110 practice final (page 1)

1b) 3

1c) 4.00×10^{-4} N

1d) 400 μN

2a) $D = \frac{C}{A^2+B^2}$ do denom first $A^2 = (7.7)^2 = 59.29$

$B^2 = 62.41$

A^2	59.29
$+B^2$	62.41
?	121.7

$D = \frac{C}{A^2+B^2} = \frac{0.0365}{121.7} = 0.000299918$

(2a) 3.00×10^{-4} grams

(2b) 300 μgrams

(3a) $m = \rho V$

$Vol = (\pi r^2 - s^2)t$

$\Rightarrow$ (a) $m = \rho t(\pi r^2 - s^2)$ $= \rho t\left(\frac{\pi d^2}{4} - s^2\right)$

(*) problem gave $d = 2r$ — *use this

* 3c) if $\sqrt{2}s = r$ hole touches edge!

$\Rightarrow r = \frac{s}{\sqrt{2}}$

$m = \rho t s^2\left[\frac{\pi}{2} - 1\right]$

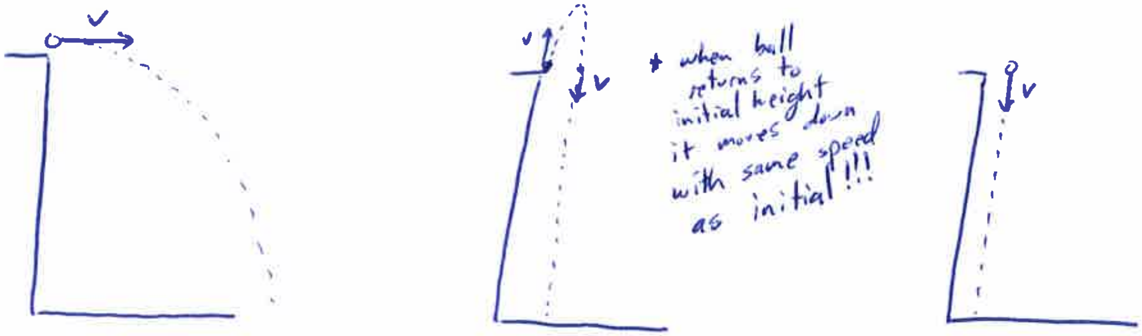
$m \approx 0.571 \rho t s^2$

(b) surface area $= 2(\pi r^2) + t(2\pi r) - 2(s^2) + t(4s)$

- $2(\pi r^2)$: 2 faces
- $t(2\pi r)$: edge
- $2(s^2)$: 2 faces of square
- $t(4s)$: inside edge (of square

110 practice final (page 2)

(4) Case 1 Case 2 Case 3



(4a) Case 3 hits first, then case 1, then case 2

(4b) notice case 2 + case 3 both hit ground with greatest speed

$\boxed{\text{in y-dir}}$

$V_f^2 = V_o^2 + 2(-g)(-h)$

$V_f = -\sqrt{v^2 + 2gh}$

- cuz going down @ time of impact

Special Note: this is valid for both case 2 + case 3

final speed is $V_f = \sqrt{v^2 + 2gh}$

*Case 1

$V_{fy}^2 = 0 + 2(-g)(-h)$

$V_{fy} = \sqrt{2gh}$

$V_{fx} = V_{ox} = v$

$V_f = \sqrt{V_{fx}^2 + V_{fy}^2}$

final speed $= \sqrt{v^2 + (\sqrt{2gh})^2}$

Same!

WOW!!! All three hit with same speed.

we will learn more about this in chapter on work + energy...

note: if (4) v 30° and (5) 30° v are included $t_2 < t_5 < t_1 < t_4 < t_3$

(4c) time between 1^{st} + last impact

long time corresponds to thrown upwards

* neg cuz going ↓ @ impact!!!

