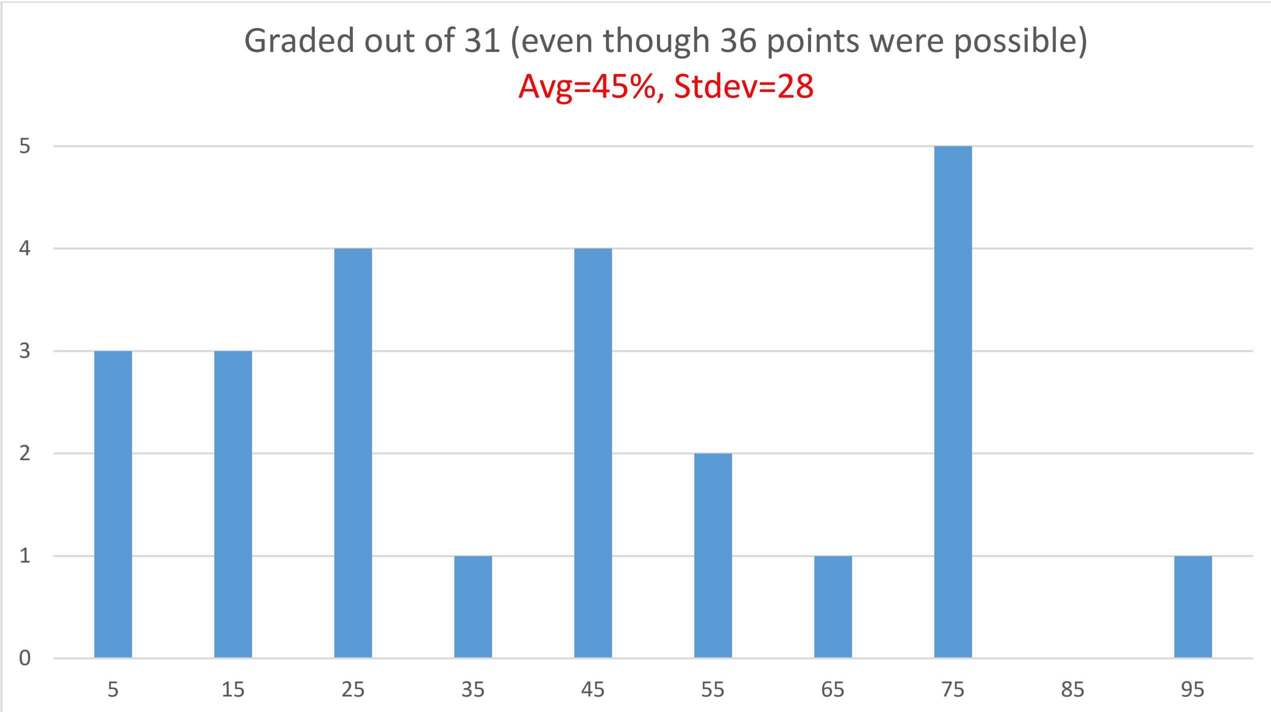
161sp22t1bSoln

Distribution on this page. Solutions begin on the next page.



**1) Remember to cube all numbers in the conversion factor 1 m = 100 cm…

$$1.914 \times 10^1 \frac{\text{BTU}}{\text{cm}^3} = 1.91 \times 10^1 \frac{\text{BTU}}{\text{cm}^3}$$

Note: you may be wondering why I am willing to accept the option which included 4 sig figs. Often engineers include an extra sig fig when the first digit of a number is 1.

**2) The units of any one term match the units of every other term.

$$[F] = \left[\sqrt{\frac{\alpha x^5}{v^3}}\right]$$

Square both sides.

$$[F]^2 = \left[\frac{\alpha x^5}{v^3}\right]$$

$$[\alpha] = \frac{[F]^2 \cdot [v^3]}{[x^5]}$$

$$[\alpha] = \frac{\left(\text{kg} \cdot \frac{\text{m}}{\text{s}^2}\right)^2 \cdot \left(\frac{\text{m}}{\text{s}}\right)^3}{\text{m}^5}$$

$$[\boldsymbol{\alpha}] = \frac{\mathbf{kg^2}}{\mathbf{s^7}}$$

**3) Use

$$A_{tot} = \frac{1}{2} A_{circle} - A_{square}$$

$$A_{tot} = \frac{1}{2}\left(\frac{\pi D^2}{4}\right) - s^2$$

Think, the main diagonal of the square has length $\frac{D}{2} = \sqrt{2}\, s \;\rightarrow\; s = \frac{D}{2\sqrt{2}}$.

