161 Spring 2022 Test 2a Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

| $V_{\text {sphere }}=\frac{4}{3} \pi R^{3}$ | $V_{\text {box }}=L W H$ | $V_{c y l}=\pi R^{2} H$ | $\rho=\frac{M}{V}$ |
| :---: | :---: | :---: | :---: |
| $A_{\text {sphere }}=4 \pi R^{2}$ | $V=\left(A_{\text {base }}\right) \times($ height $)$ | $A_{\text {circle }}=\pi R^{2}$ | $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ |
| $C=2 \pi R$ | $A_{\text {rect }}=L W$ | $A_{\text {cylside }}=2 \pi R \mathrm{H}$ |  |
| $160 \underline{9} \mathrm{~m}=1 \mathrm{mi}$ | $12 \mathrm{in}=1 \mathrm{ft}$ | $60 \mathrm{~s}=1 \mathrm{~min}$ | $1000 \mathrm{~g}=1 \mathrm{~kg}$ |
| $2.54 \mathrm{~cm}=1 \mathrm{in}$ | $1 \mathrm{cc}=1 \mathrm{~cm}^{3}=1 \mathrm{~mL}$ | $60 \mathrm{~min}=1 \mathrm{hr}$ | $100 \mathrm{~cm}=1 \mathrm{~m}$ |
| $1 \mathrm{~cm}=10 \mathrm{~mm}$ | 1 yard $=3 \mathrm{ft}$ | $3600 \mathrm{~s}=1 \mathrm{hr}$ | $1 \mathrm{~km}=1000 \mathrm{~m}$ |
| 1 furlong = 220 yards | $528 \underline{\mathrm{ft}}=1 \mathrm{mi}$ | $24 \mathrm{hrs}=1$ day | $1 \mathrm{rev}=2 \pi \mathrm{rad}=360^{\circ}$ |
| $g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ | $G=6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}}$ | $P_{0}=1.0 \times 10^{5} \mathrm{~Pa}$ | $1 \mathrm{eV}=1.60 \underline{2} \times 10^{-19} \mathrm{~J}$ |
| $1 \mathrm{~N}=1 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}$ | $1 \mathrm{~J}=1 \mathrm{~N} \cdot \mathrm{~m}$ | $1 \mathrm{~Pa}=1 \frac{\mathrm{~N}}{\mathrm{~m}^{2}}$ |  |
| $x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$ | $v_{f x}^{2}=v_{i x}^{2}+2 a_{x}(\Delta x)$ | $v_{f x}=v_{i x}+a_{x} t$ | $r=\sqrt{x^{2}+y^{2}}$ |
| $\vec{A} \cdot \vec{B}=A B \cos \theta_{A B}$ | $\\|\vec{A} \times \vec{B}\\|=A B \sin \theta_{A B}$ | $\begin{aligned} & \sin (A \pm B) \\ & =\sin A \cos B \pm \cos A \sin B \end{aligned}$ | $\begin{aligned} & \cos (A \pm B) \\ & =\cos A \cos B \mp \sin A \sin B \end{aligned}$ |
| $\vec{v}_{a e}+\vec{v}_{e b}=\vec{v}_{a b}$ | $\hat{r}=\cos \theta \hat{\imath}+\sin \theta \hat{\jmath}$ | $\hat{\theta}=-\sin \theta \hat{\imath}+\cos \theta \hat{\jmath}$ |  |
| $a_{t a n}=r \alpha$ | $a_{c}=\frac{v^{2}}{r}=r \omega^{2}$ | $\vec{a}=a_{r} \hat{r}+a_{\text {tan }} \hat{\theta}$ | $\vec{a}=a_{c}(-\hat{r})+a_{t a n} \hat{\theta}$ |
| $\Sigma \vec{F}=m \vec{a}$ | $f \leq \mu n$ | $F_{G}=\frac{G m M}{r^{2}}(-\hat{r})$ | $U_{G}=-\frac{G m M}{r}$ |
| $T K E=\frac{1}{2} m v^{2}$ | $R K E=\frac{1}{2} I \omega^{2}$ | $U_{S}=S P E=\frac{1}{2} k x^{2}$ | $U_{G}=G P E=m g h$ |
|  | $\Delta K E=W_{\text {ext.\& }}$ non-con | $W=F d \cos \theta=F_{\\| \\|} d$ | $W=\int F_{x} d x$ |
| $\Delta U=-W=-\int_{i}^{f} \vec{F} \cdot d \vec{s}$ | $F_{x}=-\frac{d}{d x} U(x)$ | $\mathcal{P}_{\text {inst }}=\frac{d E}{d t}=\vec{F} \cdot \vec{v}$ | $\mathcal{P}_{\text {avg }}=\frac{\Delta E}{\Delta t}=\frac{\text { Work }}{\text { time }}$ |
| $\vec{J}=\Delta \vec{p}=\vec{F} \Delta t$ | $\vec{p}=m \vec{v}$ | $x_{\mathrm{CM}}=\frac{m_{1} x_{1}+m_{2} x_{2}}{m_{1}+m_{2}}$ | $x_{\mathrm{CM}}=\frac{\int x d m}{\int d m}$ |
| $\vec{\tau}=\vec{r} \times \vec{F}$ | $\Sigma \vec{\tau}=I \vec{\alpha}$ | $L=I \omega=m v r_{\perp}$ | $\mathcal{P}_{\text {inst }}=\vec{\tau} \cdot \vec{\omega}$ |
| $s=r \Delta \theta$ | $v=r \omega$ | $a_{\text {tan }}=r \alpha$ | $a_{c}=\frac{v^{2}}{r}=r \omega^{2}$ |
| $I_{\\| \text {laxis }}=I_{\mathrm{CM}}+m d^{2}$ | $I_{z z}=I_{x x}+I_{y y}$ | $I=\int r^{2} d m$ | $\frac{F}{A}=E \frac{\Delta L}{L_{0}}$ |
| $P=\frac{F}{A}$ | $P_{\text {gauge }}=P_{\text {abs }}-P_{\text {ambient }}$ | $B=\rho_{f} V_{\text {disp }} g$ | $A_{1} v_{1}=A_{2} v_{2}$ |
| $P(h)=P_{0}+\rho g h$ | $P+\frac{1}{2} \rho v^{2}+\rho g h=$ constant | $R=\frac{\pi r^{4} \Delta P}{8 \eta L}$ | $F=\eta A \frac{\Delta v_{x}}{\Delta y}$ |