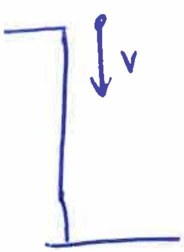
$$V_{fy} = -\sqrt{V_0^2 + 2gh} \qquad V_0 = V_{0y} = +V$$

$$V_{fy} = V_{0y} + (-g)t$$

$$-gt = V_{fy} - V_{0y}$$

$$t = \frac{V_{0y} - V_{fy}}{g} = \frac{V - (-\sqrt{V^2+2gh})}{g}$$

$$t_{long} = \frac{V + \sqrt{V^2+2gh}}{g}$$



t_{short} corresponds to thrown down. here $V_{0y} = -V$

same math as above gives

$$t = \frac{V_{0y} - V_{fy}}{g} \quad \text{where } V_{fy} = -\sqrt{V^2+2gh}$$

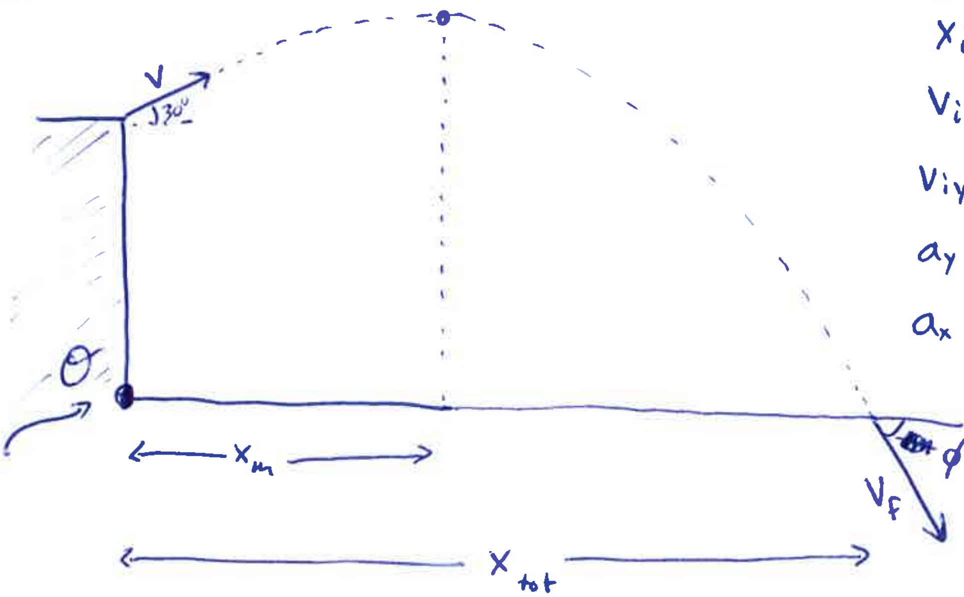
$$V_{0y} = -V$$

$$t_{short} = \frac{-V - (-\sqrt{V^2+2gh})}{g}$$

time between? $t_{long} - t_{short} = \frac{V + \sqrt{V^2+2gh}}{g} - \frac{-V + \sqrt{V^2+2gh}}{g} = \frac{2V}{g}$

$$\boxed{t_{long} - t_{short} = \frac{2V}{g}}$$

(5a)



$y_i = h$

$x_i = 0$

$V_{ix} = v\cos 30 = v\cos\theta$

$V_{iy} = v\sin 30 = v\sin\theta$

$a_y = -g$ *

$a_x = 0$

* $a = -9.8\,\frac{m}{s^2}$, $g = +9.8\,\frac{m}{s^2}$

to get y_m use $V_{fy}^2 = V_{oy}^2 + 2(-g)(y_m - y_i)$

@ y_{max} $V_{fy} = 0$ (not V_f... V_{fy}) $\Rightarrow -V_{oy}^2 = -2g(y_m - y_i)$ ← neg signs cancel!!!