$$A_{tot} = \frac{\pi D^2}{8} - \frac{D^2}{8}$$

You were told: **Answer as a decimal number with three sig figs times D^2 for credit.**

$$\boldsymbol{A_{tot} = 0.268 D^2}$$

*****4) I used

$$v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x$$

Acceleration is to the left with magnitude a ($a_x = -a$).
Object moves to the left distance d ($\Delta x = -d$).
Object initially moving to the left and down ($v_{ix} = -v\cos\theta$).

$$v_{fx}^2 = (-v\cos\theta)^2 + 2(-a)(-d)$$

$$v_{fx}^2 = v^2\cos^2\theta + 2ad$$

$$v_{fx} = -\sqrt{v^2\cos^2\theta + 2ad}$$

Here I used the *negative* root since the final horizontal component of velocity should point to the *left*.
Now use

$$v_{fx} = v_{ix} + a_x t$$

$$t = \frac{v_{fx} - v_{ix}}{a_x}$$

$$t = \frac{\left(-\sqrt{v^2\cos^2\theta + 2ad}\right) - (-v\cos\theta)}{(-a)}$$

$$\boldsymbol{t = \frac{\sqrt{v^2\cos^2\theta + 2ad} - v\cos\theta}{a}}$$

Here it probably makes sense to factor out $\frac{v\cos\theta}{a}$ since that has the correct units (dimensions of time).

$$\boldsymbol{t = \frac{v\cos\theta}{a}\left(\sqrt{1 + \frac{2ad}{v^2\cos^2\theta}} - 1\right)}$$

**5a) On a plot of position versus time we can get displacement using $x_f - x_i$.
Notice the object reverse direction at $t = 1.00$ s.
Determine displacement from 0 → 1.00 s and separately determine displacement from 1.00 s until end of interval shown.
Take absolute value of each displacement to get each distance…then sum.
I found $\mathbf{20.0}$ nm.

**5b) Slope of an xt-plot give velocity.
Pick points *just* before and *just* after the time of interest to ensure your estimate is decent.
I used $t_i = 0.40$ s and $t_f = 0.60$ s.
Check the unit on each axis!

$$v_x = slope$$

$$v_x \approx \frac{x_f - x_i}{t_f - t_i}$$

$$v_x \approx \frac{(-1.5\text{ nm}) - (1.8\text{ nm})}{(0.60\text{ s}) - (0.40\text{ s})}$$

$$\boldsymbol{v_x \approx -1\underline{6}.5\frac{\text{nm}}{\text{s}}}$$

5c) Remember we are looking at a xt-plot.
Here velocity corresponds to slope and acceleration corresponds to concavity.
Moving backwards means $v < 0$ (negative slope).
This occurs between $\boldsymbol{t = 0.00}$ s → $\mathbf{1.00}$ s.

5d) Moving forwards and slowing down (on an xt-plot) corresponds to positive slope with downwards concavity.
This doesn't occur during the time interval shown.

*****6) Without loss of generality, we may assume the race runs to the right.
Since there are no direction reversals in this problem, we may set $x_i = 0$ and use distance traveled and displacement interchangeably. Remember, when an object reverse direction (as in problem 5) we cannot use this simplification.
We have tow constraints in this problem.

$$32.25\text{ s} = t_1 + t_2$$
$$\mathbf{200.0\ m} = \boldsymbol{\Delta x_1 + \Delta x_2}$$

Final velocity of stage 1 relates to initial velocity of stage 2. Let

$$v = v_{stage2i} = v_{stage1f}$$
$$v = at_1$$

Now plug in displacement equations for each Δx in the bold constraint equation:

$$\mathbf{200.0\ m} = \boldsymbol{\frac{1}{2}at_1^2 + vt_2}$$

$$200.0\text{ m} = \frac{1}{2}at_1^2 + (at_1)t_2$$

Replace t_2 using the other constraint equation.

$$200.0\text{ m} = \frac{1}{2}at_1^2 + (at_1)(32.25\text{ s} - t_1)$$

Plug in numbers and clean it up to get a quadratic of the form

$$t_1^2 + (-64.5\text{ s})t_1 + 145.45\text{ s}^2 = 0$$

From there I found $\boldsymbol{t_1 = 2.340}$ s which gave $\boldsymbol{v = 6.44\frac{\text{m}}{\text{s}}}$.

