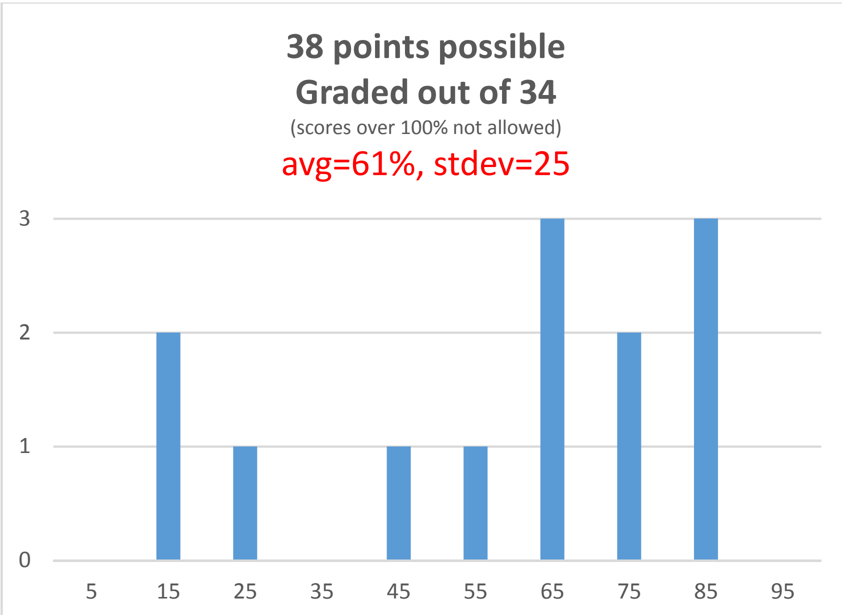
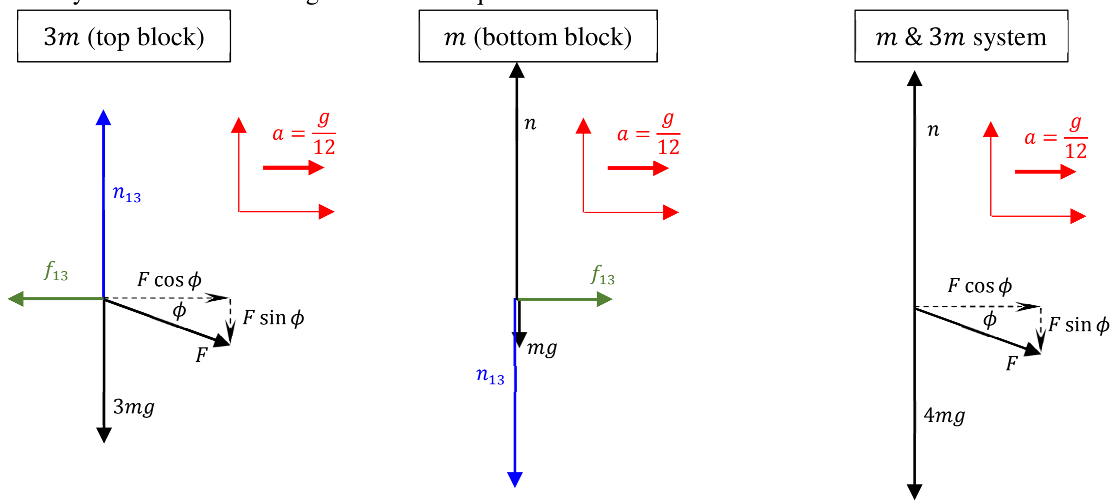
161sp22t2aSoln

Distribution on this page. Solutions begin on next page.



1a) Force arrows not drawn to scale.
I threw in a system FBD even though it was not required.



$3m$ (top block)	m (bottom block)	m & $3m$ system
X: $F\cos\phi - f_{13} = (3m)\frac{g}{12}$ Y: $n_{13} = F\sin\phi + 3mg$	X: $f_{13} = m\frac{g}{12}$ Y: $n = n_{13} + mg$	X: $F\cos\phi = (4m)\frac{g}{12}$ Y: $n = F\sin\phi + 4mg$

1b) $F = \frac{(4m)\frac{g}{12}}{\cos 15.0°} = 0.345mg$

1c) $f_{13} = m\frac{g}{12} = 0.0833mg$
In this problem, there is no indication of the upper block being on the verge of slipping.
Most common mistake is to think $f = \mu_s n$ applies.