$V_{oy}^2 = 2g(y_m - y_i)$

$$y_m = y_i + \frac{V_{oy}^2}{2g}$$

$$y_m = h + \frac{V_o^2 \sin^2\theta}{2g}$$

← remember, $g = +9.8\,\frac{m}{s^2}$

part a find (x,y) of max height

$$(x_m, y_m) = \left(\frac{V_o^2 \sin\theta\cos\theta}{g},\ h + \frac{V_o^2 \sin^2\theta}{2g}\right)$$

x_{max}? need <u>time</u> to max height

* y-dir

$V_{fy}^{\,0} = V_o \sin\theta + (-g)t$

$\Rightarrow t = \frac{V_o \sin\theta}{g} \Rightarrow \Delta x = V_o \cos\theta\, t$ (Δx = $x_{max} - 0$)

$\Rightarrow x_{max} = \frac{V_o^2 \sin\theta\cos\theta}{g}$

problem 5 continued

speed @ max height is not zero

$$V_{max} = \sqrt{(V_{max_x})^2 + (V_{max_y})^2}$$

this is $V_0 \cos\theta$!!! — this term is zero

(5a) $$\boxed{V_{max} = V_0 \cos\theta}$$

(5b) ~~@~~ acceleration is a vector

$\vec{a} = -g\hat{j}$ (or $\vec{a} = g$ downwards)

magnitude — direction

(5c) impact location? use entire flight ($\Delta y = -h$) ← neg cuz net downwards displacement

$$V_{fy}^2 = V_{oy}^2 + 2(-g)(-h)$$

$$V_{fy} = -\sqrt{V_{oy}^2 + 2gy}$$

neg cuz moving ↓ @ impact!!!

$$V_{fy} = V_{oy} + (-g)\,t_{tot}$$

watch signs!

$g = +9.8\frac{m}{s^2}$	$a_y = -g$

$$\Rightarrow t_{tot} = \frac{V_{oy} - V_{fy}}{g} = \frac{V_{oy} + \left(\sqrt{V_{oy}^2 + 2gy}\right)}{g}$$

minus a neg. makes this +!!!

$* V_{oy} = V_0 \sin\theta$

(5c) $$\Rightarrow \Delta x = x_F = V_{ox} t = V_0 \cos\theta \, \frac{V_{oy} + \sqrt{V_{oy}^2 + 2gy}}{g}$$

cuz $x_i = 0$

(5d) @ impact

$$V_{fx} = V_0 \cos\theta$$

$$V_{fy} = -\sqrt{V_0^2 \sin^2\theta + 2gh}$$

$$V_f = \sqrt{V_{fx}^2 + V_{fy}^2} = \sqrt{V_0^2 \cos^2\theta + \left(\sqrt{V_0^2 \sin^2\theta + 2gh}\right)^2}$$

(V_f ↑ speed)

$$= \sqrt{\underbrace{V_0^2 \cos^2\theta + V_0^2 \sin^2\theta}_{V_0^2 \underbrace{(\cos^2\theta + \sin^2\theta)}_{1}} + 2gh}$$

$$V_f = \sqrt{V_0^2 + 2gh}$$

same as every case in ~~part~~ problem (4)!

WOW!

(6a) $X_i = -4.0\,m$

$V_i = -2.0\,\frac{m}{s}$

if $t = 4.0\,s$ then $x_f = 0$

assuming const. accel

$$X_f = X_i + V_{ix}t + \frac{1}{2}a_x t^2$$

$$\Rightarrow \frac{1}{2}a_x t^2 = X_f - X_i - V_{ix}t$$

$$a_x = \frac{X_f - X_i - V_{ix}t}{\frac{1}{2}t^2} = \frac{0-(-4)-(-2)(4.0)}{\frac{1}{2}(4.0)^2} = 1.5\,\frac{m}{s^2}$$

