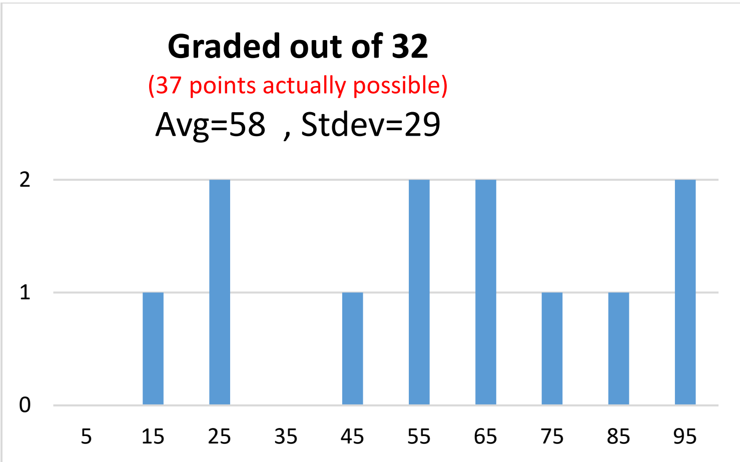
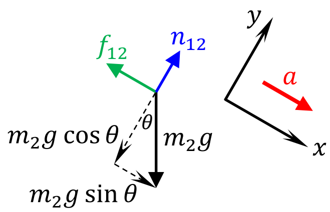
110fa23t3aSoln

Distribution on this page.
Solutions begin on the next page.

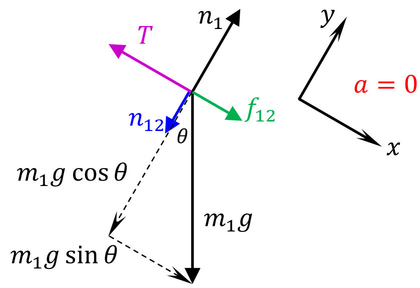


1a) Arrows not drawn to scale.

FBD for $\boldsymbol{m_2}$



FBD for $\boldsymbol{m_1}$



ΣF_x: $m_2 g \sin\theta - f_{12} = m_2 a$	ΣF_x: $m_1 g \sin\theta + f_{12} - T = 0$
ΣF_y: $n_{12} - m_2 g \cos\theta = 0$	ΣF_y: $n_1 - n_{12} - m_1 g \cos\theta = 0$

1b) We are told m_2 is sliding.
Therefore $f_{12} = \mu_k n_{12}$.
Using ΣF_y for m_2 gives $n_{12} = m_2 g \cos\theta$.

$$f_{12} = \mu_k m_2 g \cos\theta$$

Using ΣF_x for m_2 gives

$$m_2 g \sin\theta - f_{12} = m_2 a$$

$$m_2 g \sin\theta - \mu_k m_2 g \cos\theta = m_2 a$$

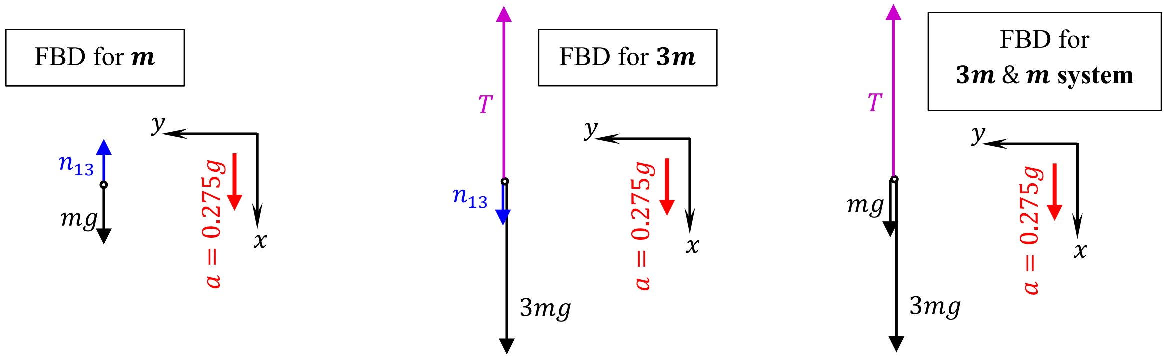
$$\boldsymbol{a = g(\sin\theta - \mu_k \cos\theta)}$$

1c) Using ΣF_x for m_1 gives

$$T = m_1 g \sin\theta + f_{12}$$

$$\boldsymbol{T = m_1 g \sin\theta + \mu_k m_2 g \cos\theta}$$

2a & 2b) In this situation, the two massing are acerating in unison. It is appropriate to use a system FBD.
I drew all three FBDs below, but you really only need two of these to get the job done.
In this case, I would avoid the middle FBD since it has two unknown forces.
That said, choose any two and the algebra is straightforward.



