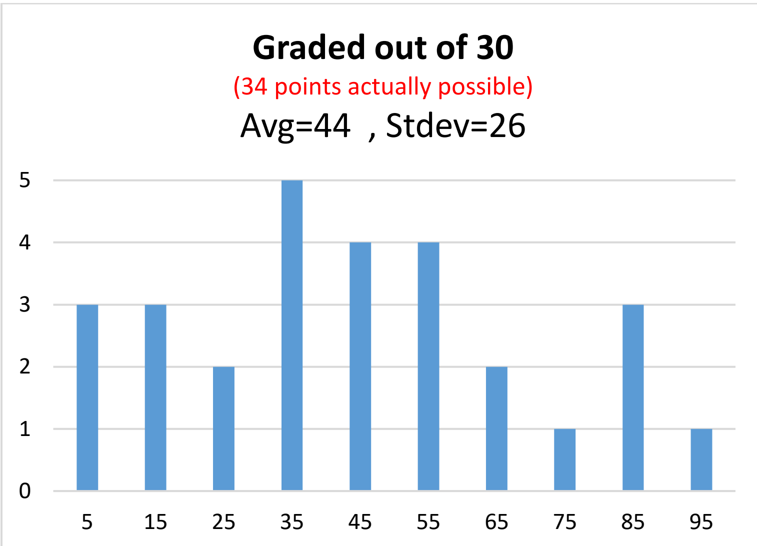
110fa23t1aSoln

Distribution on this page.
Solutions begin on the next page.



Note: the scores shown in the distribution above do NOT include the extra credit associated with the syllabus quiz.
You can add your score from the Canvas syllabus quiz to your final *percentage* on the exam.

For example:

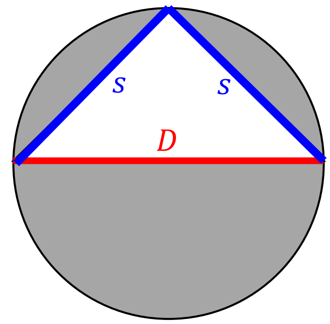
Suppose you got 4.83 out of 5 on the Canvas syllabus quiz.
You also got 15 check marks on the test (check is good, x is bad, check slash is half a check).
Your exam score would be:

$$\frac{15}{30} = 50\%$$

$$\text{then add } 4.83\%$$

$$\textbf{Exam 1 score} = \mathbf{54.83\%}$$

1) We frequently use



$$\rho = \frac{m_{after\ hole\ cut}}{V_{after\ hole\ cut}}$$

The trick here is to figure out the volume in terms of the given parameters D & t.
From experience, one *probably* knows that for a 45-45-90 triangle

$$\sqrt{2} \times side\ length = hypotenuse$$

If you don't know this, you could always use the Pythagorean Theorem.

$$s^2 + s^2 = D^2$$

$$2s^2 = D^2$$

$$s^2 = \frac{D^2}{2} \quad \textbf{OR} \quad s = \frac{D}{\sqrt{2}}$$

Now square root each side to find:

$$\sqrt{2}s = D$$

We have a circular plate of thickness (or height) t which is essentially a cylinder with a triangular hole.

$$V_{after\ hole\ cut} = V_{cyl} - V_{tri}$$

$$V_{after\ hole\ cut} = \pi r^2 h - (A_{tri}) \times (thickness)$$

$$V_{after\ hole\ cut} = \frac{\pi D^2}{4} t - \frac{1}{2} s^2 t$$

$$V_{after\ hole\ cut} = \frac{\pi D^2}{4} t - \frac{1}{2}\left(\frac{D^2}{2}\right) t$$

$$V_{after\ hole\ cut} = D^2 t \left(\frac{\pi - 1}{4}\right)$$

Now plug this into the density formula

$$\rho = \frac{m_{after\ hole\ cut}}{V_{after\ hole\ cut}} = \frac{m}{D^2 t \left(\frac{\pi - 1}{4}\right)} = \left(\frac{4}{\pi - 1}\right) \frac{m}{D^2 t} \approx \mathbf{1.868} \frac{\boldsymbol{m}}{\boldsymbol{D^2 t}}$$

2) When looking for the units of k we can completely ignore the second term on the right hand side!

$$[F] = \frac{[a] \cdot [t]^3}{[3] \cdot [m] \cdot [k]^2}$$

Since the number 3 has no units, we can ignore that.

$$[k]^2 = \frac{[a] \cdot [t]^3}{[m] \cdot [F]}$$

$$[k] = \sqrt{\frac{\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \mathrm{s}^3}{\mathrm{kg} \cdot \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2}}}$$

$$[k] = \sqrt{\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \mathrm{s}^3 \cdot \frac{1}{\mathrm{kg}} \cdot \frac{\mathrm{s}^2}{\mathrm{kg} \cdot \mathrm{m}}}$$

$$[\boldsymbol{k}] = \sqrt{\frac{\mathbf{s^3}}{\mathbf{kg^2}}} = \frac{\mathbf{s^{3/2}}}{\mathbf{kg}}$$

2a) $x_i = 70\underline{0}$ mm
I include the underbar on the significant zero to clarify the ambiguity. Not all people do this but I recommend it.

