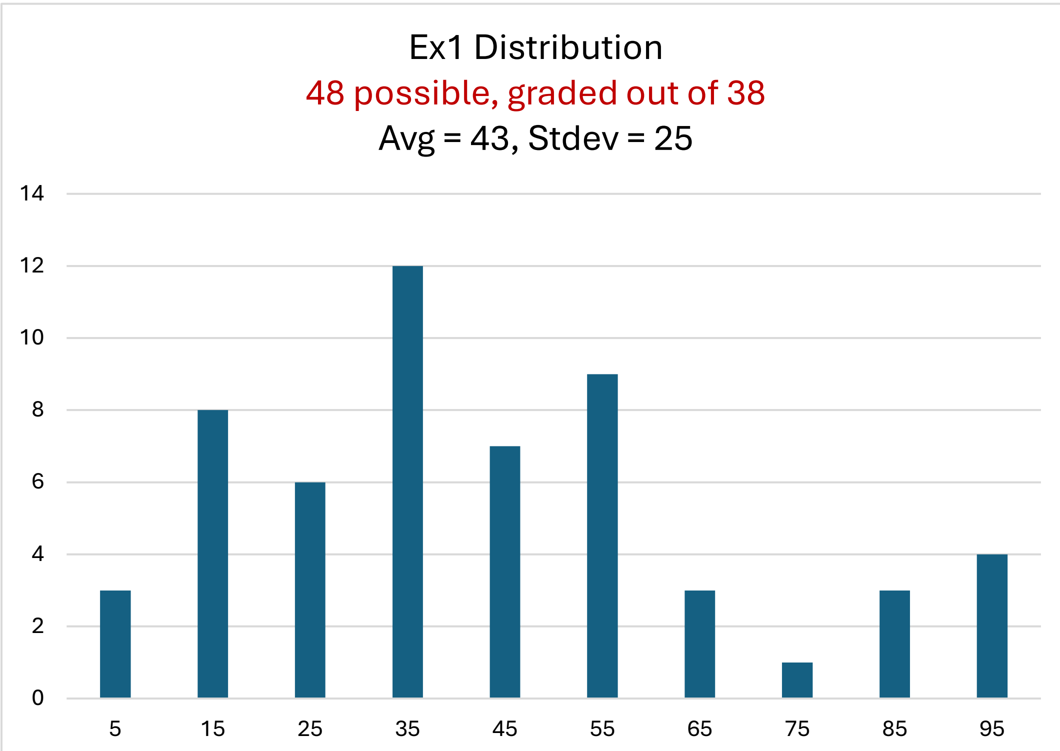
161fa26t1aSoln

Distribution on this page.
Solutions begin on the next page.



***1) Area total is area of plate minus area of hole.

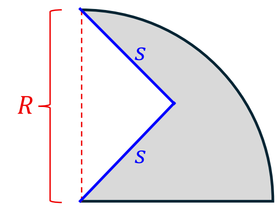
$$A = A_{qrtr\ circle} - A_{triangle}$$

$$A = \frac{1}{4}\pi R^2 - \frac{1}{2}s^2$$

$$A = \frac{1}{4}\pi R^2 - \frac{1}{2}\left(\frac{R^2}{2}\right)$$

$$A = \frac{1}{4}R^2(\pi - 1)$$

$$\boldsymbol{A = 0.53\underline{5}4R^2 = 0.535R^2}$$



Relate side length s to radius R

$$R^2 = s^2 + s^2$$

$$R^2 = 2s^2$$

$$\frac{R^2}{2} = s^2$$

In my classes I let you answer with either of the above two styles.
In Dom's classes you are expected to round (2nd style).
Note: always use **UNrounded** answers in subsequent results (even in Dom's classes).

2a) There are **FIVE sig figs** in $t = 0.000\ 000\ 020\ 200$ s

2b) $\boldsymbol{t = 20.200}$ **ns.** Notice we maintain the correct number of sig figs even though the notation has changed.
Note: if you wrote 20.200×10^{-9} ns you are double counting the 10^{-9}!!!
Notice the prefix alone is incorrect…you need ns not simply n.

