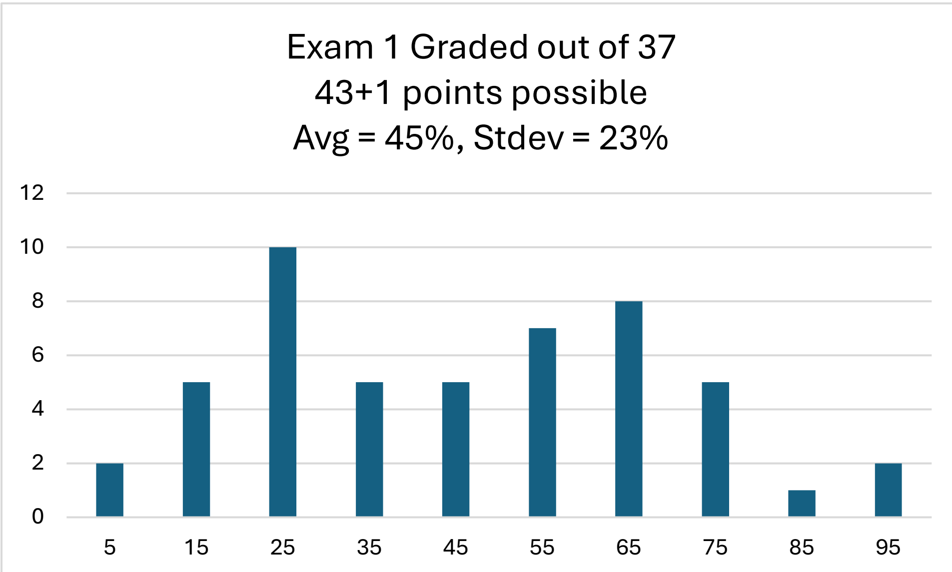
161fa25t1bSoln

Distribution on this page.
Solutions begin on the next page.



161 fa25 t1b = yellow test

1a) Outer circumference relates to outer diameter using

$$C_{outer} = 2\pi R_{outer} = \pi D_{outer} \quad \rightarrow \quad D_{outer} = \frac{C_{outer}}{\pi} = \frac{72.4 \text{ cm}}{\pi} = 23.\underline{0}46 \text{ cm} = \mathbf{23\underline{0}\ mm}$$

Note: it is crucial to keep an extra digit beyond the rounding digit for work in subsequent steps.

1b) Standard conversion. Watch out for the cubed units…remember to cube the numbers in the conversion factor.

$$1.795 \frac{\text{kg}}{\text{m}^3} \times \frac{1^3 \text{ m}^3}{100^3 \text{ cm}^3} \times \frac{1000 \text{ g}}{1 \text{ kg}} = \mathbf{1.795 \times 10^{-3}} \frac{\mathbf{g}}{\mathbf{cm^3}}$$

No need to keep the unrounded digit on this one…we won't be using this number again.

1c) Use density to convert volume to mass. Notice: $d_{inner} = 228.1 \text{ mm} = 22.81 \text{ cm} = 0.2281 \text{ m}$.

$$\rho_{air} = \frac{m_{air}}{V_{air}} \quad \rightarrow \quad m_{air} = \rho_{air} V_{air}$$

$$m_{air} = \rho_{air}\left(\frac{4}{3}\pi r_{air}^3\right)$$

$$m_{air} = \frac{\pi}{6}\rho_{air} d_{inner}^3$$

$$m_{air} = \frac{\pi}{6}\left(1.795 \times 10^{-3} \frac{\text{g}}{\text{cm}^3}\right)(22.\underline{8}1 \text{ cm})^3$$

$$\boldsymbol{m_{air}} = \mathbf{11.1\underline{5}4\ g}$$

1d) Density of the thin shell material is given by

$$\rho_{shell} = \frac{m_{shell}}{V_{shell}}$$

Be careful to ignore the mass of the air!

$$m_{shell} = m_{total} - m_{air} = 568 \text{ g} - 11.1\underline{5}4 \text{ g} = 55\underline{6}.8 \text{ g}$$

