110fa23t2aSoln

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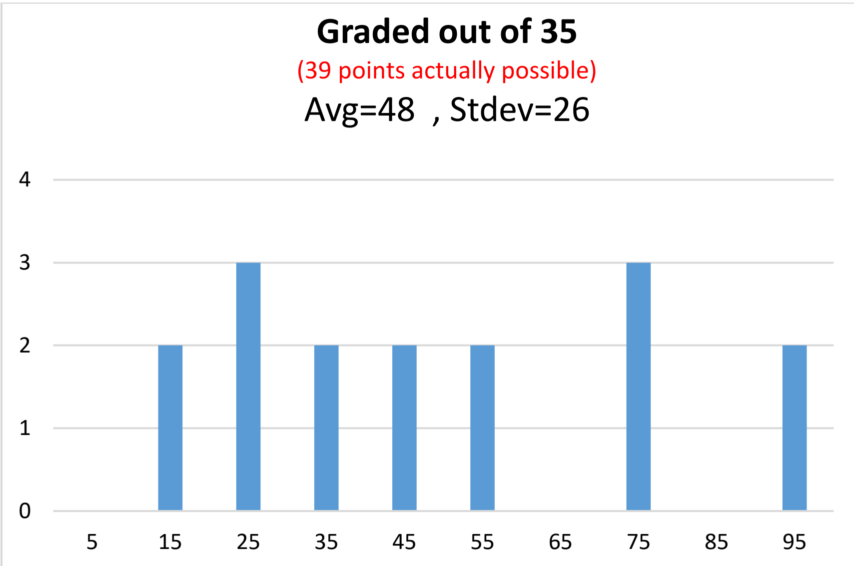
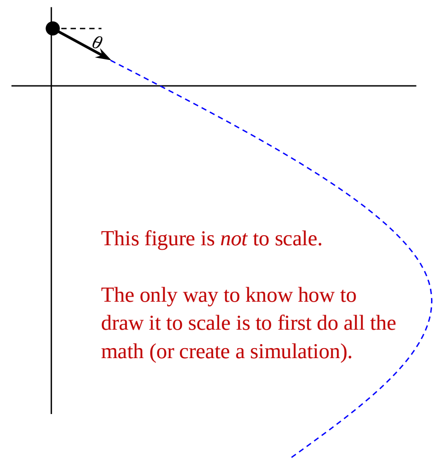


Solutions begin on the next page.



1a) For part 1a we know the following info:

- Crossing horizontal axis implies $\Delta y = -14.44 \text{ m}$
- $v_{ix} = 22.2\frac{\text{m}}{\text{s}}\cos 33.3° = 18.555\frac{\text{m}}{\text{s}}$
- $v_{iy} = -22.2\frac{\text{m}}{\text{s}}\sin 33.3° = -12.188\frac{\text{m}}{\text{s}}$
- Acceleration to the left implies $a_y = 0$ & $a_x = -5.55\frac{\text{m}}{\text{s}^2}$

Want time to horizontal axis.

$$\Delta y = \frac{1}{2}a_y t^2 + v_{iy}t$$

Plug in zeros. Then solve algebraically. Then plug in numbers.

$$\Delta y = v_{iy}t$$

$$t = \frac{\Delta y}{v_{iy}} = \frac{-14.44 \text{ m}}{-12.188\frac{\text{m}}{\text{s}}} = 1.18477 \text{ s} = \mathbf{1.185\ s}$$

Note: by convention we often include an extra sig fig when the first digit of a result is a 1.
Note: I like the *unrounded* result written on my paper as well (to use in any subsequent calculations).

1b) Note: we still have the same known quantities as in part 1a since we are still going from initial position to the point where the particle crosses the horizontal axis. We want *speed*. *Speed* is *magnitude of velocity*. I'll get both velocity components then use the Pythagorean formula to get the magnitude!

$$v_{fx} = v_{ix} + a_x t = \left(18.555\frac{\text{m}}{\text{s}}\right) + \left(-5.55\frac{\text{m}}{\text{s}^2}\right)(1.18477 \text{ s}) = 11.980\frac{\text{m}}{\text{s}}$$

NOTICE: In part b we should use the *unrounded* time from part a!