***3) Notice the prefix for b! We were told

$a\ \left(\frac{\text{m}}{\text{s}}\right)$	b (mm)	c (m)	d (m)
8.75×10^{-7}	25.6	4.90×10^{1}	52.0

$$u = \frac{ab}{c - d}$$

$$u = \frac{\left(8.75 \times 10^{-7}\ \frac{\text{m}}{\text{s}}\right)(25.6 \times 10^{-3}\ \text{m})}{(49.0\ \text{m}) - (52.0\ \text{m})}$$

WATCH OUT! When doing the subtraction in the basement we keep the *rightmost column* (not *number of sig figs*).

$$u = \frac{2.2\underline{4} \times 10^{-8}\ \frac{\text{m}^2}{\text{s}}}{-3.\underline{0}\ \text{m}}$$

$$\boldsymbol{u = -7.\underline{4}67 \times 10^{-9}\ \frac{\text{m}}{\text{s}}}$$

$$\boldsymbol{u = -7.5 \times 10^{-9}\ \frac{\text{m}}{\text{s}}}$$

Notice you must change the mode on your calculator to get the extra sig fig!!!
Typically one does NOT include a prefix for scientific notation but DOES include a prefix for engineering notation.
Again, in my classes I let you answer with either of the above two styles.
In Dom's classes you will probably be asked to do something different.

**4) Sole for r:

$$U_0 = \frac{3\alpha}{r^2} + \frac{1}{2}\beta x^3$$

$$U_0 - \frac{1}{2}\beta x^3 = \frac{3\alpha}{r^2}$$

Cross-multiply.

$$r^2 = \frac{3\alpha}{U_0 - \frac{1}{2}\beta x^3}$$

Multiply every term in both the numerator & denominator by 2 to reduce the number of fractions.

$$r = \pm\sqrt{\frac{6\alpha}{2U_0 - \beta x^3}}$$

We were specifically told "**no fractions should appear in the numerator or denominator of the fraction used in your final result.**"
When taking the square root you should always indicate the $\pm$ sign unless there is good reason not to do so.
Note: we were not specifically told r was a radius in this case.
As such we do not know if the $\pm$ might be relevant.

WHY SO PICKY? We are trying to make things as easy as possible for people who read our work.
Getting into this habit now will hopefully make you look better later in your career.

Most people lost a lot of points on this page even though they could do the computation.
The way you write your results matters in engineering classes. Pay attention to detail!

5a) We were asked to determine B (the magnitude of $\vec{B}$).

$$B = \sqrt{(-4.00\text{ m})^2 + (5.00\text{ m})^2 + (-3.00\text{ m})^2} = \mathbf{7.0\underline{7}1\text{ m}} \approx \mathbf{7.07\text{ m}}$$

Notice I did not give you exact values; my values imply three sig figs on each term!

If you wrote $\sqrt{50}$ m $= 5\sqrt{2}$ m that is unacceptable because it implies I gave exact values.

5b) Determine unit vector $\hat{B}$.

While it may not matter for *this* problem, ensure you use the UNrounded result for B.

Notice the units *always* cancel out when computing *any unit vector*.

Notice we still have the unit vectors $\hat{\imath}, \hat{\jmath}, \& \hat{k}$ in the final result!

Lastly, watch out for $\pm$ signs.

$$\hat{B} = \frac{\vec{B}}{B} = \frac{(-4.00\hat{\imath} + 5.00\hat{\jmath} - 3.00\hat{k})\text{ m}}{7.0\underline{7}1\text{ m}}$$

$$\hat{B} = -0.566\hat{\imath} + 0.707\hat{\jmath} - 0.424\hat{k}$$

I gave you decimal numbers in the parameters. Switching to exact results (and implying infinite sig figs) is wrong.

5c) Do the cross-product in the correct order to avoid a minus sign error.

Remember to put correct units on the final answer!

Notice the wheel of pain is often faster than the determinant of a 3 by 3 matrix in many cases…

$$\vec{A} \times \vec{B} = (7.00\hat{\jmath}) \times (-4.00\hat{\imath} + 5.00\hat{\jmath} - 3.00\hat{k})\frac{\text{m}^2}{\text{s}}$$

Notice we can ignore the $\hat{\jmath} \times \hat{\jmath}$ term!

$$\vec{A} \times \vec{B} = [(7.00\hat{\jmath}) \times (-4.00\hat{\imath}) + (7.00\hat{\jmath}) \times (-3.00\hat{k})]\frac{\text{m}^2}{\text{s}}$$

$$\vec{A} \times \vec{B} = [-28.0(\hat{\jmath} \times \hat{\imath}) - 21.0(\hat{\jmath} \times \hat{k})]\frac{\text{m}^2}{\text{s}}$$

$$\vec{A} \times \vec{B} = [-28.0(-\hat{k}) - 21.0(+\hat{\imath})]\frac{\text{m}^2}{\text{s}}$$

Clean up the signs and put things in the standard order for full credit.

$$\boldsymbol{\vec{A} \times \vec{B} = (-21.0\hat{\imath} + 28.0\hat{k})\frac{\text{m}^2}{\text{s}}}$$

5d) Remember to put correct units on the final answer!