Correctly compute the volume of material comprising the shell.

$$V_{shell} = V_{outer} - V_{inner}$$

$$V_{shell} = \frac{4}{3}\pi(r_{outer}^3 - r_{inner}^3)$$

$$V_{shell} = \frac{\pi}{6}(d_{outer}^3 - d_{inner}^3)$$

$$V_{shell} = \frac{\pi}{6}\left[(23.\underline{0}46 \text{ cm})^3 - (22.\underline{8}1 \text{ cm})^3\right]$$

$$V_{shell} = \frac{\pi}{6}[12\underline{2}40 - 118\underline{6}8]\text{cm}^3$$

$$V_{shell} = \frac{\pi}{6}[\underline{3}72]\text{cm}^3$$

$$V_{shell} = \underline{1}95 \text{ cm}^3$$

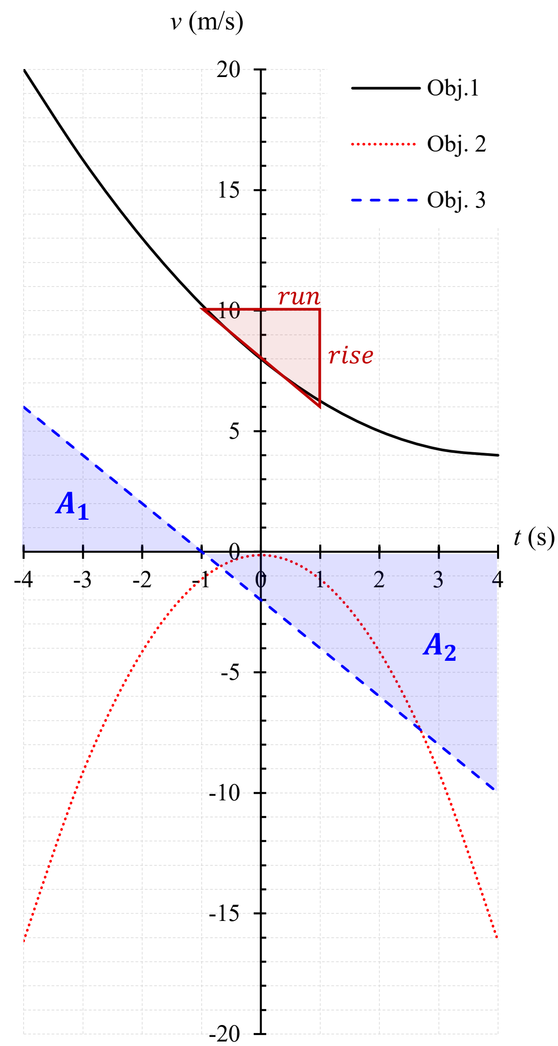
Finally, plug this garbage into the density equation.

$$\rho_{shell} = \frac{m_{shell}}{V_{shell}}$$

$$\rho_{shell} = \frac{55\underline{6}.8 \text{ g}}{\underline{1}95 \text{ cm}^3}$$

$$\rho_{shell} = \underline{2}.86 \frac{\text{g}}{\text{cm}^3}$$

$$\boldsymbol{\rho_{shell}} = \mathbf{3} \frac{\mathbf{g}}{\mathbf{cm^3}}$$



2a) Acceleration is given by the slope on a vt-plot.
Use times just before & just after the time of interest for accuracy.

$$a = slope \approx \frac{rise}{run} = \frac{\left(6.0\frac{\text{m}}{\text{s}}\right) - \left(10.0\frac{\text{m}}{\text{s}}\right)}{(1.0\text{ s}) - (-1.0\text{ s})} = \mathbf{-2.0\,\frac{m}{s^2}}$$

Did you remember to include units?

2b) Displacement is given by area under the curve on a vt-plot.
Object 1 has positive area under the curve for the entire time.