✓units $\frac{m - m - \frac{m}{s}\cdot s}{s^2} = \frac{m}{s^2}$

(a) $a = 1.5\,\frac{m}{s^2}$

(b) $t_{rev} = 1.333\,s$

(c) $V_{4sec} = 4.0\,\frac{m}{s}$

(b) $\cancel{V_f}^{0} = V_i + a t$ when $V_f = 0$ object reverses direction

$$\frac{-V_i}{a} = t_{reverse} = \frac{-(-2.0\,\frac{m}{s})}{+1.5\,\frac{m}{s^2}} = 1.33\underline{3}\,s$$

(c) when $t = 4.0$, what is V_f?

$$V_f = V_i + at = (-2.0\,\tfrac{m}{s}) + (1.5\,\tfrac{m}{s^2})(4s) = 4.0\,\tfrac{m}{s}$$

(d) X_f is $x(t) = x_i + v_{ix}t + \frac{1}{2}a_x t^2$

$$x(t) = -4.0\,m - (2.0\,\tfrac{m}{s})t + (0.75\,\tfrac{m}{s^2})t^2$$

V_f is $V(t) = V_{ix} + a_x t^2$

$$V(t) = -2.0\,\tfrac{m}{s} + 1.5\,\tfrac{m}{s^2}\,t^2$$

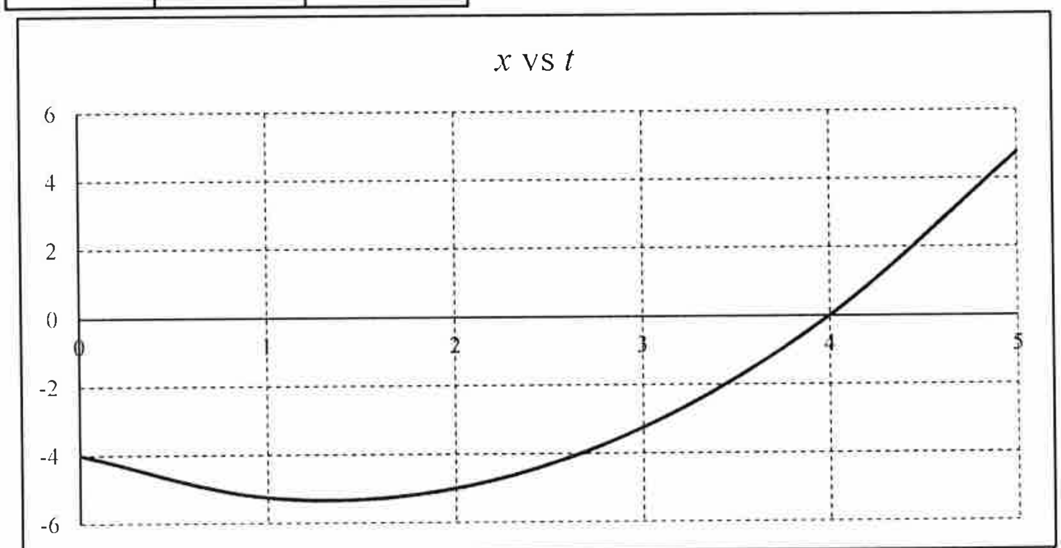
Plots for problem 6

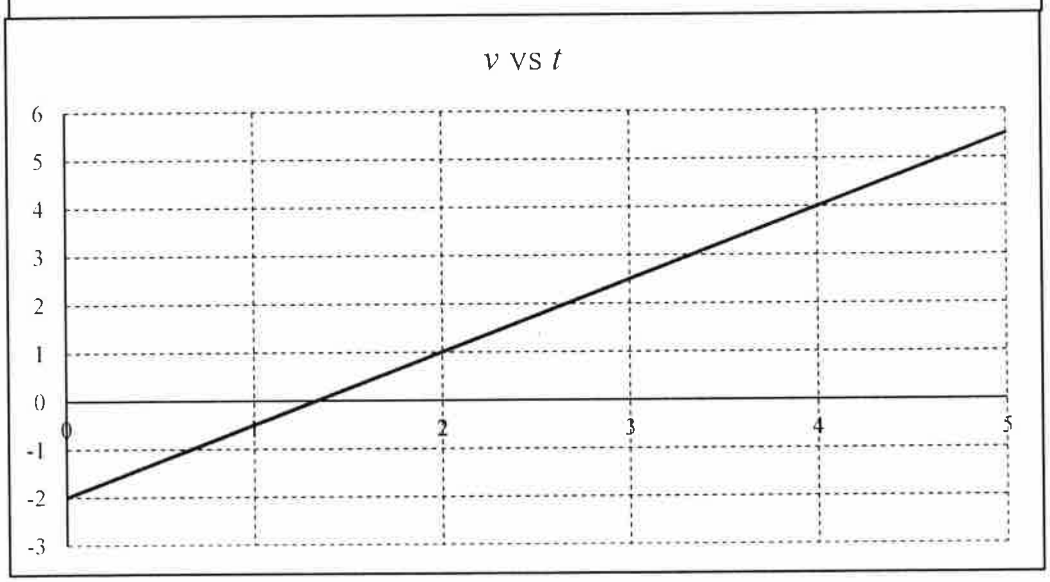
if you get stuck...

make a little table + plot the points!