| Prefix | Abbreviation | $\mathbf{1 0}^{\text {? }}$ |  | Prefix | Abbreviation | $\mathbf{1 0}^{\text {? }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Giga | G | $10^{9}$ |  | milli | m | $10^{-3}$ |
| Mega | M | $10^{6}$ |  | micro | $\mu$ | $10^{-6}$ |
| kilo | k | $10^{3}$ |  | nano | n | $10^{-9}$ |
| centi | c | $10^{-2}$ |  | pico | p | $10^{-12}$ |
|  |  |  |  | femto | f | $10^{-15}$ |

## Name:

For this problem you might want to skim the entire page before starting work so you know what I ultimately want you to figure out. Please let me know if you do scratch work elsewhere so I know to look for it.

A block of mass $3 m$ is on top of a block of mass $m$. The blocks are on a horizontal surface. Negligible friction exists between the lower block and the floor. Frictional coefficients between the upper and lower blocks are $\mu_{s}=0.800 \& \mu_{k}=0.700$. The top block is pushed to the right by an applied force. Both blocks to accelerate to the right with magnitude $a=\frac{g}{12}$. The angle is $\phi=15.0^{\circ}$. The top block does not slide relative to the lower block. ******1a) Draw FBDs for each block \& list the force equations. Include a coordinate system to ensure credit. I expect you to label the unknown force as $F \ldots$

1b) Determine the magnitude of the applied force.
Answer as a decimal number with 3 sig figs times $\boldsymbol{m g}$.
1c) Determine the frictional force (magnitude) between the blocks.
Answer as a decimal number with 3 sig figs times $\boldsymbol{m g}$.


For this problem you may assume friction is negligible everywhere.
A block of mass $m$ is placed on a ramp shaped like a quarter-circle at angle $\theta$ as shown in the figure. You may assume the block has negligible size compared to the radius of the quarter-circle. Upon being released from rest, the block slides with negligible friction until impacting a spring of constant $k$. The spring experiences maximum compression distance $d$.

****2) Determine the radius of the quarter-circle.

A particle of mass $6.00 \times 10^{-12} \mathrm{~kg}$ travels in 1D motion under the influence of a conservative force. This problem assumes the particle's motion was analyzed using a standard coordinate system of $+x$-direction to the right and $+y$-direction upwards. The plot of potential energy versus position for this force is shown at right. The particle is initially located at position $x=+35.0 \mathrm{~nm}$ and is moving left.

3a) Which of the following best describes the direction of the force acting on the particle at its initial position? Circle the best answer.

| Up | Down | Right | Left |
| :---: | :---: | :---: | :---: |
| Up \& right | Down \& right | Impossible to determine <br> without more info |  |
| Up \& left | Down \& left |  |  |


$* * * * 3 b)$ Determine the minimum initial speed required for the particle to reach position $x=-40.0 \mathrm{~nm}$. Answer using engineering notation with appropriate choice of prefix (or scientific notation).

3b

A block of negligible size slides along the interior surface of a cone. The half-angle of the cone is $\phi$ (from the cone's surface to the vertical). The circular motion of the block is in the horizontal plane distance $h$ above the cone's apex (point at the bottom). The magnitude of freefall acceleration near earth's surface is $g$. Assume friction is negligible between the block and the surface of the cone.
****4) Determine the speed of the block in terms of given parameters. Note: you may wish to introduce additional parameters when doing your work, but your final answer must be given in terms of only given parameters $g, \phi$, and/or $h$.