2c) First determine *displacements* using area under the curve.

$$A_1 = \frac{1}{2}(3.0\text{ s})\left(6.0\frac{\text{m}}{\text{s}}\right) = 9.\underline{0}\text{ m}$$

$$A_2 = \frac{1}{2}(5.0\text{ s})\left(-10.0\frac{\text{m}}{\text{s}}\right) = -2\underline{5}.0\text{ m}$$

Total distance traveled is thus:

$$distance = |A_1| + |A_2|$$

$$\boldsymbol{distance = 3\underline{4}\text{ m}}$$

2d) On a vt-plot, at rest implies the value of velocity is zero. This is straightforward on a vt-plot.

At $t = -1.0$ s object 3 has $v_2 = 0$.

Note: on at xt-plot you'd look for points of zero *slope*.

2e) Initially object 3 has positive velocity, then negative velocity.

Object 3 first moves forwards, then moves backwards.

2f) At time $t = -4.0$ s the *velocity* of object 2 is $+6.0\frac{\text{m}}{\text{s}}$ (which corresponds to *speed* $6.0\frac{\text{m}}{\text{s}}$).
At time $t = -1.0$ s the *velocity* of object 2 is $0\frac{\text{m}}{\text{s}}$ (which corresponds to *speed* $0\frac{\text{m}}{\text{s}}$).
At time $t = +4.0$ s the *velocity* of object 2 is $-10.0\frac{\text{m}}{\text{s}}$ (which corresponds to *speed* $10.0\frac{\text{m}}{\text{s}}$).

Object 3 first slows down, then speeds up.

3a) *Think:* you could write 8.77 m in the box…but use the *unrounded* result in subsequent calculations.

$$r = \|\vec{r}\| = \sqrt{(-6.00\text{ m})^2 + (5.00\text{ m})^2 + (4.00\text{ m})^2} = \mathbf{8.7\underline{7}496\ m}$$

3b) Use the dot product (not the cross product).

$$\vec{r}\cdot\hat{\imath} = \vec{r}\cdot\hat{\imath}$$

$$(r)(1)\cos\theta_x = r_x(1)$$

$$\theta_x = \cos^{-1}\left(\frac{r_z}{r}\right)$$

$$\theta_x = \cos^{-1}\left(\frac{-6.00\text{ m}}{8.7\underline{7}5\text{ m}}\right)$$

$$\boldsymbol{\theta_x = 133.1°}$$

If you want to know why I like to keep the extra sig fig when the first digit is 1, ask me in office hours.
Short explanation: It many instances it keeps percent errors in better agreement with true error propagation formulas.

3c) Wheel of pain is WAAAAAY faster than a 3×3 determinant…

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$\vec{\tau} = \left[\left(-6.00\hat{\imath} + 5.00\hat{\jmath} + 4.00\hat{k}\right)\text{ m}\right]\times\left[(-3.00\hat{\jmath})\text{ N}\right]$$

We know $\hat{\jmath}\times\hat{\jmath} = 0$…ignore those terms. Also, clean things up by putting units at the very end.

$$\vec{\tau} = \left[(-6.00\hat{\imath})\times(-3.00\hat{\jmath}) + \left(4.00\hat{k}\right)\times(-3.00\hat{\jmath})\right]\text{ N}\cdot\text{m}$$

$$\vec{\tau} = \left[18.00(\hat{\imath}\times\hat{\jmath}) - 12.00\left(\hat{k}\times\hat{\jmath}\right)\right]\text{ N}\cdot\text{m}$$

$$\vec{\tau} = \left[18.00\left(+\hat{k}\right) - 12.00(-\hat{\imath})\right]\text{ N}\cdot\text{m}$$

$$\boldsymbol{\vec{\tau} = \left(+12.00\hat{\imath} + 18.00\hat{k}\right)\ \mathbf{N\cdot m}}$$

Check: did you remember to put in standard order ($\hat{\imath}$ term then $\hat{\jmath}$ term then $\hat{k}$ term) and include units?

