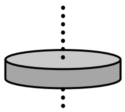
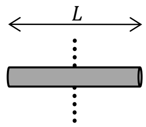
161 Fall 2025 Test 3a Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

$V_{sphere} = \frac{4}{3}\pi R^3$	$V_{box} = LWH$	$V_{cyl} = \pi R^2 H$	$\rho = \frac{M}{V}$
$A_{sphere} = 4\pi R^2$	$V = (A_{base}) \times (height)$	$A_{circle} = \pi R^2$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$C = 2\pi R$	$A_{rect} = LW$	$A_{CylSide} = 2\pi RH$	
$160\underline{9}\text{ m} = 1\text{ mi}$	$12\text{ in} = 1\text{ ft}$	$60\text{ s} = 1\text{ min}$	$1000\text{ g} = 1\text{ kg}$
$2.54\text{ cm} = 1\text{ in}$	$1\text{ cc} = 1\text{ cm}^3 = 1\text{ mL}$	$60\text{ min} = 1\text{ hr}$	$100\text{ cm} = 1\text{ m}$
$1\text{ cm} = 10\text{ mm}$	$1\text{ yard} = 3\text{ ft}$	$3600\text{ s} = 1\text{ hr}$	$1\text{ km} = 1000\text{ m}$
$1\text{ furlong} = 220\text{ yards}$	$528\underline{0}\text{ ft} = 1\text{ mi}$	$24\text{ hrs} = 1\text{ day}$	$1\text{ rev} = 2\pi\text{ rad} = 360°$
$g = 9.8\ \frac{\text{m}}{\text{s}^2}$	$G = 6.67 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	$P_0 = 1.0 \times 10^5\text{ Pa}$	$1\text{ eV} = 1.60\underline{2} \times 10^{-19}\text{ J}$
$1\text{ N} = 1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$	$1\text{ J} = 1\text{ N}\cdot\text{m}$	$1\text{ Pa} = 1\ \frac{\text{N}}{\text{m}^2}$	$m_e = 9.11 \times 10^{-31}\text{ kg}$
$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$	$v_{fx}^2 = v_{ix}^2 + 2a_x(\Delta x)$	$v_{fx} = v_{ix} + a_x t$	$r = \sqrt{x^2 + y^2}$
$\vec{A}\cdot\vec{B} = AB\cos\theta_{AB}$	$\lVert\vec{A}\times\vec{B}\rVert = AB\sin\theta_{AB}$	$\sin(A \pm B)$ $= \sin A\cos B \pm \cos A \sin B$	$\cos(A \pm B)$ $= \cos A\cos B \mp \sin A\sin B$
$\vec{v}_{ae} + \vec{v}_{eb} = \vec{v}_{ab}$	$\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$	$\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$	$\omega = 2\pi f = 2\pi/\mathbb{T}$
$a_{tan} = r\alpha$	$a_c = \frac{v^2}{r} = r\omega^2$	$\vec{a} = a_r\hat{r} + a_{tan}\hat{\theta}$	$\vec{a} = a_c(-\hat{r}) + a_{tan}\hat{\theta}$
$\Sigma\vec{F} = m\vec{a}$	$f \le \mu n$	$\vec{F}_G = \frac{GmM}{r^2}(-\hat{r})$	$U_G = -\frac{GmM}{r}$
$TKE = \frac{1}{2}mv^2$	$RKE = \frac{1}{2}I\omega^2$	$U_S = SPE = \frac{1}{2}kx^2$	$U_G = GPE = mgh$
$E_i + W_{\substack{non-con\\ or\ ext}} = E_f$	$\Delta KE = W_{ext.\&\,non-con}$	$W = Fd\cos\theta = F_{\parallel}d$	$W = \int F_x dx$
$\Delta U = -W = -\int_i^f \vec{F}\cdot d\vec{s}$	$F_x = -\frac{d}{dx}U(x)$	$\mathcal{P}_{inst} = \frac{dE}{dt} = \vec{F}\cdot\vec{v}$	$\mathcal{P}_{avg} = \frac{\Delta E}{\Delta t} = \frac{Work}{time}$
$\vec{J} = \Delta\vec{p} = \vec{F}\Delta t$	$\vec{p} = m\vec{v}$	$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$	$x_{\text{CM}} = \frac{\int x\,dm}{\int dm}$
$\vec{\tau} = \vec{r}\times\vec{F}$	$\Sigma\vec{\tau} = I\vec{\alpha}$	$L = I\omega = mvr_{\perp}$	$\mathcal{P}_{inst} = \vec{\tau}\cdot\vec{\omega}$
$s = r\Delta\theta$	$v = r\omega$	$a_{tan} = r\alpha$	$a_c = \frac{v^2}{r} = r\omega^2$
$I_{\parallel axis} = I_{\text{CM}} + md^2$	$I_{zz} = I_{xx} + I_{yy}$	$I = \int r^2 dm$	$\frac{F}{A} = E\frac{\Delta L}{L_0}$
$P = \frac{F}{A}$	$P_{gauge} = P_{abs} - P_{ambient}$	$B = \rho_f V_{disp} g$	$A_1v_1 = A_2v_2$
$P(h) = P_0 + \rho g h$	$P + \frac{1}{2}\rho v^2 + \rho g h$ $= \text{constant}$	$R = \frac{\pi r^4 \Delta P}{8\eta L}$	$F = \eta A\frac{\Delta v_x}{\Delta y}$