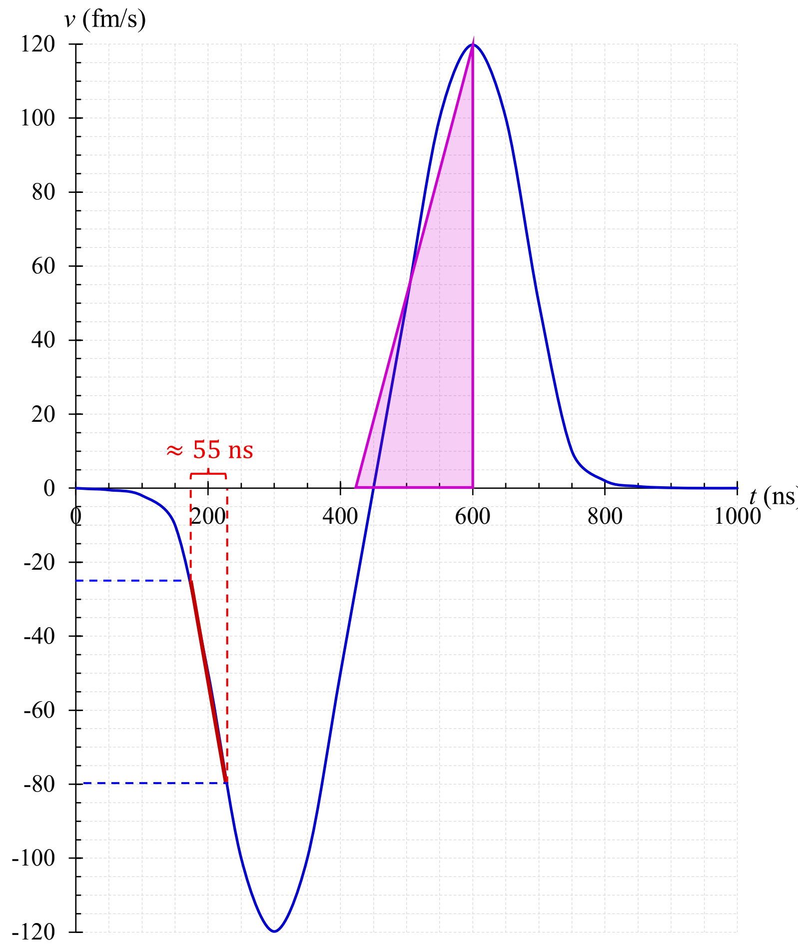
$$\vec{A} \cdot \vec{B} = (7.00\hat{\jmath}) \cdot (-4.00\hat{\imath} + 5.00\hat{\jmath} - 3.00\hat{k})\frac{\text{m}^2}{\text{s}}$$

Notice we can ignore the $\hat{\jmath} \cdot \hat{\imath}$ & $\hat{\jmath} \cdot \hat{k}$ terms!

$$\vec{A} \cdot \vec{B} = (7.00\hat{\jmath}) \cdot (5.00\hat{\jmath})\frac{\text{m}^2}{\text{s}}$$

$$\boldsymbol{\vec{A} \cdot \vec{B} = 35.0\frac{\text{m}^2}{\text{s}}}$$

6a) First, notice we have a vt-plot.
Recall speed increases when velocity (value on vt-plot) and acceleration (slope of vt-plot) have the same sign.
On this particular vt-plot we see speeding up occurs **from $\mathbf{0-300\ ns\ \&\ 450-600\ ns}$**.
Check, did you remember to include units?
If not, you lost easy points.