If you are curious, one can determine $n_{13} = F\sin\phi + 3mg = 3.089mg$.
The *maximum possible* friction between the two blocks (using $F = 0.345mg$) is thus $f_{13max} = \mu_s n_{13} = 2.47mg$.
Again, we see we only require $f_{13} = 0.0833mg$ to cause the lower block to accelerate at rate $\frac{g}{12}$ (even though more friction is possible between the two blocks).

2) Before and after pictures shown below.

Normal force does zero work on the block (normal force always perpendicular to displacement).
Friction does zero work (we were told friction was negligible in problem statement).

$$K_i + U_{Gi} + U_{Si} + W_{\substack{ext \\ n.c.}} = K_f + U_{Gf} + U_{Sf}$$

$$0 + U_{Gi} + 0 + 0 = 0 + 0 + U_{Sf}$$

$$U_{Gi} = U_{Sf}$$

$$mgR(1 - \cos\theta) = \frac{1}{2}kd^2$$

$$\boldsymbol{R = \frac{kd^2}{2mg(1 - \cos\theta)}}$$

3a) Remember the analogy with a marble on a track is not perfect.
While a marble moves in both the x and y directions, the forces on our particle are only left or right.
At $x = 35.0$ nm the force is to the *left* on our particle ($slope > 0$ implies $F_x < 0$).

3b) The particle starts at $x = +35.0$ nm and is moving left.
The particle must reach position $x = +10.0$ nm with some non-zero speed to make it to $x = -40.0$ nm.
To find *minimum* initial speed, do energy conservation assuming *exactly* zero speed at $x = +10.0$ nm.
Get the values of U_i & U_f from the plot!
Note: unless otherwise specified, on these types of problems assume the conservative force is the *only* force acting on the particle. As such, we needn't worry about work done by non-conservative or external forces.

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + U_i = 0 + U_f$$

$$\frac{1}{2}mv_i^2 = U_f - U_i$$

$$\frac{1}{2}mv_i^2 = \Delta U$$

$$v_i = \sqrt{\frac{2}{m}\Delta U}$$

No need for $\pm$ here since we are determining speed (a positive parameter).

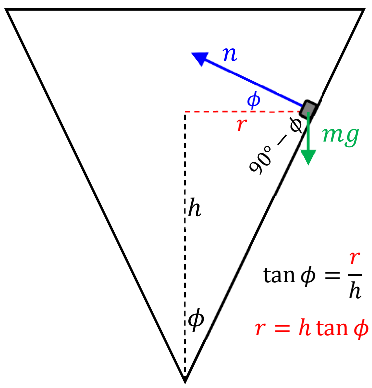
Again, we are concerned with ΔU between $x_i = 35$ nm and the trouble spot $x_f = 10$ nm.

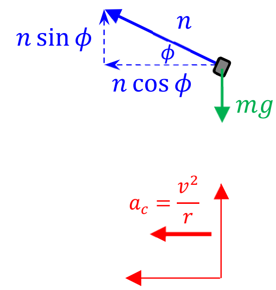
$$v_i = \sqrt{\frac{2}{6.00 \times 10^{-12}\ \text{kg}}(200 \times 10^{-3}\ J)}$$

$$\boldsymbol{v_i = 2.58 \times 10^5\ \frac{m}{s} = 258\ \frac{km}{s}}$$

4) I find the solution to this problem really interesting for some reason…

Consider the side view and associated FBD shown below.





Force equations give

$$n\sin\phi = mg$$

$$n\cos\phi = ma_c$$

Taking the ratio of these equations gives

$$\tan\phi = \frac{g}{a_c}$$

$$a_c = \frac{g}{\tan\phi}$$

$$\frac{v^2}{r} = \frac{g}{\tan\phi}$$

Don't forget to write the answer in terms of given parameters!

In this case we must replace r in terms of the knowns h & ϕ…

$$\frac{v^2}{h\tan\phi} = \frac{g}{\tan\phi}$$

Here's the cool part…the angle drops out!!!

This means the result would be the same for any cone angle, as long as we are distance h above the cone's apex!

$$\boldsymbol{v = \sqrt{gh}}$$

5a) Our given equation was

$$v_x(t) = v_0 + 555t^3$$

This will only makes sense if we assume

$$[555] = \frac{\mathrm{m}}{\mathrm{s}^4}$$

5b) Answer: $\mathbf{179.8}$ kN.

