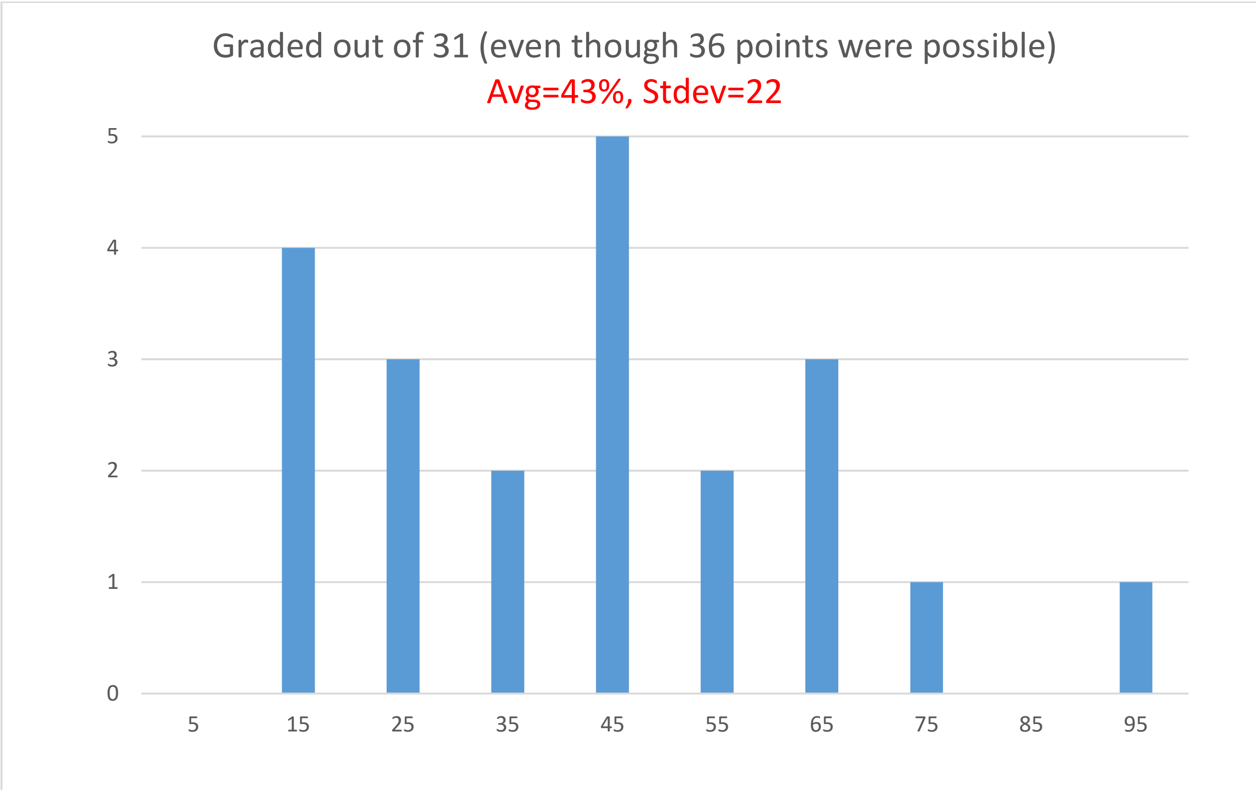
161sp22t1dSoln

Distribution on this page. Solutions begin on the next page.



**1a)

$$\vec{L} = \vec{r} \times (m\vec{v})$$

$$\vec{L} = (-5.00\hat{j} - 8.00\hat{k})\mathrm{m} \times \left((1.00\ \mathrm{kg})(-4.44\hat{\imath} + 7.77\hat{k})\frac{\mathrm{m}}{\mathrm{s}}\right)$$

$$\vec{L} = (-5.00\hat{j} - 8.00\hat{k})\mathrm{m} \times (-4.44\hat{\imath} + 7.77\hat{k})\ \frac{\mathrm{kg{\cdot}m^2}}{\mathrm{s}}$$

$$\vec{L} = (-38.9\hat{\imath} + 35.2\hat{j} - 22.2\hat{k})\ \frac{\mathrm{kg{\cdot}m^2}}{\mathrm{s}}$$