A rocket of mass $m=300.0 \mathrm{~kg}$ is initially moving with speed $v_{0}=2.00 \times 10^{3} \frac{\mathrm{~m}}{\mathrm{~s}}$ in the positive $x$-direction. The rocket is in deep space, far from any possible sources of gravitational forces. For this problem, you may assume both gravitational and frictional forces are negligible. At time $t=0$, a thruster causes the rocket to speed up for exactly 1.000 s . Video analysis determines the velocity as a function of time the rocket is

$$
v_{x}(t)=v_{0}+555 t^{3}
$$

This equation is valid for the entire 1.000 s time interval.

5a) What units are implied on the number 555 ?
***5b) What force (magnitude) is exerted by the thruster at $t=600.0 \mathrm{~ms}$ ?

## Answer in engineering notation with best choice of prefix.

**5c) WATCH OUT! In this part I used a different time to simplify the math! What power is delivered by the thruster at $t=1.000 \mathrm{~s}$ ?
Answer in engineering notation with best choice of prefix.

| 5 a |  |
| :--- | :--- |
| 5 b |  |
|  |  |
| 5 c |  |
|  |  |

A rotating rod (not a string) has two point masses connected to it as shown. The rod rotates with constant rate 55.5 RPMs in the direction shown. The rod has length 44.4 cm . Point masses $\mathbf{1} \& 2$ are 33.3 g each and have negligible size compared to the rod. They are numbered to simplify communication later. The rod has negligible mass compared to the point masses.
Notice the additional questions at the bottom of the page...
Read parts a \& b before attempting part a...it may help you think about the problem. ***6a) For the instant shown, determine the force magnitude exerted by the rod on mass 1. $6 b)$ For the instant shown, determine the force direction exerted by the rod on mass 1 . In particular, is the rod pushing up or pulling down on mass $\mathbf{1}$. Hint: pick a direction then do the
 math. If you get a negative sign, the force is opposite the direction you chose to draw it!


6c) During one full revolution, which best describes the signs of work done by gravity on mass $\mathbf{1}$ ?

| Positive work <br> for entire revolution | Positive work on the way down, <br> negative work on the way up | Zero work done |
| :---: | :---: | :---: |
| Negative work <br> for entire revolution | Negative work on the way down, <br> positive work on the way up | Impossible to determine <br> without more info |

6d) During one full revolution, which best describes the signs of work done by the rod's force on mass $\mathbf{1}$ ?

| Positive work <br> for entire revolution | Positive work on the way down, <br> negative work on the way up | Zero work done |
| :---: | :---: | :---: |
| Negative work <br> for entire revolution | Negative work on the way down, <br> positive work on the way up | Impossible to determine <br> without more info |

Two blocks have masses $m_{1}$ and $m_{2}>m_{1}$. The lower block is connected to an ideal string (massless \& inextensible) run over an ideal pulley (massless with negligible axle friction). The system is perfectly balanced such that the two blocks can be raised or lowered without the upper block slipping and falling. Frictional forces are negligible in this problem.

7a) If the blocks are lowered at constant rate, which best describes the forces exerted between blocks $1 \& 2$. Circle the best answer.

| $m_{1}$ pushes down <br> harder on $m_{2}$ than <br> $m_{2}$ pushes up on $m_{1}$ | $m_{2}$ pushes up harder <br> on $m_{1}$ than $m_{1}$ <br> pushes down on $m_{2}$ | $m_{1}$ pushes down the <br> same amount on $m_{2}$ as <br> $m_{2}$ pushes up on $m_{1}$ | Impossible to <br> determine without <br> more info |
| :---: | :---: | :---: | :---: |



7b) If the blocks are lowered with increasing rate, which best describes the forces exerted between blocks $1 \& 2$. Note: for the rest of the problem all questions will assume the blocks are lowered with increasing rate.

| $m_{1}$ pushes down harder on $m_{2}$ than $m_{2}$ pushes up on $m_{1}$ | $m_{2}$ pushes up harder on $m_{1}$ than $m_{1}$ pushes down on $m_{2}$ | $m_{1}$ pushes down the same amount on $m_{2}$ as $m_{2}$ pushes up on $m_{1}$ | Impossible to determine without more info |
| :---: | :---: | :---: | :---: |

7c) If the blocks are lowered with increasing rate, which best describes the work done on $m_{2}$ by the normal force between the blocks?

| positive work | negative work | zero work | Impossible to <br> determine without <br> more info |
| :---: | :---: | :---: | :---: |

**7d) Again assume the blocks are lowered with increasing rate.
For this question, assume the action force is the weight of block $m_{2}$.
Describe the reaction force by filling in the sentence below with:

- The object exerting the reaction force
- The type of force (normal, frictional, tension, gravitational, etc)
- The direction of the reaction force
- The object experiencing the reaction force
$\qquad$ exerts a $\qquad$ force directed $\qquad$ on $\qquad$ .

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