2b) **3 –** In scientific notation, notice the trailing zeros were to the *right* of a decimal point!

2c) $x_f = 6.8 \times 10^{-1}$ m (6.8×10^2 mm is NOT acceptable)
We should assume the trailing zero is NOT significant since it is to the *left* of the decimal point in 680 mm!
Standard *scientific* notation typically *avoids prefixes* to simplify calculations in calculators & computers.

2d) **2**

2e) $40\underline{0}$ μs
Again, we want to clarify the trailing zeros *are* significant since they came to the *right* of the decimal point.

2f) **3 –** Leading zeros are never significant. These trailing zeros are significant since they were to the right of the decimal point.

2g) First one solves algebraically for a.
This reduces the amount of writing since we don't have to keep writing all those decimal places and units!
Also, it is easier to fix mistakes in algebra compared to a giant wall of numerical work.

$$a = \frac{2\Delta x}{t^2} = \frac{2(x_f - x_i)}{t^2}$$

When you do addition or subtraction while tracking sig figs, it is wise to have all numbers in the same notation.
Ideally, floating point notion is nice if there aren't excessive leading or trailing zeros.
Floating point notation is the "normal" way of writing numbers (without any $\times 10^{power}$ stuff).

$$a = \frac{2(0.6\underline{8}\text{ m} - 0.70\underline{0}\text{ m})}{(0.00040\underline{0}\text{ s})^2}$$

$$a = \frac{2(-0.0\underline{2}\text{ m})}{(0.00040\underline{0}\text{ s})^2}$$

$$a = -\underline{2}50000\,\frac{\text{m}}{\text{s}^2}$$

Traditional rounding says to round this to $a = -\underline{3}00000\,\frac{\text{m}}{\text{s}^2}$
Many people (banking, chemistry, stats, IEEE, etc.) round numbers whose rightmost digit is 5 to the nearest even.
In my work I tend to leave the next digit but also indicate the rounding digit (in case I need to use this number in a subsequent calculation).
In any event, we were asked to use engineering notation with best choice of prefix.

I would accept any of these answers even though the rightmost column seems unusual to some students at first.		
$\boldsymbol{a = -\underline{2}50\,\frac{\text{km}}{\text{s}^2}}$	$\boldsymbol{a = -\underline{3}00\,\frac{\text{km}}{\text{s}^2}}$	$\boldsymbol{a = -\underline{2}00\,\frac{\text{km}}{\text{s}^2}}$

3a) I can tell the units should be m so I will leave them off until the end.
Magnitude of $\vec{r}$ is given by

$$r = \|\vec{r}\| = \sqrt{(7.50)^2 + (-5.00)^2} = 9.0\underline{1}4 \text{ m}$$

I keep the extra sig fig written down to avoid intermediate rounding on subsequent parts of the problem.

3b) Notice the units cancel out! Notice I used the *unrounded* answer for r to avoid intermediate rounding.

$$\hat{r} = \frac{\vec{r}}{r} = \frac{(7.50\hat{\jmath} - 5.00\hat{k}) \text{ m}}{9.0\underline{1}4 \text{ m}} = \mathbf{0}\hat{\boldsymbol{\imath}} + \mathbf{0.832}\hat{\boldsymbol{\jmath}} - \mathbf{0.555}\hat{\boldsymbol{k}}$$

Style notes: it is common to explicitly state $0\hat{\imath}$ to ensure the reader knows you didn't forget the $\hat{\imath}$ computation.
You can also use brackets like this:

$$\hat{r} =< 0, 0.832, -0.555 >$$

Some engineers do NOT use these brackets (perhaps to clarify which type of unit vector since they use many types).
Lastly, since I do not expect to be using this result in subsequent calculations I can round and avoid underbar usage.

3c) Follow the standard procedure outlined in the videos, workbook, and homework.

$$\theta_{rP} = \cos^{-1}\left(\frac{\vec{r} \cdot \vec{P}}{rP}\right)$$

I notice I need the magnitude P which I get in similar fashion to part a.
Also, I notice the units will drop out…so I will leave them out of the calculation entirely.
Lastly, be sure to use the unrounded values for r & P to avoid intermediate rounding.