**1b) Notice the units will cancel out in a unit vector!

$$\hat{v} = \frac{\vec{v}}{v} = \frac{(-4.44\hat{\imath} + 7.77\hat{k})\frac{\mathrm{m}}{\mathrm{s}}}{\sqrt{\left(-4.44\frac{\mathrm{m}}{\mathrm{s}}\right)^2 + \left(7.77\frac{\mathrm{m}}{\mathrm{s}}\right)^2}}$$

$$\hat{v} = -0.496\hat{\imath} + 0.868\hat{k}$$

**1c) To get the angle between two vectors we use our standard approach.

$$\theta = \cos^{-1}\left(\frac{\vec{r}\cdot\vec{v}}{rv}\right)$$

Notice units will all drop out in this expression...as such I will ignore units for the rest of this calculation. Notice we need

$$r = \|\vec{r}\| = \sqrt{(-5.00)^2 + (-8.00)^2} = 9.4\underline{3}4$$

$$v = \|\vec{v}\| = \sqrt{(-4.44)^2 + (7.77)^2} = 8.9\underline{4}9$$

The only surviving term in $\vec{r}\cdot\vec{v}$ will be $r_z v_z$.

$$\theta = \cos^{-1}\left(\frac{r_z v_z}{rv}\right)$$

$$\theta = \cos^{-1}\left(\frac{(-8.00)(7.77)}{(9.4\underline{3}4)(8.9\underline{4}9)}\right)$$

$$\boldsymbol{\theta = 137.4°}$$

**2) Notice the two triangular sections can be combined to make a rectangle with sides $\frac{s}{2}$ & s

$$A_{total} = s^2 - \left(\frac{s}{2}\right)(s)$$

$$A_{total} = \frac{1}{2}s^2$$

Get volume by multiplying by thickness $\frac{s}{8}$

$$V_{total} = A \times thickness = \left(\frac{1}{2}s^2\right)\frac{s}{7}$$

$$V_{total} = \frac{s^3}{14}$$

You were told "**Answer as a decimal number with three sig figs times s^3 for credit.**"

$$\boldsymbol{V_{total} = 0.0714s^3}$$

**3) Watch the units carefully ($918\,\frac{\text{cm}}{\text{s}^2} \rightarrow 9.18\,\frac{\text{m}}{\text{s}^2}$ & $2.00 \times 10^4\,\frac{\text{cm}}{\text{s}} \rightarrow 20\underline{0}\,\frac{\text{cm}}{\text{s}}$). Also, you were told to track sig figs on this one. Be extra careful with sig figs any time you have addition or subtraction as the least significant *column* of sig figs matters for those operations! Rearrange to get

$$k = \frac{v^2}{a - g}$$

$$k = \frac{\left(20\underline{0}\,\frac{\text{cm}}{\text{s}}\right)^2}{\left(9.18\,\frac{\text{m}}{\text{s}^2}\right) - \left(9.806\,\frac{\text{m}}{\text{s}^2}\right)}$$

$$k = \frac{40\underline{0}00\,\frac{\text{m}^2}{\text{s}^2}}{-0.6\underline{2}6\,\frac{\text{m}}{\text{s}^2}}$$

$$k = -6\underline{3}898\ \text{m}$$

$$\boldsymbol{k = -64\ \text{km}}$$

**4) The units of any one term match the units of every other term. In this case $[a] = \left[\sqrt{\frac{cx^7}{v^5}}\right]$. Square both sides.

$$[c] = \frac{1}{\text{s}^9}$$

**5a) WATCH THOSE UNITS! On a plot of velocity versus time, displacement is area under the curve.
Areas below the time axis are negative.
Notice the object reverses direction at $t = 0.40$ s.
Get the displacement (area) from $t = 0.00\ \text{s} \rightarrow 0.40$ s and also the displacement (area) from $t = 0.40\ \text{s} \rightarrow 1.00$ s.
Get absolute value each displacement then sum them to get distance traveled.

