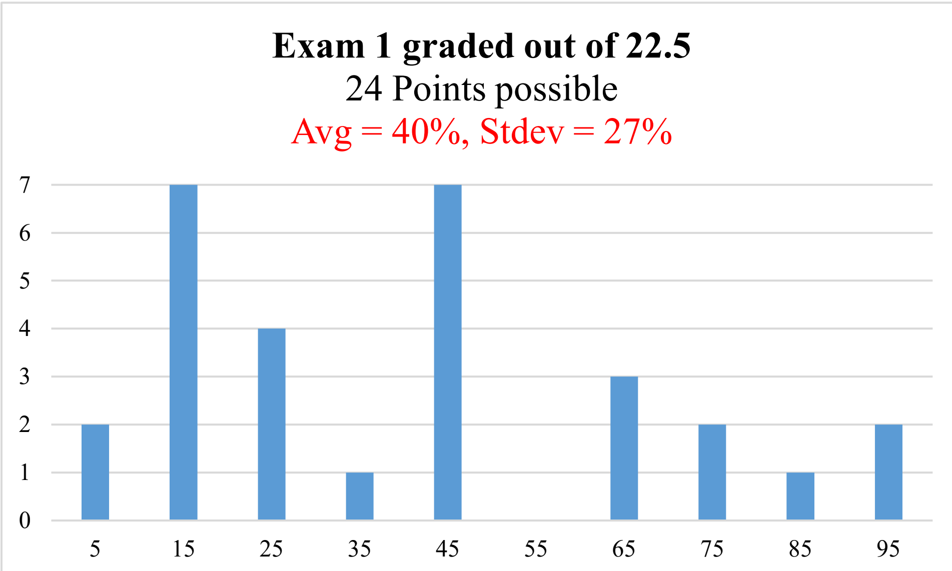
110fa25t1bSoln
Distribution on this page.
Solutions begin on the next page.



1a) $C = \|\vec{C}\| = 7.2\underline{1}1$ & $D = \|\vec{D}\| = 6.403$.
Think: we will probably use these values in later parts of this problem.
It is wise to keep an extra digit (beyond the rounding digit) to avoid intermediate rounding in subsequent steps.

1b) Doing the cross-product:

$$\vec{C} \times \vec{D} = (4.00\hat{\imath} - 6.00\hat{\jmath}) \times (-4.00\hat{\imath} + 5.00\hat{\jmath})$$

Note: ignore $\hat{\imath} \times \hat{\imath} = 0$ and $\hat{\jmath} \times \hat{\jmath} = 0$.

$$\vec{C} \times \vec{D} = (4.00\hat{\imath}) \times (5.00\hat{\jmath}) + (-6.00\hat{\jmath}) \times (-4.00\hat{\imath})$$

$$\vec{C} \times \vec{D} = 20.\underline{0}0(\hat{\imath} \times \hat{\jmath}) + 24.\underline{0}0(\hat{\jmath} \times \hat{\imath})$$

Remember $\hat{\jmath} \times \hat{\imath} = -\hat{k}$.

$$\vec{C} \times \vec{D} = 20.\underline{0}0(\hat{k}) + 24.\underline{0}0(-\hat{k})$$

$$\boldsymbol{\vec{C} \times \vec{D} = -4.\underline{0}0(\hat{k})}$$

Note: you were not required to track sig figs on this problem but I did it to help you practice.
Our default on exam results is to use three sig figs.
Exception: when the first digit of a result is 1, keep 4 sig figs.