WATCH OUT! If you drew upwards as the positive y-direction, in your force equations all forces AND a will flip signs compared to mine. For example, you'd get $T - mg - 3mg = (3m + m)(-a)$

2a) My force equation for the system FBD gives:

$$-T + mg + 3mg = (3m + m)a$$

$$-T + 4mg = 4m(0.275g)$$

$$\boldsymbol{T = 2.90mg}$$

2b) My force equation mass m gives:

$$mg - n_{13} = m(0.275g)$$

$$\boldsymbol{n_{13} = 0.725mg}$$

2c) The two forces described ($\vec{n}_{up}$ & $\vec{n}_{down}$) are an action reaction pair.
These two forces have equal magnitude whether they are accelerating or not.

2d) The two forces described ($\vec{n}_{up}$ & $\vec{n}_{down}$) are an action reaction pair.
These two forces have equal magnitude whether they are accelerating or not.

2e) The problem statement implied the ACTION force was the weight force associated with $3m$.
You were supposed to determine the REACTION listed below.

ACTION	**The earth** exerts a **gravitational** force on **mass $\boldsymbol{3m}$** directed **downwards**.
REACTION	**Mass $\boldsymbol{3m}$** exerts a **gravitational** force on **the earth** directed **upwards**.

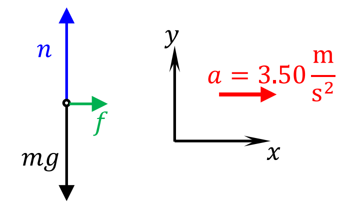
3a) FBD is sneaky if you don't think carefully about the friction direction.
As the truck accelerates forwards, the box should tend to slide *backwards* relative to the truck.
This implies friction is backwards on the truck.
By Newton's third law, it implies friction is forwards on the box!

Think: if no friction was present, the box would not move at all as the truck drove forwards.

$$\Sigma F_x: \quad f = ma$$

$$\Sigma F_y: \quad n = mg$$

FBD for box



Problem asks for minimum coefficient of friction to prevent sliding.

This implies the box is on verge of slipping.

$$f = \mu_s n = \mu_s mg$$

Now plug into $f = ma$ to find

$$\boldsymbol{\mu_s = \frac{a}{g} \approx 0.357}$$

Notice units drop out as expected.
Coefficient of friction μ_s is dimensionless while frictional *force* f has units of N.

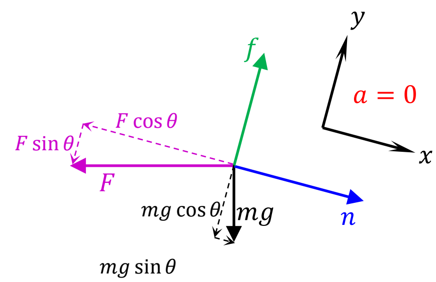
3b) First, notice changing acceleration in the x-direction has no effect on the vertical force equation.
We expect *normal force remains unchanged*.
Next, if the truck accelerates at a *slower* rate, less friction is required to keep the box accelerating with the truck.
The box would no longer be on the verge of slipping.
We would expect frictional force changes from $f = \mu_s n$ to $f < \mu_s n$.
Friction would decrease.
As a check, notice there is no friction at all if acceleration decreases to zero…

3d) First, notice changing acceleration in the x-direction has no effect on the vertical force equation.
We expect *normal force remains unchanged*.
Next, if the truck accelerates at a *faster* rate, friction is insufficient to keep the box accelerating with the truck.
The box would be sliding off the back of the truck!
We would expect frictional force changes from $f = \mu_s n$ to $f = \mu_k n$.
In this particular problem, the problem statement tells us $\mu_s = \mu_k$.
Friction force should remain unchanged as well.