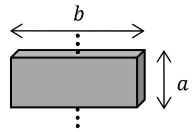
Prefix	Abbreviation	$10^?$		Prefix	Abbreviation	$10^?$
Giga	G	10^9		milli	m	10^{-3}
Mega	M	10^6		micro	µ	10^{-6}
kilo	k	10^3		nano	n	10^{-9}
centi	c	10^{-2}		pico	p	10^{-12}
				femto	f	10^{-15}



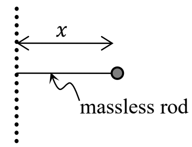
$I_{disk} = \frac{1}{2} mR^2$



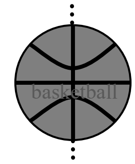
$I_{\substack{thin \\ rod}} = \frac{1}{12} mL^2$



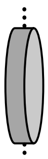
$I_{\substack{thin \\ plate}} = \frac{1}{12} mb^2$



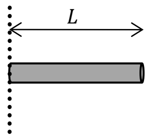
$I_{\substack{pnt \\ mass}} = mx^2$



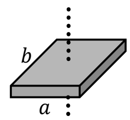
$I_{\substack{spherical \\ shell}} = \frac{2}{3} mR^2$



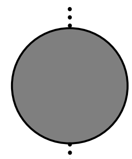
$I_{disk} = \frac{1}{4} mR^2$



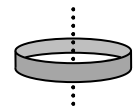
$I_{\substack{thin \\ rod}} = \frac{1}{3} mL^2$



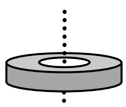
$I_{\substack{thin \\ plate}} = \frac{1}{12} m(b^2 + a^2)$



$I_{\substack{solid \\ sphere}} = \frac{2}{5} mR^2$



$I_{\substack{thin \\ ring}} = mR^2$

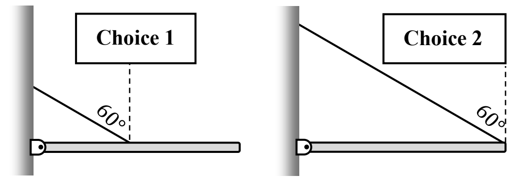


$I_{\substack{thick \\ ring}} = \frac{1}{2} m(R_{inner}^2 + R_{outer}^2)$

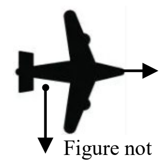
Name:____

1) A beam is connected to an ideal pivot and supported by a cable. An engineer has two choices to support the beam as shown at right. Which choice creates *more* tension in the support cable? Circle the best answer.

1	2	It's a tie	Impossible to determine without more info



A spaceship (shaped like an airplane) flies in deep space with constant velocity. Air resistance and gravitational forces are negligible. A gunner in the ship fires a single shot. The figure at right shows a top view of the instant just *after* the bullet was fired. The following questions refer to this scenario. In this after picture, assume the bullet moves perfectly in the $-\hat{j}$ while the ship moves perfectly in the $\hat{\imath}$ direction. This is *just* after the bullet was fired (1 nanosecond after the bullet left the gun barrel). Figure not to scale.