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{\left(18.555\frac{\text{m}}{\text{s}}\right)^2 + \left(-12.188\frac{\text{m}}{\text{s}}\right)^2} = \mathbf{17.09\frac{m}{s}}$$

1c) Still under the same set of knowns and unknowns since we are still interested in going from initial position to the point where the particle crosses the horizontal axis. Again, in part b we should use the *unrounded* time from part a!

$$\Delta x = \frac{1}{2}a_x t^2 + v_{ix}t = \frac{1}{2}\left(-5.55\frac{\text{m}}{\text{s}^2}\right)(1.18477 \text{ s})^2 + \left(18.555\frac{\text{m}}{\text{s}}\right)(1.18477 \text{ s}) = \mathbf{18.09\ m}$$

1d) Part 1d is essentially asking about when the particle reverses direction as it travels horizontally. Since this describes a different final point in time, we need to relist our knowns:

- Reversing direction horizontally implies $v_{fx} = 0$
- $v_{ix} = 22.2\frac{\text{m}}{\text{s}}\cos 33.3° = 18.555\frac{\text{m}}{\text{s}}$
- $v_{iy} = -22.2\frac{\text{m}}{\text{s}}\sin 33.3° = -12.188\frac{\text{m}}{\text{s}}$
- Acceleration to the left implies $a_y = 0$ & $a_x = -5.55\frac{\text{m}}{\text{s}^2}$

$$v_{fx} = v_{ix} + a_x t$$

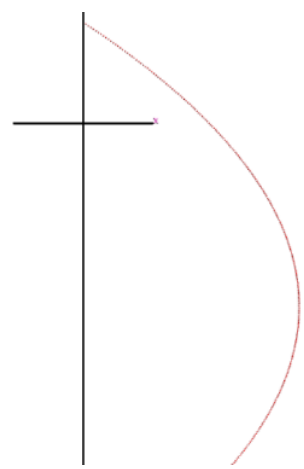
Plug in zeros first. Then solve algebraically. Then plug in numbers.

$$0 = v_{ix} + a_x t$$

$$t = -\frac{v_{ix}}{a_x} = \mathbf{3.34\ s}$$

Side note: One could use this time to find the (x, y) coordinates of where the object reverses direction then draw the trajectory to scale based on the object initially 14.44 m from the origin.

A better way to know the trajectory is to write a simulation in Python (this is shown at right).



2a) We are assuming air resistance is negligible as indicated by the problem statement.
During the entire flight, from just after the initial kick until just before impact, acceleration is constant.
This is true for all points in the flight (including at max height).

$$\boldsymbol{a_x = 0} \quad \& \quad \boldsymbol{a_y = -g}$$

Recall $g = 9.8\,\frac{\text{m}}{\text{s}^2}$. This is the *magnitude* of the acceleration due to gravity.
Remember to account for the direction (according to the given coordinate system $-\hat{\jmath}$) which gives the negative sign.

2b) Once in the air, the *horizontal* component of the projectile doesn't change (when air resistance is negligible).
Since it is initially moving to the right, it moves to the right throughout the entire flight.
At max height the particle reverses direction *vertically*. At this instant the *vertical* velocity component goes to zero.

$$\boldsymbol{v_x > 0} \quad \& \quad \boldsymbol{v_y = 0}$$

2c) Only on a level ground trajectory will time to max height equal exactly half of total flight time.
In this scenario, the first half of the flight covers more than half the distance.
We expect the time to max height should be more than half the flight time.

$$\boldsymbol{1.0\ \text{s} < t_{max} < 2.0\ \text{s}}$$

2d) Similarly, only on a level ground trajectory will impact speed equal launch speed.
Just before impact, final vertical velocity component is *smaller* than the launch vertical velocity component.
We know the horizontal component is unchanged.
If one velocity component gets *smaller* while the other remains constant, we expect the speed to also get smaller.

$$\boldsymbol{v_f < v_0}$$

2e) We know the horizontal component is unchanged.

$$|\boldsymbol{v_{fx}}| = |\boldsymbol{v_{0x}}|$$

2f) The vertical component of velocity is smaller at impact.

$$|\boldsymbol{v_{fy}}| < |\boldsymbol{v_{0y}}|$$

2g) The final vertical position is above the initial vertical position. Don't overthink it.