6b) Acceleration is the slope of the vt-plot.
Brack the time of interest with points nearby.
Typically, I expect your results should be within 10% of mine.
This one was hard to read so I allowed up to 20% difference.

$$a_x = \frac{rise}{run}$$

$$a_x = \frac{\left(-80\,\frac{\text{fm}}{\text{s}}\right) - \left(-25\,\frac{\text{fm}}{\text{s}}\right)}{230\text{ ns} - 175\text{ ns}}$$

$$a_x = \frac{-55 \times 10^{-15}\,\frac{\text{m}}{\text{s}}}{55 \times 10^{-9}\text{ s}}$$

$$a_x = -1.00 \times 10^{-6}\,\frac{\text{m}}{\text{s}^2}$$

$$\boldsymbol{a_x = -1.00\,\frac{\mu\text{m}}{\text{s}^2}}$$

6c) Displacement is area under a vt-curve.
I estimated the area is about twice the area of the triangle shown in the figure.
Typically, I expect your results should be within 10% of mine. This one was hard to read so I allowed up to 20% difference.

$$Displacement \approx 2A_{triangle}$$

$$Displacement \approx 2\left(\frac{1}{2}base \times height\right)$$

$$Displacement \approx base \times height$$

$$Displacement \approx (180\text{ ns}) \times \left(+120\,\frac{\text{fm}}{\text{s}}\right)$$

$$Displacement \approx (180 \times 10^{-9}\ ) \times (+120 \times 10^{-15})\text{ m}$$

$$\boldsymbol{Displacement \approx +2.2 \times 10^{-20}\text{ m}}$$

6d) The particle undergoes negative displacement until 450 ns then positive displacement until 1000 ns.
It first moves left the moves back to the right.

6e) Areas below & above the curve are approximately equal (implying zero *net* displacement). The particle returns to its initial location (approximately).

7a) We were given $\vec{v}(t) = (-36.0t^2 + 48.0)\hat{\imath} + 24.0t\hat{\jmath}$.
From this we can determine

$$v_x = -36.0t^2 + 48.0 \quad \& \quad v_y = 24.0t$$
$$a_x = -72.0t \quad \& \quad a_y = 24.0$$
$$a_x \neq constant \quad \& \quad a_y = constant$$

7b) Set time equal to zero to find

$$\vec{v}(0) = 48.0\hat{\imath} + 0\hat{\jmath}$$

Initially the object moves to the right.

7c)

$$[v] = [-36.0][t^2] \quad \rightarrow \quad [-36.0] = \frac{[v]}{[t^2]} = \frac{\frac{\text{m}}{\text{s}}}{\text{s}^2} = \frac{\text{m}}{\text{s}^2}\cdot\frac{1}{\text{s}^2} = \mathbf{\frac{m}{s^3}}$$

7d) If velocity is a function of *position*, *separate the variables* before you *integrate*.
Fortunately, in this problem velocity is a function of time…we can integrate without thinking too much.
Notice you are still separating the variables before you integrate…it just is really easy to get all the t's on one side…

$$\frac{dx}{dt} = -36.0t^2 + 48.0 \quad \& \quad \frac{dy}{dt} = 24.0t$$

$$dx = (-36.0t^2 + 48.0)\, dt \quad \& \quad dy = 24.0t\, dt$$

$$\int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} (-36.0t^2 + 48.0)\, dt \quad \& \quad \int_{y_i}^{y_f} dy = \int_{t_i}^{t_f} 24.0t\, dt$$

$$x_f - x_i = -12.0t^3 + 48.0t \quad \& \quad y_f - y_i = 12.0t^2$$

$$x_f = x_i - 12.0t^3 + 48.0t \quad \& \quad y_f = y_i + 12.0t^2$$

$$x_f = 0 - 12.0t^3 + 48.0t \quad \& \quad y_f = -7.00 + 12.0t^2$$

$$\boldsymbol{\vec{r}(t) = (-12.0t^3 + 48.0t)\hat{\imath} + (-7.00 + 12.0t^2)\hat{\jmath}}$$