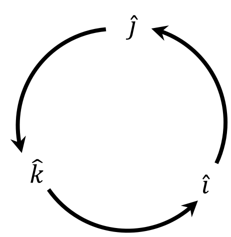
$$\theta_{rP} = \cos^{-1}\left(\frac{(7.50\hat{\jmath} - 5.00\hat{k}) \cdot (-3.33\hat{\imath} - 4.44\hat{\jmath})}{(9.0\underline{1}4)(5.55)}\right)$$

Think: when doing *dot* products we know $\hat{\jmath} \cdot \hat{\jmath} = 1$ but $\hat{\jmath} \cdot \hat{\imath} = 0, \hat{k} \cdot \hat{\jmath} = 0$ & $\hat{k} \cdot \hat{\imath} = 0$.

$$\theta_{rP} = \cos^{-1}\left(\frac{(7.50) \cdot (-3.33)}{(9.0\underline{1}4)(5.55)}\right)$$

$$\boldsymbol{\theta_{rP} = 131.7°}$$

3d) I get out my trusty wheel of pain as shown at right. No one else calls it that but it is fun to say.



$$\vec{M} = (7.50\hat{\jmath} - 5.00\hat{k}) \text{ m} \times (-3.33\hat{\imath} - 4.44\hat{\jmath}) \text{ kg} \cdot \frac{\text{m}}{\text{s}^2}$$

Remember, when doing *cross* products $\hat{\jmath} \times \hat{\jmath} = 0$…ignore that term. Notice the units!

$$\vec{M} = [(7.50\hat{\jmath}) \times (-3.33\hat{\imath}) + (-5.00\hat{k}) \times (-3.33\hat{\imath}) + (-5.00\hat{k}) \times (-4.44\hat{\jmath})] \text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2}$$

$$\vec{M} = [(-24.\underline{9}8)(\hat{\boldsymbol{\jmath}} \times \hat{\boldsymbol{\imath}}) + (16.\underline{6}5)(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{\imath}}) + (22.\underline{2}0)(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{\jmath}})] \text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2}$$

$$\vec{M} = [(-24.\underline{9}8)(-\hat{\boldsymbol{k}}) + (16.\underline{6}5)(\hat{\boldsymbol{\jmath}}) + (22.\underline{2}0)(-\hat{\boldsymbol{\imath}})] \text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2}$$

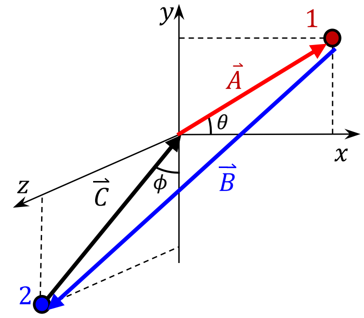
Get all the signs correct then put in standard order.
Don't forget the units!!! They don't cancel in this computation…

$$\boldsymbol{\vec{M} = [-22.2\hat{\imath} + 16.65\hat{\jmath} + 25.0\hat{k}] \text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2}}$$

We are told the particle travels in the following way:

- First from the origin to point 1 (call this vector $\vec{A}$)
- Next from point 1 to point 2 (call this vector $\vec{B}$)
- Finally from point 2 to the origin (call this vector $\vec{C}$)

See the vectors I drew in the figure at right.



4a) As usual, I show rounding digit but keep unrounded answer.

$$\vec{A} = (7.00\cos 33.3°\,\hat{\imath} + 7.00\sin 33.3°\,\hat{\jmath})\text{ m}$$

$$\boldsymbol{\vec{A} = \left(5.8\underline{5}1\hat{\imath} + 3.8\underline{4}3\hat{\jmath}\right)\textbf{ m}}$$

4b) Notice the angle is adjacent to the y-axis and opposite the z-axis.
Notice the y-component uses cosine while the z-component uses sine…
Finally, notice $\vec{C}$ goes in the negative z-direction as it goes towards the origin…minus sign on z-component!

$$\vec{C} = \left(8.00\cos 22.2°\,\hat{\jmath} - 8.00\sin 33.3°\,\hat{k}\right)\text{ m}$$

$$\boldsymbol{\vec{C} = \left(7.4\underline{0}7\hat{\jmath} - 3.0\underline{2}3\hat{k}\right)\textbf{ m}}$$

4c) I can tell the final answer will have units of meters so I leave units off until the final answer.

$$\vec{A} + \vec{B} + \vec{C} = 0$$

$$\vec{B} = -\left(\vec{A} + \vec{C}\right)$$

$$\vec{B} = -\left[\left(5.8\underline{5}1\hat{\imath} + 3.8\underline{4}3\hat{\jmath}\right) + \left(7.4\underline{0}7\hat{\jmath} - 3.0\underline{2}3\hat{k}\right)\right]$$

$$\vec{B} = -\left[5.8\underline{5}1\hat{\imath} + 11.250\hat{\jmath} - 3.0\underline{2}3\hat{k}\right]$$

$$\vec{B} = \left(-5.8\underline{5}1\hat{\imath} - 11.250\hat{\jmath} + 3.0\underline{2}3\hat{k}\right)\text{ m}$$

Think: this is a good time to check the signs on each component.
Verify they match the directions associated with the arrow in the figure (e.g. left $\Rightarrow\ -\hat{\imath}$).
Finally, be sure you actually answer what was requested…I asked for the *magnitude* of this vector.

$$B = \left\|\vec{B}\right\| = \sqrt{\left(-5.8\underline{5}1\right)^2 + (-11.250)^2 + \left(3.0\underline{2}3\right)^2}$$

$$\boldsymbol{B = 13.04\textbf{ m}}$$

Remember, it is often customary to include an extra sig fig when the first digit of a number is 1.