$$\boldsymbol{\Delta y > 0} \quad (\boldsymbol{positive})$$

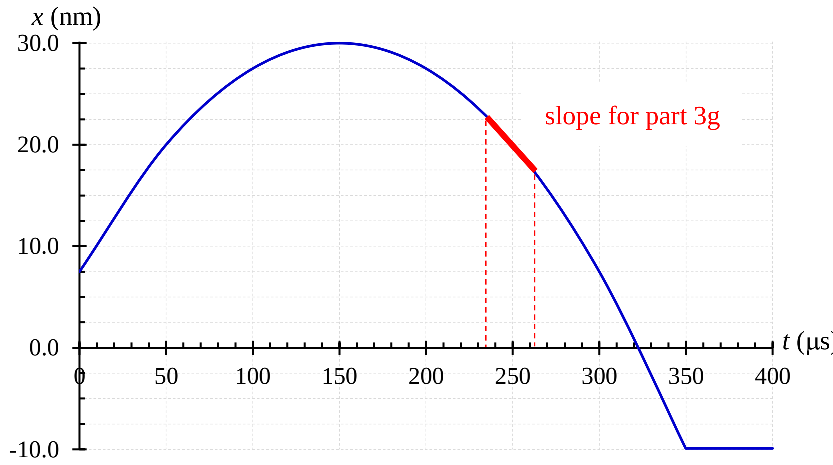
3a) Moving left implies $v_x < 0$.
On an xt-plot, this implies $slope < 0$.

From 150 → 350 μs.

Check, did you remember to include units?

3b) Speeding up implies acceleration and velocity have same sign. On an xt-plot, concavity indicates the sign of acceleration. Here, concave down implies negative acceleration. Notice we have negative acceleration during all times of negative slope! In a rare coincidence, the answer is exactly the same as the previous result!

From 150 → 350 μs.



3c) Symbol x represents **position** (not displacement, not distance). Circle **"None of the other answers is correct."**

3d) Between $t = 0 \rightarrow 150$ μs the object travels 22.5 nm to the *right* (from $x_0 = 7.5$ nm to $x_{150} = 30$ nm).
Between $t = 150 \rightarrow 350$ μs the object travels 40 nm to the *left* (from $x_{150} = 30$ nm to $x_{350} = -10$ nm).
Between $t = 350 \rightarrow 400$ μs the object doesn't move (no slope on xt-plot implies no velocity implies no motion).
Total distance traveled is 62.5 nm. Check: did you include the units?

3e) Don't overthink this one. Total displacement is

$$x_f - x_i = -10 \text{ nm} - 7.5 \text{ nm} = \mathbf{-17.5\ nm}$$

Total displacement is −17.5 nm. Check: did you include both the minus sign and the units?

3f) Average speed is given by *total distance* over *total time*.

$$speed_{avg} \approx \frac{62.5 \text{ nm}}{400\ \mu\text{s}} = \frac{62.5 \times 10^{-9} \text{ m}}{400 \times 10^{-6} \text{ s}} = \mathbf{1.56 \times 10^{-4} \frac{m}{s}}$$

Did you remember to convert the prefixes and answer in the units requested in scientific notation?
Think: because this result came from numbers read off a graph it is probably reasonable to only use two sig figs.
However, since the first digit is a 1, I'll include an extra digit.

3g) *Velocity* (not *speed*) is given by the slope of the xt-plot.
In the graph shown above I drew the line I used to calculate the slope.
In trying to get an accurate slope for 250 μs, I used initial & final points which bracket 250 μs.
In real life, the numbers rarely land exactly on grid coordinates.
However, notice I chose points such that the x values on the vertical axis were easy.
(A fine alternative is to have the times line up well)
Either way, a bit of estimation is required. As such, I'll allow for about 10% slop compared to my answer.

$$v_x \approx \frac{x_f - x_i}{t_f - t_i} = \frac{17.5 \text{ nm} - 22.5 \text{ nm}}{262\ \mu\text{s} - 235\ \mu\text{s}} = \mathbf{-1.85 \times 10^{-4} \frac{m}{s}}$$

Did you remember to convert the prefixes and answer in the units requested in scientific notation?
Did you remember the minus sign? This is *velocity* (not *speed*) so the sign of the answer matters!
Think: because this result came from numbers read off a graph it is probably reasonable to only use two sig figs.
However, since the first digit is a 1, I'll include an extra digit.

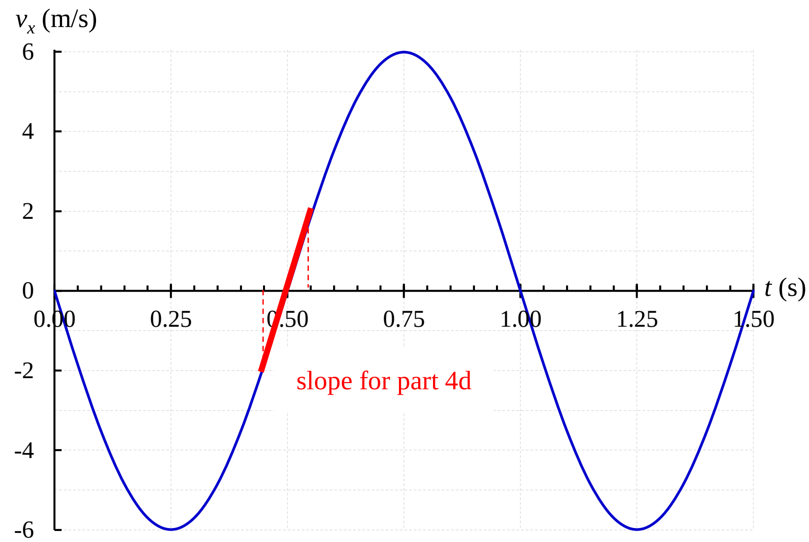
4a) Moving left implies $v_x < 0$.
On a vt-plot, this implies $value < 0$.
From $0 \rightarrow 0.50$ s & $1.00 \rightarrow 1.50$ s.
Check, did you remember to include units?