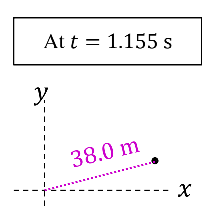
7e) As discussed in part 7a we found

$$\boldsymbol{\vec{a}(t) = -72.0t\hat{\imath} + 24.0\hat{\jmath}}$$

7f) Determine time to reverse direction horizontally by setting $v_x = 0$. I found $t = \sqrt{\frac{48.0}{36.0}} = 1.1547$ s.
Keep extra digits to avoid intermediate rounding error. Since I'd usually keep 4 (when first digit is 1) I keep 5 instead.
Plug this into the position equation to find

$$\vec{r}(t) = \left(-12.0(1.1547\text{ s})^3 + 48.0(1.1547\text{ s})\right)\hat{\imath} + \left(-7.00 + 12.0(1.1547\text{ s})^2\right)\hat{\jmath}$$

$$\vec{r}(t) = 36.95\hat{\imath} + 9.000\hat{\jmath}$$

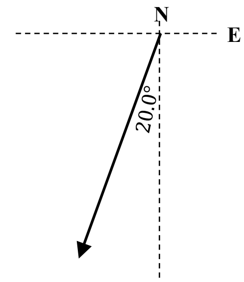


Consider the figure at right. Notice distance from the origin is

$$r = \sqrt{(36.95)^2 + (9.000)^2} = \mathbf{38.0\ m}$$

Note: here it was ok to leave off units because I stated as much in the problem statement.

8) This is a standard relative velocity problem. I'll leave off units until the final result to reduce clutter.



We are given info about the speed of the drone relative to earth.

$$\vec{v}_{de} = 44.4\ directed\ 20.0°\ \mathrm{W\ of\ S}$$

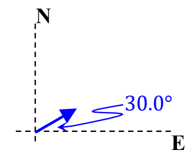
$$\vec{v}_{de} = -44.4 \sin 20.0°\,\hat{\imath} - 44.4 \cos 20.0°\,\hat{\jmath}$$

Notice the horizontal component uses the sine function…notice the angle is to a vertical axis.

$$\vec{v}_{de} = -15.\underline{1}86\hat{\imath} - 41.\underline{7}2\hat{\jmath}$$

Out of habit I keep an extra unrounded digit on the term whose first digit is 1.

It is reasonable to expect you to understand wind speed is speed of air relative to the earth.



$$\vec{v}_{ae} = 8.88\ directed\ 30.0°\ \mathrm{N\ of\ E}$$

$$\vec{v}_{ae} = 8.88 \cos 30.0°\,\hat{\imath} + 8.88 \sin 30.0°\,\hat{\jmath}$$

$$\vec{v}_{ae} = 7.6\underline{9}0\hat{\imath} + 4.4\underline{4}0\hat{\jmath}$$

Now write a correct velocity addition equation. There are multiple ways to do this.
Here was my thought process.
Asking for velocity of drone "in still air" is akin to asking for velocity of drone relative to air.

$$\vec{v}_{da} = \vec{v}_{de} + \vec{v}_{ea}$$

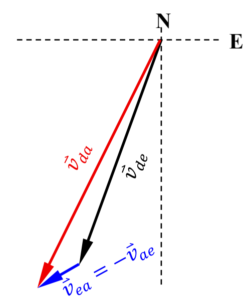
Recall $\vec{v}_{ea} = -\vec{v}_{ae}$.

$$\vec{v}_{da} = \vec{v}_{de} - \vec{v}_{ae}$$

$$\vec{v}_{da} = \left(-15.\underline{1}86\hat{\imath} - 41.\underline{7}2\hat{\jmath}\right) - (7.6\underline{9}0\hat{\imath} + 4.4\underline{4}0\hat{\jmath})$$

$$\vec{v}_{da} = -22.\underline{8}8\hat{\imath} - 46.\underline{1}6\hat{\jmath}$$

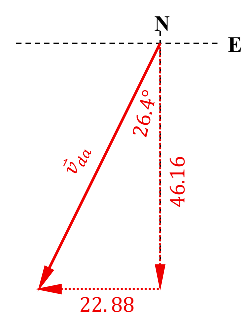
For 2D vectors it is always wise to draw a sketch to better understand things.
While not perfectly to scale, my figures are approximately drawn to scale.
The upper figure at right shows the graphical vector addition.
The lower figure at right shows the desired result.
Find speed using Pythagorean theorem and SOH CAH TOA (I used inverse tangent).