$$d = \left|\frac{1}{2}(0.40\ \text{s})\left(200\,\frac{\text{mm}}{\text{s}}\right)\right| + \left|\frac{1}{2}(0.60\ \text{s})\left(-100\,\frac{\text{mm}}{\text{s}}\right)\right|$$

$$d = 7\underline{0}\ \text{mm}$$

**5b) Acceleration is slope of the vt curve. Generally speaking, select points in time just before or just after the instant of interest to ensure a decent estimate for the slope. Here, because the plot is linear, it isn't so bad.

$$a = slope = \frac{rise}{run}$$

$$a = \frac{\left(-100\,\frac{\text{mm}}{\text{s}}\right) - \left(200\,\frac{\text{mm}}{\text{s}}\right)}{(0.60\ \text{s}) - (0.00\ \text{s})}$$

$$\boldsymbol{a = -5\underline{0}0\,\frac{\text{mm}}{\text{s}^2}}$$

5c) Moving backwards implies $v < 0$.
This occurs between $t = 0.40\ \text{s} \rightarrow 1.00\ \text{s}$.

5d) Moving backwards implies $v < 0$.
Slowing down implies a has opposite sign as v (in this case $a > 0$).
Recall, acceleration is the slope of the vt curve.
Therefore we are looking for times with negative velocity and positive slope.
This occurs between $t = 0.60\ \text{s} \rightarrow 1.00\ \text{s}$.

*****6) For the fall $\Delta y = -h, a_y = -g,$ and $v_{iy} = 0$ (rock dropped, not thrown).

$$\Delta y = \frac{1}{2}a_y t_1^2 + v_{iy}t_1$$

$$-h = \frac{1}{2}(-g)t_1^2$$

$$\boldsymbol{h = \frac{g}{2}t_1^2}$$

For the sound returning to the rock dropper after the splash

$$h = v_{sound}t_2$$

$$h = v_{sound}(t_{total} - t_1)$$

From here you could go a couple of different ways.
You could isolate t_1 in the lower equation, plug into the upper equation, and solve the quadratic for h.
Alternatively, you could set the equations equal, solve the quadratic for t_1, then use the upper equation to get h.
I kept a couple extra sig figs on the intermediate calculations since the quadratic formula involves a subtraction.

$$t_1 = t_{total} - \frac{h}{v_{sound}}$$

$$h = \frac{g}{2}\left(t_{total} - \frac{h}{v_{sound}}\right)^2$$

$$\frac{2h}{g} = t_{total}^2 + \frac{h^2}{v_{sound}^2} - \frac{2ht_{total}}{v_{sound}}$$

$$\frac{2h}{g}v_{sound}^2 = v_{sound}^2 t_{total}^2 + h^2 - 2hv_{sound}t_{total}$$

$$0 = h^2 - h\left(\frac{2v_{sound}^2}{g} + 2v_{sound}t_{total}\right) + v_{sound}^2 t_{total}^2$$

$$0 = h^2 - h\left(\frac{2\left(343\,\frac{\text{m}}{\text{s}}\right)^2}{9.8\,\frac{\text{m}}{\text{s}^2}} + 2\left(343\,\frac{\text{m}}{\text{s}}\right)(5.00\text{ s})\right) + \left(343\,\frac{\text{m}}{\text{s}}\right)^2(5.00\text{ s})^2$$

$$0 = h^2 - h(2.744 \times 10^4\text{ m}) + 2.9412 \times 10^6\text{ m}^2$$

$$\boldsymbol{h = 107.6\text{ m}}$$

*****7) There were several good ways to solve this problem. Alternate methods described on the next page…

First method:

Since the ball starts at the origin and moves into the first quadrant, we can use $\Delta x = x$ and $\Delta y = y$ for this problem.