4) The units of any two terms in a sum or difference match. Constants such as $\frac{3}{4}$ & π have no effect on the units.

$$[F] = [Q]\sqrt{\frac{[m][a]}{[v][x]}}$$

Isolate $[E]$ with some cross-multiplication…

$$[Q] = [F]\sqrt{\frac{[v][x]}{[m][a]}}$$

Once we start plugging in units, we can drop the square brackets.

$$[Q] = \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}\sqrt{\frac{\frac{\mathrm{m}}{\mathrm{s}} \cdot \mathrm{m}}{\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}}}$$

It is easy to make mistakes with so many fractions. I recommend using horizontal fraction bars and explicitly showing the step of multiplying by the reciprocal for fractions in the basement.

$$[Q] = \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}\sqrt{\frac{\frac{\mathrm{m}}{\mathrm{s}} \cdot \mathrm{m} \cdot \frac{\mathrm{s}^2}{\mathrm{kg} \cdot \mathrm{m}}}{\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2} \cdot \frac{\mathrm{s}^2}{\mathrm{kg} \cdot \mathrm{m}}}}$$

$$[Q] = \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}\sqrt{\mathrm{m} \cdot \frac{\mathrm{s}}{\mathrm{kg}}}$$

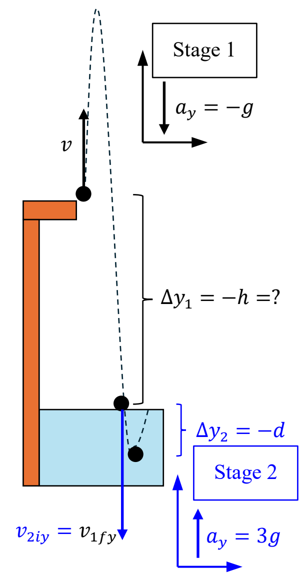
To simplify this, bring terms outside the square root to the inside (or eliminate the root using fractional exponents).

$$[Q] = \sqrt{\left(\mathrm{kg}^2 \cdot \frac{\mathrm{m}^2}{\mathrm{s}^4}\right)\mathrm{m} \cdot \frac{\mathrm{s}}{\mathrm{kg}}}$$

$$[\boldsymbol{Q}] = \sqrt{\mathbf{kg} \cdot \frac{\mathbf{m}^3}{\mathbf{s}^3}} = \mathbf{kg}^{1/2} \cdot \frac{\mathbf{m}^{3/2}}{\mathbf{s}^{3/2}}$$

WATCH OUT! I am picky about those pesky square brackets in your final results!

Again, $\boldsymbol{Q} \neq \sqrt{\mathbf{kg} \cdot \frac{\mathbf{m}^3}{\mathbf{s}^5}}$. The variable $\boldsymbol{Q}$ has both a numerical value and units. $[\boldsymbol{Q}]$ = units of $\boldsymbol{Q} = \sqrt{\mathbf{kg} \cdot \frac{\mathbf{m}^3}{\mathbf{s}^3}}$.



5a) Before trying to answer anything, I'm going to draw a sketch. Figure not to scale. Motion is purely vertical but I included a slight x-component to make things easier to label.

During the flight (in air) we were told air resistance was negligible.
This is freefall (gravity is the only force acting on the ball).
For freefall near earth's surface, we assume gravitational force is constant.
Acceleration is constant & directed downwards (negative direction in coordinate system).

The best answer is "`Always negative`".

5b) The ball spends more time travelling downwards.

5c) If you drew separate pictures for each stage that's totally reasonable.
We know information about stage 2; I will start with that part of the problem

$$v_{2fy}^2 = v_{2iy}^2 + 2a_{2y}\Delta y_2$$

Think: at max depth the ball is reversing direction. This implies $v_{2fy} = 0$.

$$0 = v_{2iy}^2 + 2a_{2y}\Delta y_2$$

$$v_{2iy}^2 = -2a_{2y}\Delta y_2$$

$$v_{2iy} = \pm\sqrt{-2a_{2y}\Delta y_2}$$

Think: use the *minus* sign; the ball is moving *downwards* as it begins stage 2 (ends stage 1).

$$v_{2iy} = -\sqrt{-2(3g)(-d)}$$

$$v_{2iy} = -\sqrt{6gd}$$

Now do a stage 1 problem with using the fact $v_{1fy} = v_{2iy}$.

$$v_{1fy}^2 = v_{1iy}^2 + 2a_{1y}\Delta y_1$$

$$\Delta y_1 = \frac{v_{1fy}^2 - v_{1iy}^2}{2a_{1y}}$$

$$-h = \frac{\left(-\sqrt{6gd}\right)^2 - v^2}{2(-g)}$$

$$\boldsymbol{h = \frac{6gd - v^2}{2g}}$$

$$\boldsymbol{h = 3d - \frac{v^2}{2g}}$$

Either of these last two forms seems simplified a reasonable amount.