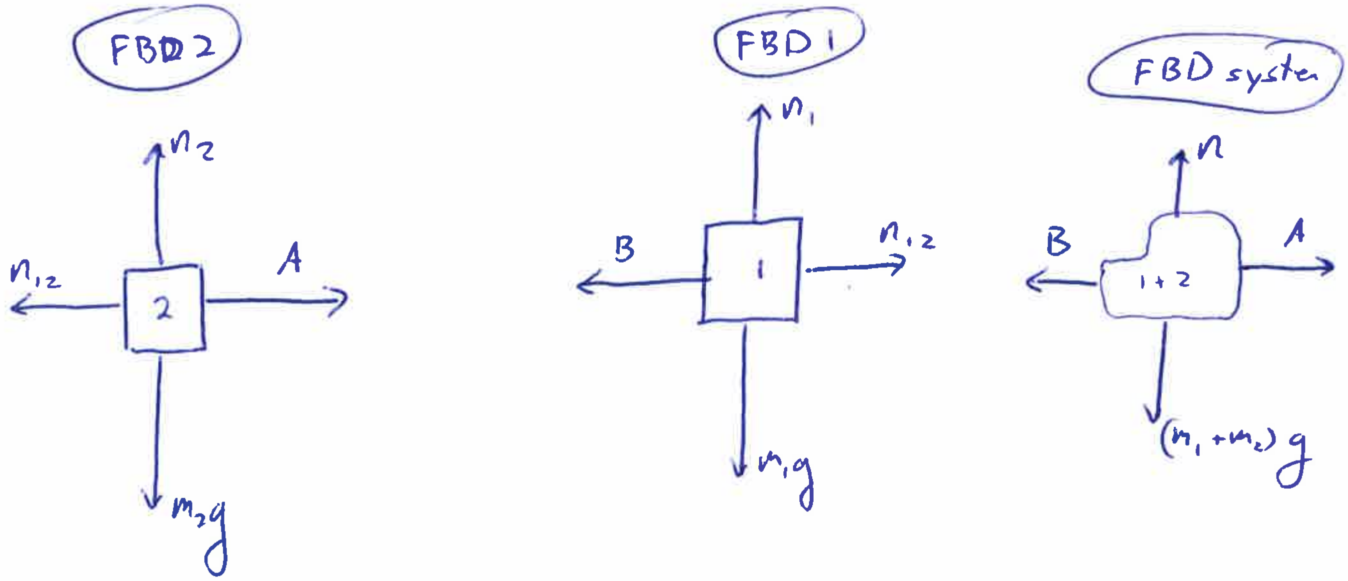
x_i (m)	v_i (m/s)	a (m/s^2)
-4	-2	1.5

t (s)	x_f (m)	v_i (m/s)
0	-4	-2
1	-5.25	-0.5
2	-5	1
3	-3.25	2.5
4	0	4
5	4.75	5.5





7a



7a) + 7b) if blocks accel. to the right

$A > n_{12}$

$n_{12} > B$

$A > B$

7c + 7d @ const speed $a = 0$ => $A = n_{12} = B$

Action	Reaction
(1) Block 2 ~~to~~ pushes Right on 1 using normal force n_{12}	~~B~~ 1 pushes Left on 2 using normal force n_{12}
(2) Floor pushes up on 1 using normal force n_1	1 pushes down ~~on~~ on floor using normal force n_1
(3) Bob pushes Left on 1 using some kind of push/normal	1 pushes Right on Bob with same kind of push/normal force.
(4) Earth pulls down on 1 using gravitational force	1 pulls up on Earth using gravitational force

(8a) horizontal axis represents time

(8b) assume horizontal is $t(s)$ while vertical is $v(m/s)$

(8c) slows down on vt plot $\Rightarrow$ $v>0$ and $a<0$ (positive values of v, neg slope)

OR

$V<0$ and $a>0$ (neg. values of v, pos slope)

between 0 – 0.33 s and 4 – 6 s slowing

(8d) when $v=0$ on vt plot this occurs when line crosses time axis

(note: on xt plot occurs when slope = 0)

(8e) speeding up implies v and a have same sign

from about 0.33 → 2 sec