$$\boldsymbol{\vec{v}_{da} = 51.5\frac{m}{s}}\ \textbf{directed 26.4° W of S}$$

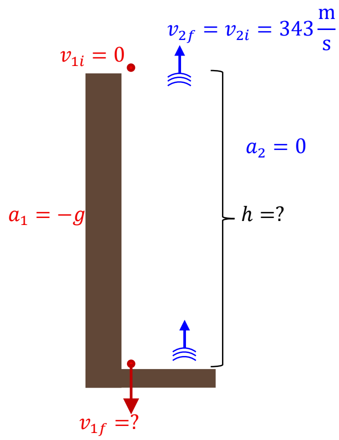
$$\boldsymbol{\vec{v}_{da} = 51.5\frac{m}{s}}\ \textbf{directed 63.6° S of W}$$

Remember to include units on the final result!
Think about the result.
In this case the drone was having to fight the wind.
We expect the drone velocity in still air should be slightly higher (and it is).
This is a good check on our result.



9a) Notice we have a classic two-stage 1D motion problem.
Every figure will look a little bit different. One way to draw it is shown below.
NOTE: The problem said, "sound from impact travels upwards with rate $343\,\frac{\text{m}}{\text{s}}$". This is a clear indication speed is constant for stage 2.

Δy_1	$-h$
v_{1i}	0
v_{1f}	?
a_1	$-g = -9.8\,\frac{\text{m}}{\text{s}^2}$
t_1	?



Δy_2	$+h$
v_{2i}	$343\,\frac{\text{m}}{\text{s}}$
v_{2i}	$343\,\frac{\text{m}}{\text{s}}$
a_2	0
t_2	?

Falling Rock	**Sound Travels Upwards**
$\Delta y_1 = v_{1i}t_1 + \frac{1}{2}a_1 t_1^2$ Plug in zeros but work algebraically as long as reasonable. $-h = \frac{1}{2}(-g)t_1^2$ $h = \frac{1}{2}g t_1^2$	$\Delta y_2 = v_{2i}t_2 + \frac{1}{2}a_2 t_2^2$ Plug in zeros but work algebraically as long as reasonable. $+h = v_{2i}t_2$

I notice h is unknown. Meanwhile the problem mentions at least *something* about time ($t_{total} = 4.00\text{ s}$).
This makes me think I should eliminate h. I choose to do this by setting the equations equal.

$$\frac{1}{2}g t_1^2 = v_{2i}t_2$$

Now use the constraint connecting the two parts of the problem. Again, work algebraically as long as reasonable.

$$t_{total} = t_1 + t_2$$

$$t_2 = t_{total} - t_1$$

Plug in the constraint for one of the unknown times and we have one equation with one unknown…EUREKA!

$$\frac{1}{2}g t_1^2 = v_{2i}(t_{total} - t_1)$$

Plug in numbers and work towards doing the quadratic formula. For me, I'm going to first rearrange algebraically and check the units. Then, when I plug in numbers, I've already ensured the units will work out. I choose to keep the ½ on the t_1^2 term for reasons which should become obvious shortly.

$$\frac{1}{2}t_1^2 + \frac{v_{2i}}{g}t_1 - \frac{v_{2i}}{g}t_{total} = 0$$

$$\frac{1}{2}t_1^2 + \frac{343\,\frac{\text{m}}{\text{s}}}{9.8\,\frac{\text{m}}{\text{s}^2}}t_1 - \frac{343\,\frac{\text{m}}{\text{s}}}{9.8\,\frac{\text{m}}{\text{s}^2}}(4.00\text{ s}) = 0$$

Notice the units work out (s^2 on each term). Notice once I get t_1 height will be easy to get using the Δy_1 equation.

Solution to 9a) continues on the next page…

Solution to 9a) continues…

$$\frac{1}{2}t_1^2 + \frac{343\,\frac{\mathrm{m}}{\mathrm{s}}}{9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}}t_1 - \frac{343\,\frac{\mathrm{m}}{\mathrm{s}}}{9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}}(4.00\ \mathrm{s}) = 0$$

$$\frac{1}{2}t_1^2 + 35.0t_1 - 140.0 = 0$$

$$t_1 = \frac{-(35.0) \pm \sqrt{(35.0)^2 - 4\left(\frac{1}{2}\right)(-140.0)}}{2\left(\frac{1}{2}\right)}$$

$$t_1 = -35.0 \pm \sqrt{1505}$$

$$t_1 = -35.0 \pm 38.794$$

Notice, we expect time must be positive so use the + root.
Also notice I want my final answer correct to use three sig figs (exam default). I'll keep an *extra* unrounded digit when subtracting two numbers close together; otherwise I will lose a sig fig!

$$t_1 = 3.794\ \mathrm{s}$$

As a quick intermediate check, THINK! We expect sound travels much faster than the rock. This implies t_1 should be close to the total time but a little bit smaller. Looks good.

