161 Spring 2022 Test $1 D$ Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

| $V_{\text {sphere }}=\frac{4}{3} \pi R^{3}$ | $V_{\text {box }}=L W H$ | $V_{c y l}=\pi R^{2} H$ | $\rho=\frac{M}{V}$ |
| :---: | :---: | :---: | :---: |
| $A_{\text {sphere }}=4 \pi R^{2}$ | $V=\left(A_{\text {base }}\right) \times($ height $)$ | $A_{\text {circle }}=\pi R^{2}$ | $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ |
| $C=2 \pi R$ | $A_{\text {rect }}=L W$ | $A_{\text {CylSide }}=2 \pi R H$ |  |
| $1609 \mathrm{~m}=1 \mathrm{mi}$ | $12 \mathrm{in}=1 \mathrm{ft}$ | $60 \mathrm{~s}=1 \mathrm{~min}$ | $1000 \mathrm{~g}=1 \mathrm{~kg}$ |
| $2.54 \mathrm{~cm}=1 \mathrm{in}$ | $1 \mathrm{cc}=1 \mathrm{~cm}^{3}=1 \mathrm{~mL}$ | $60 \mathrm{~min}=1 \mathrm{hr}$ | $100 \mathrm{~cm}=1 \mathrm{~m}$ |
| $1 \mathrm{~cm}=10 \mathrm{~mm}$ | 1 yard $=3 \mathrm{ft}$ | $3600 \mathrm{~s}=1 \mathrm{hr}$ | $1 \mathrm{~km}=1000 \mathrm{~m}$ |
| 1 furlong $=220$ yards | $528 \underline{\mathrm{ft}}=1 \mathrm{mi}$ | $24 \mathrm{hrs}=1$ day | $1 \mathrm{rev}=2 \pi \mathrm{rad}=360^{\circ}$ |
| $g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ | $G=6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}}$ | $P_{0}=1.0 \times 10^{5} \mathrm{~Pa}$ | $1 \mathrm{eV}=1.60 \underline{2} \times 10^{-19} \mathrm{~J}$ |
| $1 \mathrm{~N}=1 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}$ | $1 \mathrm{~J}=1 \mathrm{~N} \cdot \mathrm{~m}$ | $1 \mathrm{~Pa}=1 \frac{\mathrm{~N}}{\mathrm{~m}^{2}}$ |  |
| $x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$ | $v_{f x}^{2}=v_{i x}^{2}+2 a_{x}(\Delta x)$ | $v_{f x}=v_{i x}+a_{x} t$ | $r=\sqrt{x^{2}+y^{2}}$ |
| $\vec{A} \cdot \vec{B}=A B \cos \theta_{A B}$ | $\\|\vec{A} \times \vec{B}\\|=A B \sin \theta_{A B}$ | $\begin{aligned} & \sin (A \pm B) \\ & =\sin A \cos B \pm \cos A \sin B \end{aligned}$ | $\begin{aligned} & \cos (A \pm B) \\ & =\cos A \cos B \mp \sin A \sin B \end{aligned}$ |
| $\vec{v}_{a e}+\vec{v}_{e b}=\vec{v}_{a b}$ | $\hat{r}=\cos \theta \hat{\imath}+\sin \theta \hat{\jmath}$ | $\hat{\theta}=-\sin \theta \hat{\imath}+\cos \theta \hat{\jmath}$ |  |
| $a_{t a n}=r \alpha$ | $a_{c}=\frac{v^{2}}{r}=r \omega^{2}$ | $\vec{a}=a_{r} \hat{r}+a_{t a n} \hat{\theta}$ | $\vec{a}=a_{c}(-\hat{r})+a_{t a n} \hat{\theta}$ |
| $\Sigma \vec{F}=m \vec{a}$ | $f \leq \mu n$ | $F_{G}=\frac{G m M}{r^{2}}(-\hat{r})$ | $U_{G}=-\frac{G m M}{r}$ |
| $T K E=\frac{1}{2} m v^{2}$ | $R K E=\frac{1}{2} I \omega^{2}$ | $U_{S}=S P E=\frac{1}{2} k x^{2}$ | $U_{G}=G P E=m g h$ |
|  | $\Delta K E=W_{\text {ext. \& non-con }}$ | $W=F d \cos \theta=F_{\\|} d$ | $W=\int F_{x} d x$ |
| $\Delta U=-W=-\int_{i}^{f} \vec{F} \cdot d \vec{s}$ | $F_{x}=-\frac{d}{d x} U(x)$ | $\mathcal{P}_{\text {inst }}=\frac{d E}{d t}=\vec{F} \cdot \vec{v}$ | $\mathcal{P}_{\text {avg }}=\frac{\Delta E}{\Delta t}=\frac{\text { Work }}{\text { time }}$ |
| $\vec{J}=\Delta \vec{p}=\vec{F} \Delta t$ | $\vec{p}=m \vec{v}$ | $x_{\mathrm{CM}}=\frac{m_{1} x_{1}+m_{2} x_{2}}{m_{1}+m_{2}}$ | $x_{\text {CM }}=\frac{\int x d m}{\int d m}$ |
| $\vec{\tau}=\vec{r} \times \vec{F}$ | $\Sigma \vec{\tau}=I \vec{\alpha}$ | $L=I \omega=m v r_{\perp}$ | $\mathcal{P}_{\text {inst }}=\vec{\tau} \cdot \vec{\omega}$ |
| $s=r \Delta \theta$ | $v=r \omega$ | $a_{t a n}=r \alpha$ | $a_{c}=\frac{v^{2}}{r}=r \omega^{2}$ |
| $I_{\\| \text {axis }}=I_{\mathrm{CM}}+m d^{2}$ | $I_{z z}=I_{x x}+I_{y y}$ | $I=\int r^{2} d m$ | $\frac{F}{A}=E \frac{\Delta L}{L_{0}}$ |
| $P=\frac{F}{A}$ | $P_{\text {gauge }}=P_{\text {abs }}-P_{\text {ambient }}$ | $B=\rho_{f} V_{\text {disp }} g$ | $A_{1} v_{1}=A_{2} v_{2}$ |
| $P(h)=P_{0}+\rho g h$ | $P+\frac{1}{2} \rho v^{2}+\rho g h=\text { constant }$ | $R=\frac{\pi r^{4} \Delta P}{8 \eta L}$ | $F=\eta A \frac{\Delta v_{x}}{\Delta y}$ |