6a) Checking the units:

$$[k] = \frac{[a]}{[v]^3} = \frac{\frac{\mathrm{m}}{\mathrm{s}^2}}{\frac{\mathrm{m}^3}{\mathrm{s}^3}} = \frac{\mathrm{m}}{\mathrm{s}^2} \cdot \frac{\mathrm{s}^3}{\mathrm{m}^3} = \boldsymbol{\frac{\mathrm{s}}{\mathrm{m}^2}}$$

6b) Use separation of variables.

$$\frac{dv}{dt} = kv^3$$

$$\frac{dv}{v^3} = k\,dt$$

$$\int_i^f v^{-3}\,dv = \int_i^f k\,dt$$

$$\left[\frac{v^{-2}}{-2}\right]_i^f = k\left(t_f - t_i\right)$$

Shift $t_i \to 0$ & $t_f \to t$. Cross multiply by -2.

$$[v^{-2}]_i^f = -2kt$$

$$\frac{1}{v_f^2} - \frac{1}{(-v_0)^2} = -2kt$$

Think: we were told $v_i = -v_0$ in the problem statement!

$$\frac{1}{v_f^2} = \frac{1}{v_0^2} - 2kt$$

$$v_f^2 = \frac{1}{\frac{1}{v_0^2} - 2kt}$$

$$v_f^2 = \frac{1}{\frac{1}{v_0^2} - 2kt} \cdot \frac{v_0^2}{v_0^2}$$

$$v_f^2 = \frac{v_0^2}{1 - 2v_0^2 kt}$$

$$v_f = \pm\sqrt{\frac{v_0^2}{1 - 2v_0^2 kt}}$$

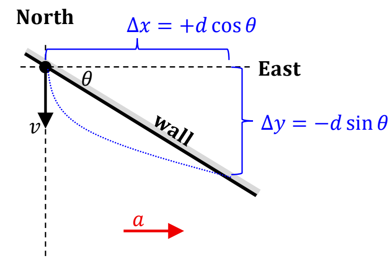
Think: we were told the initial velocity was negative.

Notice we require the *negative* root (plug in $t = 0$ to check)!

Identify v_f as $v(t)$.

$$\boldsymbol{v(t) = -\frac{v_0}{\sqrt{1 - 2v_0^2 ct}}}$$

7) Made it worth 5…This problem is nearly identical to the ski slope problem!



$\Delta x = +d\cos\theta$	$\Delta y = -d\sin\theta$
$v_{ix} = 0$	$v_{iy} = -v$
$v_{fx} = ?$	$v_{fy} = -v$
$a_x = +a$	$a_y = 0$
$t = ?$	

Listing the kinematic equations and plugging in given parameters.

$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$ $d\cos\theta = \frac{1}{2}(a)t^2$ $d\cos\theta = \frac{1}{2}at^2$	$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ $-d\sin\theta = -vt$ $t = \frac{d\sin\theta}{v}$

Notice we can determine time from the Δy equation and sub that into the Δx equation!

$$d\cos\theta = \frac{1}{2}a\left(\frac{d\sin\theta}{v}\right)^2$$

$$d\cos\theta = \frac{1}{2}a\frac{d^2\sin^2\theta}{v^2}$$

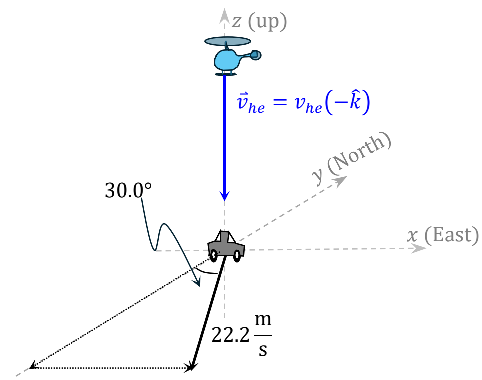
$$2v^2\cos\theta = ad\sin^2\theta$$

$$\boldsymbol{d = \frac{2v^2\cos\theta}{a\sin^2\theta}}$$

Another nice way to write this is shown below. Either of these last two forms is fine.