$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$ $y = v \sin\theta \, t - \frac{1}{2} g t^2$	$\Delta x = v_{ix} t + \frac{1}{2} a_x t^2$ $x = v \cos\theta \, t$

Now use the constraint equation.

$$y = kx$$

$$v \sin\theta \, t - \frac{1}{2} g t^2 = k(v \cos\theta \, t)$$

Notice we can cancel a t in each term then solve for t.

$$v \sin\theta - \frac{1}{2} g t = k(v \cos\theta)$$

$$t = \frac{2v}{g}(\sin\theta - k\cos\theta)$$

From here we can use

$$v_{fy} = v_{iy} + a_y t$$

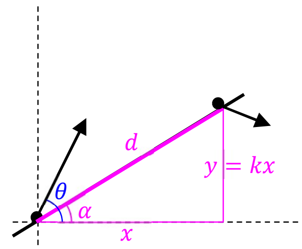
$$v_{fy} = v \sin\theta + (-g)\left[\frac{2v}{g}(\sin\theta - k\cos\theta)\right]$$

$$v_{fy} = v \sin\theta - 2v(\sin\theta - k\cos\theta)$$

$$\boldsymbol{v_{fy} = 2kv\cos\theta - v\sin\theta}$$

Alternate method 1:

You were told the distance along the ramp was d.
Consider the figure at right.
Notice you could use



$$x^2 + y^2 = d^2$$

$$x^2 + (kx)^2 = d^2$$

$$x = \frac{d}{\sqrt{1+k^2}}$$

From there one could determine $y = kx = \frac{kd}{\sqrt{1+k^2}}$ then plug into the $v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$ eqt'n to determine v_{fy}.

$$v_{fy}^2 = \pm\sqrt{v^2\sin^2\theta - \frac{2gkd}{\sqrt{1+k^2}}}$$

Discussion of $\pm$ sign follows after next alternate method…

Alternate method 2:

Consider the figure at right.
Notice

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1} k$$

We also know impact height is $y = d\sin\alpha = d\sin(\tan^{-1} k)$.

While ugly, you could plug this into the $v_{fy}^2 = v_{iy}^2 + 2a_y\Delta y$ eqt'n to determine v_{fy}.

$$v_{fy}^2 = \pm\sqrt{v^2\sin^2\theta - 2gd\sin(\tan^{-1} k)}$$

Think about the $\pm$ symbol:

Consider what happens if k is a big number versus a tiny number.
Under what circumstances, in general, should you use the positive root?
Notice the first solution, on the previous page, did not require this kind of thought process.
I argue that makes the first page solution a better solution.

****8) Everything is in meters for this problem. I will ignore units until the final answer. In this problem we know

$$\vec{d}_1 + \vec{d}_2 = \vec{r}$$

$$\vec{d}_2 = \vec{r} - \vec{d}_1$$

Watch out! Notice the angle for final position vector is given to the negative y axis.
We expect negative components for both $\hat{\imath}$ & $\hat{\jmath}$. We expect the $\hat{\jmath}$ term to use cos 22.2°…

$$\vec{d}_2 = \left(8.88\sin 22.2°\,(-\hat{\imath}) + 8.88\cos 22.2°\,(-\hat{\jmath})\right) - (6.66\hat{\imath} + 4.44\hat{k})$$

$$\vec{d}_2 = \left(-3.3\underline{5}\underline{5}\hat{\imath} - 8.2\underline{2}\underline{2}\hat{\jmath}\right) - (6.66\hat{\imath} + 4.44\hat{k})$$

$$\vec{d}_2 = -10.0\underline{1}5\hat{\imath} - 8.2\underline{2}2\hat{\jmath} - 4.44\hat{k}$$

The distance traveled during the second displacement is the magnitude of this vector.

$$\boldsymbol{d_2 = 13.70\ \text{m}}$$

Note: in general we include an extra sig fig on a number if the first digit is a 1.
Also, units are required on the final answer for full credit.

****9) We are given

$$a = \frac{bt}{v^6}$$

To get velocity as a function of time, use the separation of variables technique.

$$\frac{dv}{dt} = \frac{bt}{v^6}$$

$$v^6 \, dv = bt \, dt$$

$$\int_{v_0}^{v_f} v^6 \, dv = \int_{t_i=0}^{t_f} bt \, dt$$

$$\frac{v_f^7}{7} - \frac{v_0^7}{7} = \frac{bt_f^2}{2} - 0$$

After integration, we can change $t_f \to t$. At this point, v_f is equivalent to $v(t)$…

$$v_f = v(t) = \sqrt[7]{v_0^7 - \frac{7}{2}bt^2}$$