| Prefix | Abbreviation | $\mathbf{1 0}^{?}$ |  | Prefix | Abbreviation | $\mathbf{1 0}^{?}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Giga | G | $10^{9}$ |  | milli | m | $10^{-3}$ |
| Mega | M | $10^{6}$ |  | micro | $\mu$ | $10^{-6}$ |
| kilo | k | $10^{3}$ |  | nano | n | $10^{-9}$ |
| centi | c | $10^{-2}$ |  | pico | p | $10^{-12}$ |
|  |  |  |  | femto | f | $10^{-15}$ |

## Name:

Angular momentum $(\vec{L})$ is defined by the equation

$$
\vec{L}=\vec{r} \times(m \vec{v})
$$

where $\vec{r}$ is a position vector, $m$ is mass, and $\vec{v}$ is velocity. For this problem:

- $\vec{r}=(-5.00 \hat{\jmath}-8.00 \hat{k}) \mathrm{m}$
- $\quad m=1.00 \mathrm{~kg}$
- $\vec{v}=(-4.44 \hat{\imath}+7.77 \hat{k}) \frac{\mathrm{m}}{\mathrm{s}}$
**1a) Determine angular momentum (in Cartesian form).
**1b) Determine $\hat{v}$ (in Cartesian form).
**1c) Determine the angle between $\vec{r} \& \vec{v}$.

| 1a |  |
| :--- | :--- |
| 1 C |  |
|  |  |
| 1 c |  |
|  |  |

A metal widget is built by making several cuts from a square metal plate of side length $s$ and thickness $\frac{s}{7}$. In the figure, thickness is the dimension into the page. Two triangular sections are removed from the square plate to create the widget (see figure). The black dots in the figure indicate the midpoints of the sides of the square.
**2) Determine an algebraic expression for the volume of the widget (after the cuts are made).
Answer as a decimal number with three sig figs times $\boldsymbol{s}^{\mathbf{3}}$ for credit.

**3) A made-up physics equation is given by

$$
a=g+\frac{v^{2}}{k}
$$

A data table of measurements is shown at right.
Determine a value for $k$.
Use correct sig figs in engineering notation with appropriate prefix.

| $v\left(\frac{\mathrm{~cm}}{\mathrm{~s}}\right)$ | $g\left(\frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)$ | $a\left(\frac{\mathrm{~cm}}{\mathrm{~s}^{2}}\right)$ |
| :---: | :---: | :---: |
| $2.00 \times 10^{4}$ | 9.806 | 918 |

**4) Suppose you are told an acceleration equation is given by

$$
a=\sqrt{\frac{c x^{7}}{v^{5}}}-\frac{k v^{4}}{x^{2}}
$$

In this equation $x$ represents horizontal position, $v$ represents speed, and $c \& k$ are positive constants. Determine the units required for the constant $c$. Write your final answer in terms of $\mathrm{kg}, \mathrm{m}, \& \mathrm{~s}$. Note: it is possible for some or all of the units cancel out.

An object travelling in 1D motion is visualized with the plot of velocity versus time shown at right.
**5a) Determine distance traveled for the entire time interval shown.

**5b) Determine acceleration at 0.40 s .

5c) Over what time interval(s), or at what instant(s) in time, is this object moving backwards. If this doesn't occur in the time interval shown, state "doesn't occur).

5d) Over what time interval(s), or at what instant(s) in time, is this object moving backwards and slowing down. If this doesn't occur in the time interval shown, state "doesn't occur).

A person drops a stone into a deep well. The person hears the sound of the splash 5.00 s after releasing the stone from rest. To be clear, in this problem the well is so deep the time required for the sound to travel is NOT negligible. You may assume the sound wave from the splash travels upwards with constant speed $343 \frac{\mathrm{~m}}{\mathrm{~s}}$.


A ball is initially located at the origin shown in the figure. The ball is thrown with initial speed $v$ and launch angle $\theta$. The ball impacts an incline distance $d$ from the origin. To be clear, the angle $\theta$ is measured from the horizontal axis shown in the figure with a dotted line. The surface of the incline can be modeled by the equation $y=k x$ where $x$ is the horizontal position (relative to the origin shown) and $k$ is an known constant. The magnitude of acceleration due to gravity is $g$.

*****7) Determine the vertical component of impact velocity.


A drone travels in two straight line displacements. In the first stage, the drone's displacement vector is described by $\vec{d}_{1}=(6.66 \hat{\imath}+4.44 \hat{k}) \mathrm{m}$. After the second stage, the drone's final position is 8.88 m angled $22.2^{\circ}$ west of south. Figure not to scale.
****8) Determine the distance traveled during the second displacement.


A particle moves in one dimensional motion with acceleration described by

$$
a=\frac{b t}{v^{6}}
$$

where $b$ is a constant, $v$ is velocity, and $t$ is time. At time $t=0$ the particle has speed $v_{0}$ moving to the right.
****9) Determine the particle's velocity as a function of time.

|  |  |
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| 9 |  |
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