***7a) It helps to first write $\vec{v}_b$ in Cartesian form.
Notice the angle is to the $\hat{\jmath}$ axis…expect the $\hat{\jmath}$ term to use $\cos(angle)$.
Furthermore, notice the $\hat{\jmath}$ component should be negative according to the coordinate system used in the figure!!!

$$\vec{v}_b = 17.77\,\frac{\mathrm{m}}{\mathrm{s}}\left(\sin 22.2°\,(+\hat{\imath}) + \cos 22.2°\,(-\hat{\jmath})\right)$$

$$\vec{v}_b = (6.714\hat{\imath} - 16.453\hat{\jmath})\,\frac{\mathrm{m}}{\mathrm{s}}$$

From there cross product can be found quickly using the FOIL method with the wheel of pain. Don't forget units!

$$\vec{\boldsymbol{v}}_{\boldsymbol{b}} \times \vec{\boldsymbol{v}}_{\boldsymbol{d}} = (6.714\hat{\imath} - 16.453\hat{\jmath})\,\frac{\mathrm{m}}{\mathrm{s}} \times \left(5.55\hat{\imath} + 8.88\hat{k}\right)\frac{\mathrm{m}}{\mathrm{s}}$$

$$\vec{\boldsymbol{v}}_{\boldsymbol{b}} \times \vec{\boldsymbol{v}}_{\boldsymbol{d}} = \left(\boldsymbol{-146.1\hat{\imath} - 59.6\hat{\jmath} + 91.3\hat{k}}\right)\frac{\mathbf{m^2}}{\mathbf{s^2}}$$

**7b) To get the angle between two vectors we use our standard approach.

$$\theta = \cos^{-1}\left(\frac{\vec{v}_d \cdot \vec{v}_b}{v_d v_b}\right)$$

Notice units will all drop out in this expression...as such I will ignore units for the rest of this calculation.
Notice we need

$$v_d = \|\vec{v}_d\| = \sqrt{5.55^2 + 8.88^2} = 10.\underline{4}72$$

The only surviving term in $\vec{v}_d \cdot \vec{v}_b$ will be $v_{dx}v_{bx}$.

$$\theta = \cos^{-1}\left(\frac{v_{dx}v_{bx}}{v_d v_b}\right)$$

$$\theta = \cos^{-1}\left(\frac{(5.55)6.714}{(10.\underline{4}72)17.77}\right)$$

$$\boldsymbol{\theta = 78.4°}$$

***7c) We know 1 hr = 3600 s. We can multiply each velocity vector by 3600 s to get each displacement.
Alternatively, we could first determine the relative velocity then multiply that by 3600 to get the relative displacement.
I choose the later method.

$$\vec{v}_{db} = \vec{v}_{de} + \vec{v}_{eb}$$

$$\vec{v}_{db} = \vec{v}_{de} - \vec{v}_{be}$$

$$\vec{v}_{db} = \left(5.55\hat{\imath} + 8.88\hat{k}\right)\frac{\mathrm{m}}{\mathrm{s}} - (6.714\hat{\imath} - 16.453\hat{\jmath})\,\frac{\mathrm{m}}{\mathrm{s}}$$

$$\vec{v}_{db} = \left(-1.164\hat{\imath} + 16.453\hat{\jmath} + 8.88\hat{k}\right)\frac{\mathrm{m}}{\mathrm{s}}$$

Using the numbers shown above one finds speed $v_{db} = 18.732\,\frac{\mathrm{m}}{\mathrm{s}}$.
Multiplying by 3600 s gives distance between.

$$\boldsymbol{d = 67.4\ \mathrm{km}}$$

**8) Watch the units carefully. Also, you were told to track sig figs on this one. Be extra careful with sig figs any time you have addition or subtraction as the least significant *column* of sig figs matters for those operations!

$$\Delta x = \frac{v_f^2 - v_i^2}{2a}$$

$$\Delta x = \frac{\left(7.65 \times 10^{-2}\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2 - \left(-80.0\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{2\left(7.00\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)}$$

$$\Delta x = \frac{0.0058\underline{5}23\,\frac{\mathrm{m}^2}{\mathrm{s}^2} - 64\underline{0}0\,\frac{\mathrm{m}^2}{\mathrm{s}^2}}{2\left(7.00\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)}$$

Notice the v_f term in blue is negligible!!!

$$\Delta x = \frac{-64\underline{0}0\,\frac{\mathrm{m}^2}{\mathrm{s}^2}}{2\left(7.00\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)}$$

Also notice we get units of m…

$$\Delta x = -45\underline{7}.1\ \mathrm{m}$$

$$\boldsymbol{\Delta x = -457\ \mathrm{m}}$$

Note: this result happens to look the exactly the same in engineering notation!!!

****9) Use separation of variables.

$$v_x = \frac{k}{x^4}$$

$$\frac{dx}{dt} = \frac{k}{x^4}$$

$$x^4\,dx = k\,dt$$

$$\int_{x_i=d}^{x_f} x^4\,dx = \int_{t_i=0}^{t_f} k\,dt$$

$$\frac{x_f^5}{5} - \frac{d^5}{5} = kt_f - 0$$

After integration, we can change $t_f \to t$. At this point, x_f is equivalent to $x(t)$…

$$\boldsymbol{x(t) = x_f = \sqrt[5]{d^5 + 5kt}}$$