If $\mu_s = \mu_k$ was not specified, it would be impossible to determine even though we normally assume $\mu_k < \mu_s$.

4a) Problem asks for minimum force to prevent sliding.
This implies the box is on verge of slipping.

$$f = \mu_s n$$



4b) FBD is shown at right. Arrows not to scale.

$$\Sigma F_x: \quad n + mg\sin\theta = F\cos\theta$$

$$\Sigma F_y: \quad f = mg\cos\theta + F\sin\theta$$

Problem asks for minimum force to prevent sliding.
This implies the box is on verge of slipping.

$$n = F\cos\theta - mg\sin\theta$$

$$f = \mu_s n = \mu_s(F\cos\theta - mg\sin\theta)$$

Plug into ΣF_y and solve for F.

$$f = mg\cos\theta + F\sin\theta$$

$$\mu_s(F\cos\theta - mg\sin\theta) = mg\cos\theta + F\sin\theta$$

$$\mu_s F\cos\theta - \mu_s mg\sin\theta = mg\cos\theta + F\sin\theta$$

$$\mu_s F\cos\theta - F\sin\theta = mg\cos\theta + \mu_s mg\sin\theta$$

$$F(\mu_s\cos\theta - \sin\theta) = mg(\cos\theta + \mu_s\sin\theta)$$

$$\boldsymbol{F = mg\frac{\cos\theta + \mu_s\sin\theta}{\mu_s\cos\theta - \sin\theta}}$$

This answer is full credit.
At this point, people usually divide every term in the fraction by $\cos\theta$.

$$\boldsymbol{F = mg\frac{1 + \mu_s\tan\theta}{\mu_s - \tan\theta}}$$

Solution continues on next page…

4c) As you decrease the angle, the y-axis of the coordinate system becomes more vertical.
You could also notice $\sin\theta$ gets slightly smaller while $\cos\theta$ gets slightly larger.

The component of weight *perpendicular* to plane ($mg\sin\theta$) becomes even more negligible.
The component of weight *parallel* to the plane ($mg\cos\theta$) increases slightly.

Regarding normal force, consider the ΣF_x force equation:

$$n + mg\sin\theta = F\cos\theta$$

$$n = F\cos\theta - mg\sin\theta$$

The $\cos\theta$ term gets slightly larger while the $\sin\theta$ term gets slightly smaller.
Normal force should get larger if you have a bigger term subtracting a smaller term!

Regarding frictional force, **WATCH OUT!!!!**
Your first instinct is probably to say friction increases since normal force increases.
Unfortunately, this is NOT true.
In this scenario, as normal force increases we are no longer on the verge of slipping.
Just like the last problem, we should no longer use $f = \mu_s n$.
Instead consider the ΣF_y force equation:

$$f = mg\cos\theta + F\sin\theta$$

This one is *a lot* tougher to figure out.
The first term gets slightly larger while the second term gets slightly smaller.
How can you tell if the net effect is an increase, decrease, or if it remains constant?
Second gut instinct might be to think it is impossible to determine. This would also be wrong.

One way to gain some intuition is to do some trial and error.
For example, assume $\mu_s = 1$ then see what happens if you use 0° instead of 10°.
I did this process and found the friction force *decreased*.

Using $\boldsymbol{\theta = 10°}$	Using $\boldsymbol{\theta = 0°}$
$F = mg\dfrac{1+(1)\tan 10°}{(1)-\tan 10°} = 1.428mg$ $f = mg\cos 10° + F\sin 10° = 1.233mg$	$F = mg\dfrac{1+(1)\tan 0°}{(1)-\tan 0°} = mg$ $f = mg\cos 0° + F\sin 0° = mg$

Optional alternative thought process using calculus:

Derivative of $\cos\theta$ near $\theta \approx 0°$ is close to 0 (slope close to 0 at $\theta \approx 0°$).
This implies the value of the cosine function will not change much as the angle changes near $\theta = 0°$.

Derivative of $\sin\theta$ near $\theta \approx 0°$ is close to 1 (slope close to 1 at $\theta \approx 0°$).
This implies the value of the sine function will change more than the cosine function for angle changes near $\theta = 0°$.

In this scenario we expect $mg\cos\theta$ essentially remain about the same while $F\sin\theta$ would decrease significantly.

Frictional force decreases despite normal force increasing!