2a) Just before the bullet was fired, which best describes the direction of motion of the ship?

$+\hat{j}$	$+\hat{\imath}$	Mostly $+\hat{j}$ but also slightly $+\hat{\imath}$	Mostly $+\hat{\imath}$ but also slightly $+\hat{j}$	Mostly $-\hat{j}$ but also slightly $+\hat{\imath}$	Mostly $-\hat{\imath}$ but also slightly $+\hat{j}$	None of the previous answers is correct
$-\hat{j}$	$-\hat{\imath}$	Mostly $+\hat{j}$ but also slightly $-\hat{\imath}$	Mostly $+\hat{\imath}$ but also slightly $-\hat{j}$	Mostly $-\hat{j}$ but also slightly $-\hat{\imath}$	Mostly $-\hat{\imath}$ but also slightly $-\hat{j}$	

2b) While the bullet is being fired, which experiences a larger acceleration: the bullet or the ship?

The bullet	The ship	It's a tie	Impossible to determine without more info

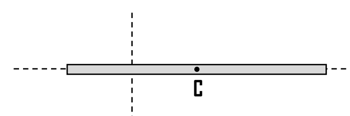
2c) While the bullet is being fired, which experiences a larger change in momentum: the bullet or the ship?

The bullet	The ship	It's a tie	Impossible to determine without more info

2d) While the bullet is being fired, which experiences a larger force: the bullet or the ship?

The bullet	The ship	It's a tie	Impossible to determine without more info

A rod has non-uniform mass density given by $\lambda = \alpha x^2$ where α is a constant. The left end of the rod is distance d from the origin. The rod has total length $4d$. The middle of the rod is labeled as point **C**. This figure is drawn approximately to scale.



3a) What units are assumed on the constant α? Write your answer in the box.

3c	

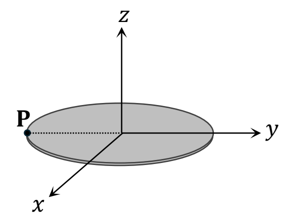
3b) Which end of the rod has more mass? Circle the best answer.

Left end	Right End	It's a tie	Impossible to determine without more info

3c) Which best describes the location of the center of mass of this rod? Circle the best answer.

At the origin	At the *left* end of the rod	At the *right* end of the rod	At point **C**	Between **C** & the left end (but not at the origin)	Between **C** & the right end	Impossible to determine without more info

At time $t = 0$, a disk with mass 1.000 kg and radius 33.3 cm rotates at $10.46\,\frac{\text{rad}}{\text{s}}$ about the positive x-axis. The disk slows to a stop at a constant rate while completing 19.25 revolutions. A point on the rim of the disk (point **P**) is labeled for ease of communication. You must express each answer using the best choice of units like we do in class (e.g., N, kg, m, s, rad, J, etc).



4a) Determine the *initial* rotation rate in **RPM**.

4b) Using a unit vector, state the *initial* tangential velocity *direction* at point **P**.

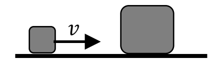
4c) Determine *initial* rotational kinetic energy of the disk.

**4d) Determine *angular* acceleration as the disk comes to rest.

***4e) At what time does *tangential* acceleration (magnitude) equal *centripetal* acceleration (magnitude) at point **P**?

4a	
4b	
4c	
4d	
4e	

A block of mass m is moves with speed v across a frictionless surface.
The block collides *elastically* with a stationary block of mass $6m$.



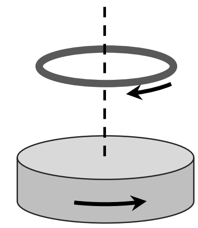
*****5a) Determine the *speed* of the small block after the collision.

5b) Determine the *direction* the small block moves after the collision.

5c) Determine the % change in kinetic energy of the small block during the collision.
To clarify, I want % change in kinetic energy of just the small block, not of the change in energy of the combined two block system. It's only worth 1 point…might do it last if you are slow at this computation.