1b) Doing the dot product:

$$\vec{C} \cdot \vec{D} = (4.00\hat{\imath} - 6.00\hat{\jmath}) \cdot (-4.00\hat{\imath} + 5.00\hat{\jmath})$$

Note: ignore $\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{\imath} = 0$. Remember $\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = 1$.

$$\vec{C} \cdot \vec{D} = (4.00\hat{\imath}) \cdot (-4.00\hat{\imath}) + (-6.00\hat{\jmath}) \cdot (+5.00\hat{\jmath})$$

$$\vec{C} \cdot \vec{D} = -16.\underline{0}0 - 30.\underline{0}0$$

$$\boldsymbol{\vec{C} \cdot \vec{D} = -46.\underline{0}0}$$

1c) Get the angle between the two vectors using:

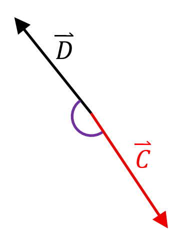
$$\theta_{CD} = \cos^{-1}\left(\frac{\vec{C} \cdot \vec{D}}{CD}\right)$$

$$\theta_{CD} = \cos^{-1}\left[\frac{-46.\underline{0}0}{(7.2\underline{1}1)(6.403)}\right]$$

$$\theta_{CD} = 17\underline{5}.1°$$

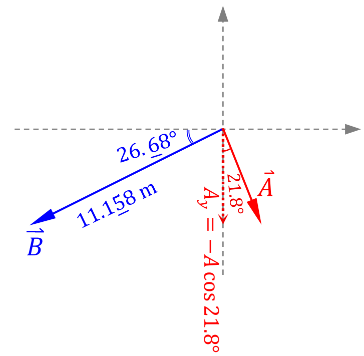
Note: if you tried to do this using the cross-product:

$$\theta_{CD} = \sin^{-1}\left(\frac{\|\vec{C} \times \vec{D}\|}{CD}\right) = \begin{cases} 4.9\underline{7}0° \\ \text{OR} \\ 180° - 4.9\underline{7}0° = 175.03° \end{cases}$$



You must know how to correctly interpret the number from calculator.
A sketch showing both vectors with tail at the origin can assist in this interpretation.

110 fa25 t1b = yellow test



In parts 2a & 2b I will indicate the appropriate sig figs with an underbar.
I do this to avoid intermediate rounding error for part 2c.

2a) $\vec{A} = \left(2.0\underline{0}2\hat{\imath} - 5.0\underline{0}45\hat{\jmath}\right)$ m

Look at the figure to see how I determined A_x.
Notice SOH CAH TOA implies $A_y = -A\,\mathbf{cos}\,33.7°$!

2b) $\vec{B} = 11.1\underline{5}8$ m @ $26.\underline{6}8°$ south of east (see figure).

Notice I keep an extra unrounded digit when the first digit is 1.
I don't think it really matters here (since we never use this unrounded result), but it is a good habit.

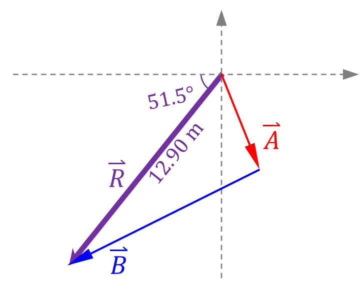
2c) Standard vector addition.

$$\vec{R} = \vec{A} + \vec{B}$$

$$\vec{R} = \left(2.0\underline{0}2\hat{\imath} - 5.0\underline{0}45\hat{\jmath}\right)\text{ m} + \left(-9.97\hat{\imath} - 5.01\hat{\jmath}\right)\text{ m}$$

$$\vec{R} = \left(-7.9\underline{6}8\hat{\imath} - 10.0\underline{1}45\hat{\jmath}\right)\text{ m}$$

$$\boldsymbol{\vec{R} = 12.90 \text{ @ } 51.5° \text{ S of W}}$$



2d) See figure at right…roughly to scale.