* note: the rate @ which speeds changes is ~~less~~ about $\approx -60 \frac{m}{s^2}$ between ~~0 → 1.0~~ 0.33 → 1.0 s

but only $-20 \frac{m}{s^2}$ between 1.0 s → 2.0 s

the acceleration magnitude decreases...

but still $v<0$ and $a<0$ so still speeding up ... just @ a lesser rate

also $v>0$ + $a>0$ between 6 → 7 s

(8f) const speed $\Rightarrow$ vt plot is flat (or xt plot linear) ∴ slope of vt is a flat $\Rightarrow$ slope $= a = 0$!

vt plot flat between 2 + 4 s AND 7 + 8 s

(8g) ~~slope = a~~ (slope of vt) = a

(slope of xt) = v

(slope of vt) $= \frac{rise}{run} = \frac{90\ m/s}{3\ s} = 30 \frac{m}{s^2}$ *remember units!

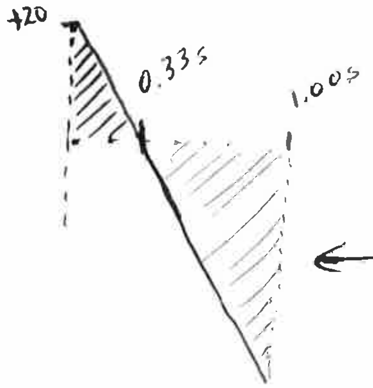
between 4 + 7 sec all points have same slope!

(8h) slope greatest magnitude between 0 – 1 s

2nd Qh?

Δx = Area

Areas above time axis are +
" below " " " −



$A_1 = \frac{1}{2}(0.33s)(+20\frac{m}{s}) = 3.3m$ ← units $\frac{m}{s} \cdot s = m$!!!
base

← $A_2 = \frac{1}{2}\left[(1.00-0.33) \text{ seconds}\right]\left[-40\frac{m}{s}\right] \approx -13.4 m$

base ... heigh ... notice $\frac{m}{s} \cdot s = m$!!!

$\Delta x = A_1 + A_2 = -10.1 m \approx -10m$

$x_f - x_i = -10m$

since @ origin after 1 sec $x_f = 0$

$\Rightarrow 0 - x_i = -10m$

$\Rightarrow x_i \approx 10m$