4b) Speeding up implies acceleration and velocity have same sign. On a vt-plot, *slope* indicates the sign of acceleration. We want

We are looking for regions with positive values & positive slope AS WELL AS negative values & negative slope! The following times match this description:

- $\mathbf{0 \rightarrow 0.25}$ **s**
- $\mathbf{0.50 \rightarrow 0.75}$ **s**
- $\mathbf{1.00 \rightarrow 1.25}$ **s**

Did you remember units?

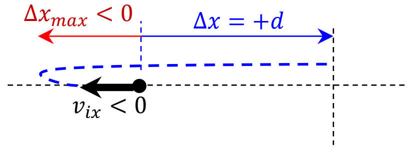


4c) Displacement on a vt-plot is given by area between the curve & the horizontal axis (including $\pm$sign).
Areas below the horizontal axis are negative.
Notice there is more negative area than positive area over the entire time interval shown.
Total displacement is *negative* (to the *left*) for the entire time interval shown.

4d) Acceleration is given by the slope on a vt-plot.
Most common mistake was trying to use too large of a time interval just because the points are easier to read.
Again, try to bracket the time of interest by choosing a point just before and just after the time of interest.

$$a_x \approx \frac{v_f - v_i}{t_f - t_i} = \frac{2\,\frac{\text{m}}{\text{s}} - \left(-2\,\frac{\text{m}}{\text{s}}\right)}{0.55\text{ s} - 0.45\text{ s}} = \mathbf{40\,\frac{m}{s^2}}$$

Did you remember the units?
Think: because this result came from numbers read off a graph it is probably reasonable to only use two sig figs.
Note: a bit of estimation is required. As such, I'll allow for about 10% slop compared to my answer.



This figure is *not* to scale.

In this problem, we are told the particle moves *horizontally*. Ignore the *vertical* displacement in the figure (used to show the path better).

5a) Based on the words in the problem statement, we expect a path similar to the one shown at right. I added a small vertical displacement just so you could see the path a bit better. In the problem statement we are told the object moves purely horizontally.

We know the object must reverse direction to reach the origin.
This can only happen if acceleration points opposite the initial velocity direction.

Accleration must be directed **to the right.**

5b) I choose to use the Δx equation.

$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$$

In this scenarios we have no zeros to plug in. Solve algebraically for v_{ix}.

$$v_{ix}t = \Delta x - \frac{1}{2}a_x t^2$$

$$v_{ix} = \frac{\Delta x}{t} - \frac{a_x t}{2}$$

Now rewrite this answer using the particular variables mentioned in the problem statement

$$\boldsymbol{v_{ix} = \frac{d}{t} - \frac{at}{2}}$$

Think: we expect this result should be negative (see figure). For stylistic reasons, I choose to put the negative number first in the equation to remind me of this fact. Either style is full credit.

$$\boldsymbol{v_{ix} = -\frac{at}{2} + \frac{d}{t}}$$

5c) To get distance traveled, consider the figure.
The object travels some distance to left, comes back to its initial position, then travels an addition distance d to reach the origin.WATCH OUT! If we use a kinematic equation to determine Δx_{max} it will be a *negative displacement*.

$$total\ distance\ traveled = d + 2|\Delta x_{max}| = d - 2\Delta x_{max}$$

I often go straight to the v_{fx}^2 equation to get Δx_{max} (because we know $v_{fx} = 0$ at the max/min positions).