5a	
5b	
5c	

A disk of mass m and radius R initially rotates with rate ω_0 about the axis shown. A thin ring with mass m and radius $\frac{3}{4}R$ rotates in the *opposite* direction (centered on the same axis of rotation). The thin ring is dropped onto the spinning disk. Eventually, both the disk and ring spin together in unison. In the end, the disk continues to spin in the same direction but with its rotation rate *reduced* by 90.0%.



To be clear, my figure is not scale; the size of the arrows in my picture has no correlation to the relative rotation rates.

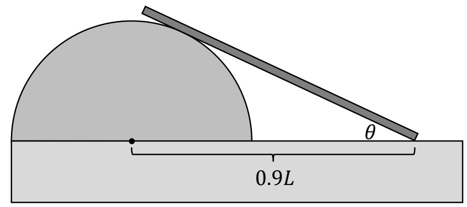
6a) Which of the following conservation laws are most appropriate to analyze this situation.
Circle the best answer.

Conservation of energy	Conservation of linear momentum	Conservation of angular momentum	None of these laws are appropriate	Impossible to determine without more info

*****6b) Determine *initial* rotation speed (ignoring direction) of the ring. Answer as a decimal number with three sig figs times ω_0.

6b	

A rod of length L and mass m is in static equilibrium. The rod is supported by a hemispherical dome and the level floor. **Assume friction is negligible between the dome & the rod.** Friction is NOT negligible between the rod & the floor. The distance between the right end of the rod and the center of the dome is provided in the figure. Assume $\theta = 20.0°$. **Figure not drawn to scale.**



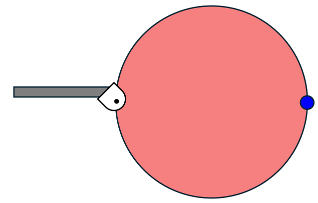
7a) Determine the radius of the dome.
Answer as a number with 3 sig figs times L.

***7b) Determine the normal force (magnitude) exerted by the dome.
Answer as a number with 3 sig figs times mg.

***7c) Determine the minimum coefficient of friction required between the rod & the level floor. Answer as a number with 3 sig figs.

7a	
7b	
7c	

A lump of clay (mass $\mathbf{2m}$) is attached to the edge of a *spherical shell* of mass $\boldsymbol{m}$ and radius R. The diametrically opposite side of the shell is attached to a low friction pivot. The system is released from rest. Assume air resistance is negligible & we are near earth's surface.



**8a) Determine the distance from the pivot point to the system center of mass.
Answer as a decimal number with three sig figs times R.

**8b) Determine moment of inertia about the axis shown.
Answer as number with 3 sig figs times an expression involving the given variables.

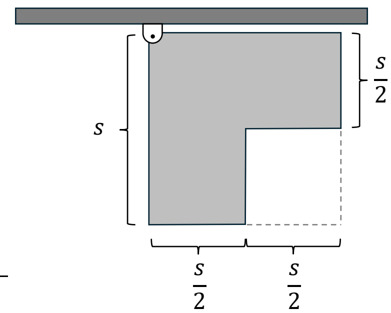
8c) **True or false:** angular acceleration (magnitude) is constant during the swing.
Circle in the box.

*8d) Determine *angular* acceleration (magnitude) at the instant the system is released. Answer as number with 3 sig figs times an expression involving the given variables.

**8e) Determine *translational* speed of the clay at the lowest point in the swing.
Answer as number with 3 sig figs times an expression involving the given variables.

8a	
8b	
8c	TRUE or FALSE
8d	
8e	

Originally problem 8 used the shape shown at right. Focus on regular credit; attempt if you finish early.



A thin plate of side s was created by removing the bottom right corner from a square plate. The mass of the plate is m after the corner was removed. The top left corner of the plate is attached to a low friction pivot.

***EC1)** Determine distance from the shape's center of mass to the pivot.
Answer in simplified form using the given variables.

***EC2)** Determine moment of inertia of the plate about the axis shown.
Answer in simplified form using the given variables.

EC1	
EC2	

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