Plug this result into the Δy_1 equation to find h.

$$h = \frac{1}{2}gt_1^2$$

$$h = \frac{1}{2}\left(9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(3.794\ \mathrm{s})^2$$

$$h = 70.54\ \mathrm{m}$$

Represent exam results with three sig figs (and include proper units on numerical results).

$$\boldsymbol{h = 70.5\ \mathrm{m}}$$

Think: this height seems reasonable. A 4.00 s freefall is 78.4 m. Our result should be slightly smaller than that since some time is required for the sound to travel!

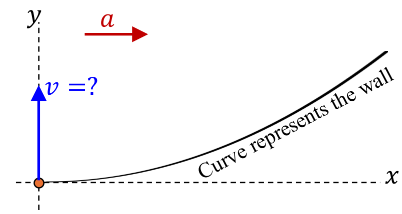
9b) Answer: $h > h_{real}$. By ignoring drag, we are essentially overestimating the magnitude of downward acceleration. This implies our calculated fall distance (h) is larger than the real distance (h_{real}).
As a way to check this, I'll recompute using $g_{effective} \approx 9.6\,\frac{\mathrm{m}}{\mathrm{s}^2}$ (air resistance lowering the effective value of g).

$$t_1 = 3.798\ \mathrm{s} \quad \rightarrow \quad h = 69.2\ \mathrm{m}$$

Notice air resistance causes the fall to take slightly *longer* even though there is *less distance* to travel.
Remember, this result is only an approximation. A better way to analyze motion with drag is to code a simulation.

A note on $g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$ versus $g = 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}$. Students always get bent out of shape about this but there is no need to stress out here. The worldwide average is often stated as $9.807\,\frac{\mathrm{m}}{\mathrm{s}^2}$. However, the value varies slightly everywhere on earth. At the equator we use $9.780\,\frac{\mathrm{m}}{\mathrm{s}^2}$ while at 35° N latitude (Santa Maria) we should use $9.797\,\frac{\mathrm{m}}{\mathrm{s}^2} \approx 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$. Usually we discuss this in labs. In fact, a routine upper division physics lab involves directly measuring g using a device known as a "Kater's Pendulum" … look it up. Furthermore, variations in the material composition of the earth can cause additional corrections. All that said, this is not something to stress over for this type of calculation.

10a) $[y] = [k] \cdot [x^2] \quad \rightarrow \quad [k] = \frac{[y]}{[x^2]} = \frac{1}{\text{m}}$



Δx	$x = ?$
v_{ix}	0
v_{fx}	?
a_x	a
t	?

Δy	$y = kx^2 = ?$
v_{iy}	v
v_{fy}	v
a_y	0

10b) My list of knowns & unknowns is given under the figure.

***x*-equations**	***y*-equations**
$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$	$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$
$x = \frac{1}{2}at^2$	$kx^2 = vt$

Now plug in x from one equation into the other equation.

$$k\left(\frac{1}{2}at^2\right)^2 = vt$$

$$\frac{ka^2}{4}t^4 = vt$$

$$\frac{ka^2}{4}t^3 = v$$

$$t^3 = \frac{4v}{ka^2}$$

$$\boldsymbol{t = \left(\frac{4v}{ka^2}\right)^{1/3} \approx 1.587\left(\frac{v}{ka^2}\right)^{1/3}}$$

This answer looks insane! This is a perfect chance to use a unit check…

$$[t] = \left[\frac{\frac{\text{m}}{\text{s}}}{\frac{1}{\text{m}} \cdot \left(\frac{\text{m}}{\text{s}^2}\right)^2}\right]^{1/3} = \left(\frac{\text{m}}{\text{s}} \cdot \text{m} \cdot \frac{\text{s}^4}{\text{m}^2}\right)^{1/3} = \text{s}$$

Looks good.

Now a couple quick reality checks on this answer.
Think: if v increases, impact should occur *farther* from the origin. We get a *longer time* for *larger* v as expected.
Think: if a increases, impact should occur *closer* to the origin. We get a *shorter time* for *larger* a as expected.