We know $F_x = ma_x$.

Take derivative of v_x with respect to time to get $a_x = 1665t^2$.

Multiply by mass to get $F_x(t) = 4.995 \times 10^5 t^2$

Plug in $t = 600.0\text{ ms} = 0.6000\text{ s}$.

5c) Answer: $\mathbf{1.276}$ GW.

Recall instantaneous power is given by

$$\mathcal{P} = \vec{F} \cdot \vec{v}$$

In 1D motion this simplifies to

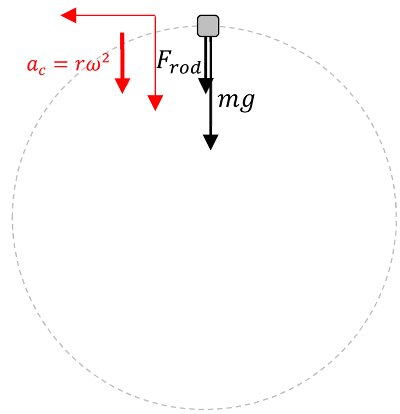
$$\mathcal{P} = F_x v_x$$

$$\mathcal{P} = (4.995 \times 10^5 t^2)\left(2.00 \times 10^3 \frac{\mathrm{m}}{\mathrm{s}} + 555t^3\right)$$

Plugging in $t = 1.000$ s gives the result.

6a) & 6b) I thought of two ways to approach this problem.

1. Don't spend all day thinking about the direction of the rod's force; pick one and see what happens…
   - If you get a negative result for your force, the force must be opposite the direction drawn.
2. Determine the critical value of rotation rate which causes *zero* force from the rod on m_1 at top of circle.
   - If our *actual* rotation rate is *larger* than the critical value, rod exerts force *downwards*.
   - If our *actual* rotation rate is *smaller* than the critical value, rod exerts force *upwards*.



I choose to assume the force exerted by the rod on m_1 was *downwards*.
According to my choices, the force equation becomes

$$F_{rod} + mg = ma_c$$

$$F_{rod} + mg = mr\omega^2$$

$$F_{rod} = mr\omega^2 - mg$$

$$F_{rod} = m(r\omega^2 - g)$$

Upon plugging in numbers…WATCH THOSE UNITS!

- $\omega = 55.5\,\frac{\text{rev}}{\text{min}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}} = 5.8\underline{1}2\,\frac{\text{rad}}{\text{s}}$
- $m = 33.3\text{ g} = 0.0333\text{ kg}$
- $r = \frac{diameter}{2} = 0.222\text{ m}$

My force equation gave the result $F_{rod} = -76.6$ mN.
Notice a *negative* force *downwards* is equivalent to a *positive* force *upwards*.
The force magnitude is 76.6 mN directed UPWARDS.

As a check, what rotation rate causes zero rod force on m_1 at the top of the circle?

$$F_{rod} + mg = ma_c$$

$$mg = mr\omega_{critical}^2$$

$$\omega_{critical} = \sqrt{\frac{g}{r}} = 6.64\,\frac{\text{rad}}{\text{s}}$$

Our *actual* rotation rate is smaller: $\omega = 5.8\underline{1}2\,\frac{\text{rad}}{\text{s}}$.

- If $\omega < \omega_{critical}$, we have a *slow* rotation. The rod must support part of the weight (F_{rod} points *up*).
- If $\omega > \omega_{critical}$, the rod must pull *down* on m_1 to keep it in the circular motion (F_{rod} points *down*).

6c) Work is positive when the force has a component pointing in the same direction as displacement.
Gravity does positive work on the way down; negative work on the way up.

6d) The rod's force on m_1 is always directed towards the center of the circle.
Since the mass is moving around the circle, this force is always perpendicular to displacement!
Work done by the rod's force is zero.

7a) The masses exert the same amount of force on each other (in accordance with Newton's 3rd law).

7b) The masses exert the same amount of force on each other (in accordance with Newton's 3rd law).
This is true if moving with constant speed or if they are accelerating.

7c) The normal force between the blocks acts downwards on m_2.
While moving downwards, this force does positive work.
This is true regardless of acceleration.

7d)
Action: The earth exerts a gravitational force directed downwards on mass 2.
Reaction: Mass 2 exerts a gravitational force directed upwards on the earth.