3a) 30.20 μN
3b) 40.1 Gg
Common errors:

- not answering with correct sig figs (as stated in class and hmwk: conversions generally shouldn't change the sig figs of the input number)
- not answering in engineering notation
- including the prefix but forgetting it *replaces* the power of ten
- forgetting the units

4a) When adding/subtracting you keep the *leftmost column* of sig figs (not the *lowest number* of sig figs).

$$\begin{array}{r} 6.03\underline{4} \\ +\,7.\underline{7} \\ \hline 13.\underline{7}37 \end{array}$$

Answers were requested in *scientific* notation!

$$\mathbf{1.37 \times 10^{1}}$$

4b) When multiplying/dividing you keep the *lowest number* of sig figs.

$$2\underline{3}21 \rightarrow \mathbf{2.3 \times 10^{3}}$$

4c) WATCH OUT! During the subtraction in the numerator you lose two sig figs!

$$z = \frac{(5.96\underline{8}) - (6.03\underline{4})}{50.5\underline{0}}$$

When adding/subtracting you keep the *leftmost column* of sig figs.

$$z = \frac{-0.06\underline{6}}{50.5\underline{0}}$$

When multiplying/dividing you keep the *lowest number* of sig figs.

$$z = -0.001\underline{3}07$$

$$\boldsymbol{z = -1.3 \times 10^{-3}}$$

5a) Remember: conversions factors are assumed to have infinite sig figs unless otherwise noted.
Unless otherwise specified, we typically assume unit conversions do *not* affect sig figs!
First conversion:

$$5.20 \times 10^{5}\,\frac{\text{ft}}{\text{s}} \times \frac{12\text{ in}}{1\text{ ft}} \times \frac{3600\text{ s}}{1\text{ hr}} = \mathbf{2.25 \times 10^{10}\,\frac{in}{hr}}$$

5b) Here is an example of a conversion factor which only has four sig figs (as indicated by the underbar).
Second conversion.

$$0.0400\text{ mi}^{2} \cdot \frac{(160\underline{9})^{2}\text{ m}^{2}}{1^{2}\text{ mi}^{2}} = \mathbf{7.77 \times 10^{4}\ m^{2}}$$

6a) I actually solved part b first. You might go through that solution first do this first one makes more sense. Apologies…

$$[v] = [A]\frac{[g][x]}{[\rho]}$$

$$[A] = \frac{[v][\rho]}{[g][x]}$$

$$[A] = \frac{\frac{\text{m}}{\text{s}} \cdot \frac{\text{kg}}{\text{m}^3}}{\frac{\text{m}}{\text{s}^2} \cdot \text{m}}$$

$$[A] = \frac{\text{m}}{\text{s}} \cdot \frac{\text{kg}}{\text{m}^3} \cdot \frac{1}{\text{m}} \cdot \frac{\text{s}^2}{\text{m}}$$

$$[\boldsymbol{A}] = \frac{\textbf{kg} \cdot \textbf{s}}{\textbf{m}^4}$$

6b) Remember this: the units of any two terms in a sum (or difference) must have the same units. Remember: writing v implies a number with units while writing $[v]$ implies only the units.

$$[v] = [B][t]^2\sqrt{\frac{[x]}{[\pi][g]}}$$

There are no units on an exact numerical parameter in a formula (e.g. π in this formula). When doing unit math, multiplying by no units is the same as multiplying by 1!

$$[v] = [B]\sqrt{\frac{[x]}{[g]}}$$

$$[B] = \frac{[v]}{[t]^2}\sqrt{\frac{[g]}{[x]}}$$

When we finally plug in units for a term, we can drop the square brackets. Notice we do NOT drop the square brackets on $[B]$ since we did not plug in units for that term!

$$[B] = \frac{\frac{\text{m}}{\text{s}}}{\text{s}^2}\sqrt{\frac{\frac{\text{m}}{\text{s}^2}}{\text{m}}}$$