$$\boldsymbol{d = \frac{2v^2}{a\sin\theta\tan\theta}}$$

8a) The angle is east of south (to the y-axis)! Watch minus signs…

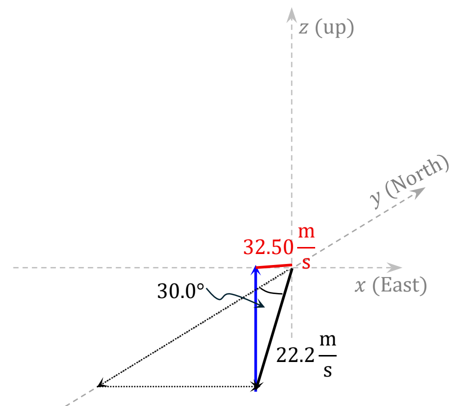


$$\vec{v}_{ce} = (+22.2 \sin 30.0°\,\hat{\imath} - 22.2 \cos 30.0°\,\hat{\jmath})\frac{\text{m}}{\text{s}}$$

$$\boldsymbol{\vec{v}_{ce} = \left(+11.\underline{1}0\hat{\imath} - 19.\underline{2}3\hat{\jmath}\right)\frac{m}{s}}$$

Since we plan to use this result later, I underline the proper rounding digit instead of giving my rounded final answer in the box. Since the first digit of each term is 1, I keep an extra digit.

8b) Watch the order of the subscripts carefully!
I will leave off units until the last step to reduce clutter.
To get the known *speed* v_{ch} involved, I first write out the *vector* $\vec{v}_{ch}$ then take the magnitude.



$$\vec{v}_{ch} = \vec{v}_{ce} + \vec{v}_{eh} +$$

$$\vec{v}_{ch} = \vec{v}_{ce} - \vec{v}_{he}$$

$$\vec{v}_{ch} = \left(+11.\underline{1}0\hat{\imath} - 19.\underline{2}26\hat{\jmath}\right) - \left(-v_{he}\hat{k}\right)$$

$$\vec{v}_{ch} = \left(+11.\underline{1}0\hat{\imath} - 19.\underline{2}26\hat{\jmath} + v_{he}\hat{k}\right)$$

$$v_{ch} = \sqrt{11.\underline{1}0^2 + 19.\underline{2}26^2 + v_{he}^2}$$

$$v_{ch}^2 = 49\underline{2}.8 + v_{he}^2$$

Note: if you were clever you noticed a right triangle in the figure and skipped to this step!

$$v_{he}^2 = v_{ch}^2 - 49\underline{2}.8$$

$$v_{he} = \pm\sqrt{v_{ch}^2 - 49\underline{2}.8}$$

Speed should be positive; use the positive root.

$$v_{he} = \sqrt{32.5^2 - 49\underline{2}.8}$$

$$\boldsymbol{v_{he} = 23.\underline{7}4\frac{m}{s}}$$

8c) To get direction use

$$\hat{v}_{ch} = \frac{\vec{v}_{ch}}{v_{ch}}$$

$$\hat{v}_{ch} = \frac{\left(+11.\underline{1}0\hat{\imath} - 19.\underline{2}26\hat{\jmath} + 23.74\hat{k}\right)\frac{\text{m}}{\text{s}}}{32.50\frac{\text{m}}{\text{s}}}$$

$$\boldsymbol{\hat{v}_{ch} = +0.342\hat{\imath} - 0.592\hat{\jmath} + 0.730\hat{k}}$$

Notice the units drop out when computing a unit vector. Direction is a dimensionless quantity.