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x_{max}$$

$$0 = v_{ix}^2 + 2a_x\Delta x_{max}$$

$$\Delta x_{max} = -\frac{v_{ix}^2}{2a_x}$$

Think: $v_{ix} < 0$ but it gets squared while $a_x > 0$. Notice $\Delta x_{max} < 0$ as expected.
Here I will be a bit clever and multiply both sides by -2 (remember we want $distance\ = d - 2\Delta x_{max}$).

$$-2\Delta x_{max} = \frac{\left(-\frac{at}{2} + \frac{d}{t}\right)^2}{a}$$

$$-2\Delta x_{max} = \frac{\frac{a^2t^2}{4} + \frac{d^2}{t^2} + 2\left(-\frac{at}{2}\right)\left(\frac{d}{t}\right)}{a}$$

$$-2\Delta x_{max} = \frac{at^2}{4} + \frac{d^2}{at^2} - d$$

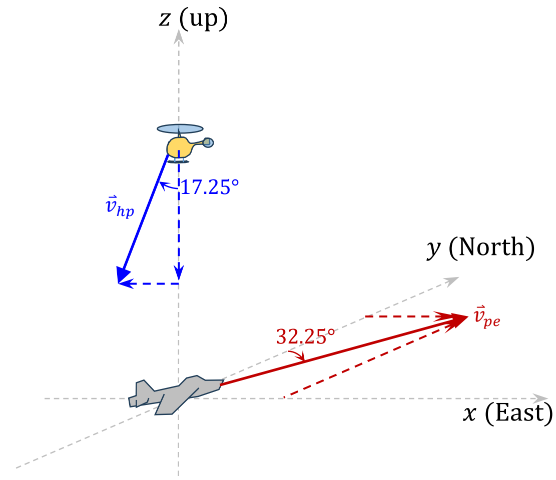
Now plug this in:

$$distance\ = d + \left(\frac{at^2}{4} + \frac{d^2}{at^2} - d\right) = \boldsymbol{\frac{at^2}{4} + \frac{d^2}{at^2}}$$

Alternate style: determine the time to reversal using $0 = v_{ix} + at_{rev}$ and use that in the Δx equation to get Δx_{max}.

6a) Pay close attention to following in these types of problems:

- location of the angles (to which axis)
- coordinate system (axis label indicates positive end of each axis)
- the $\pm$ signs & unit vectors appropriate for each vector
- velocity subscripts (relative *to earth*, relative *to plane*, etc)



We want $\vec{v}_{he}$. It makes sense to write a correct relative velocity equation.

$$\vec{v}_{he} = \vec{v}_{hp} + \vec{v}_{pe}$$

Notice $\vec{v}_{pe}$ lies in the xy-plane but angle was given to the y-axis.

$$\vec{v}_{pe} = 100.0\frac{\text{m}}{\text{s}}(\sin 32.25°\,\hat{\imath} + \cos 32.25°\,\hat{\jmath})$$

$$\vec{v}_{pe} = (53.\underline{3}6\hat{\imath} + 84.\underline{5}7\hat{\jmath})\frac{\text{m}}{\text{s}}$$

Notice $\vec{v}_{hp}$ lies in the xz-plane but angle was given to the z-axis.

$$\vec{v}_{hp} = 50.0\frac{\text{m}}{\text{s}}(-\sin 17.25°\,\hat{\imath} - \cos 17.25°\,\hat{k})$$

$$\vec{v}_{hp} = (-14.\underline{8}27\hat{\imath} - 47.\underline{7}5\hat{k})\frac{\text{m}}{\text{s}}$$

Out of habit I typically keep an extra digit along for the ride when the first digit is a one.

Now do the addition.

$$\vec{v}_{he} = (-14.\underline{8}27\hat{\imath} - 47.\underline{7}5\hat{k})\frac{\text{m}}{\text{s}} + (53.\underline{3}6\hat{\imath} + 84.\underline{5}7\hat{\jmath})\frac{\text{m}}{\text{s}}$$

$$\vec{v}_{he} = (38.\underline{5}3\hat{\imath} + 84.\underline{5}7\hat{\jmath} - 47.\underline{7}5\hat{k})\frac{\text{m}}{\text{s}}$$

Speed (of helicopter relative to earth) is the magnitude of this vector.

$$v_{he} = 104.48\frac{\text{m}}{\text{s}}$$

$$\boldsymbol{v_{he} = 104.5\frac{\text{m}}{\text{s}}}$$

Again, since the first digit is a 1 we keep an extra digit compared to our usual 3 sig figs.

Direction (of helicopter relative to earth) is the unit vector $\hat{v}_{he} = \frac{\vec{v}_{he}}{v_{he}}$.

Be sure to use the unrounded result for speed so you can avoid intermediate rounding error.

$$\hat{v}_{he} = \frac{(38.\underline{5}3\hat{\imath} + 84.\underline{5}7\hat{\jmath} - 47.\underline{7}5\hat{k})\frac{\text{m}}{\text{s}}}{104.48\frac{\text{m}}{\text{s}}}$$

$$\hat{v}_{he} = 0.369\hat{\imath} + 0.809\hat{\jmath} - 0.458\hat{k}$$