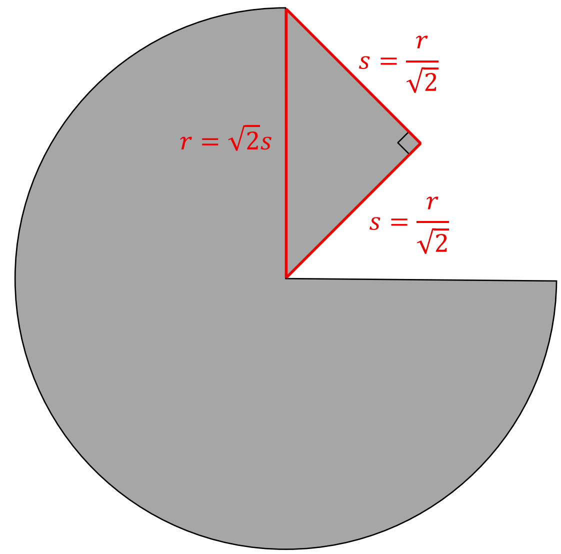
I highly recommend multiplying by the reciprocal for terms in the basement. It helps me avoid screw-ups…

$$[B] = \frac{\frac{\text{m}}{\text{s}} \cdot \frac{1}{\text{s}^2}}{\text{s}^2 \cdot \frac{1}{\text{s}^2}}\sqrt{\frac{\frac{\text{m}}{\text{s}^2} \cdot \frac{1}{\text{m}}}{\text{m} \cdot \frac{1}{\text{m}}}}$$

$$[B] = \frac{\text{m}}{\text{s}^3}\sqrt{\frac{1}{\text{s}^2}}$$

$$[\boldsymbol{B}] = \frac{\textbf{m}}{\textbf{s}^4}$$

Again, $B \neq \frac{\text{m}}{\text{s}^4}$. The variable B would have both a numerical value and units. $[B]$ = units of $B = \frac{\text{m}}{\text{s}^4}$.



7a) The combined surface area is

$$A = \frac{3}{4}A_{circle} + A_{triangle}$$

$$A = \frac{3}{4}\pi r^2 + \frac{1}{2}s^2$$

$$A = \frac{3}{4}\pi r^2 + \frac{1}{2}\left(\frac{r}{\sqrt{2}}\right)^2$$

$$A = \frac{3}{4}\pi r^2 + \frac{1}{2}\left(\frac{r^2}{4}\right)^2$$

$$A = \left(\frac{3}{4}\pi + \frac{1}{8}\right)r^2$$

$$\boldsymbol{A = 2.4\underline{8}1r^2}$$

I suspect I will use this number in the next part of the problem.
Good to show where to round while keeping the next unrounded digit.

7b) Recall: the volume of the plate is given by $V = A_{base} \times thickness$.
Furthermore, we know

$$\rho = \frac{m}{V} \quad \rightarrow \quad V = \frac{m}{\rho} \quad \rightarrow \quad A_{base} \times thickness = \frac{m}{\rho}$$

Now plug in the algebraic expression and solve for t *before* plugging in numbers (its faster this way).

$$t = \frac{m}{\rho A}$$

$$t = \frac{m}{2.4\underline{8}1\rho r^2}$$

Before plugging in umbers, think about what units you want to use and do any necessary conversions.
I chose to convert everything to grams and centimeters. You could also have chosen kg & m (or some other choice).

$$t = \frac{222\text{ g}}{2.4\underline{8}1\left(2.70\frac{\text{g}}{\text{cm}^3}\right)(9.75\text{ cm})^2}$$

$$t = \frac{222\text{ g}}{636.8\frac{\text{g}}{\text{cm}}}$$

$$t = 0.34\underline{8}6\text{ cm} = 0.00349\text{ m}$$

In engineering notation with best choice of prefix:

$$\boldsymbol{t = 3.49\text{ mm}}$$

Notice: if we start with 3 sig figs on all numbers, we *usually* end with 3 sig figs.
When doing *addition/subtraction* of two numbers close together, the number of sig figs often changes.
When doing *multiplication/division,* keeping the extra sig fig when the first digit is a 1 keeps the percent errors closer to what one finds when doing proper error analysis.