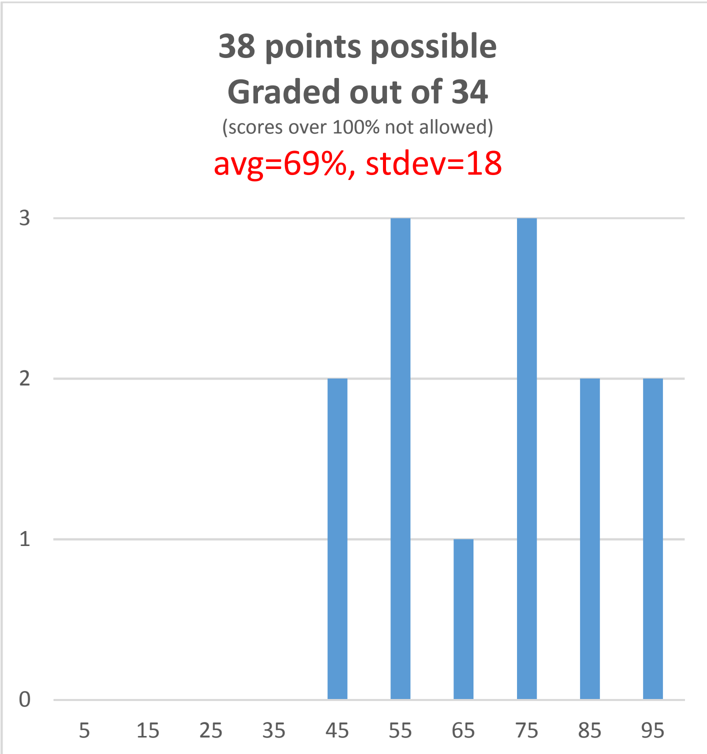
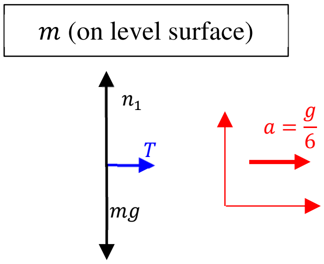
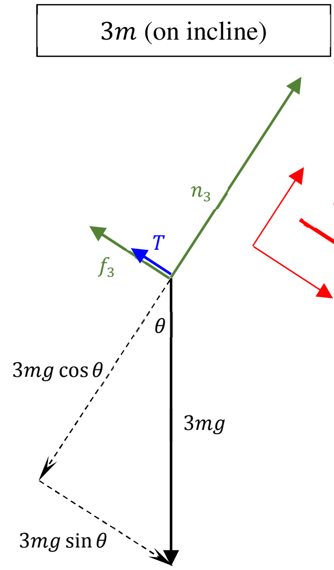
161sp22t2cSoln

Distribution on this page. Solutions begin on the next page.



1a) Force arrows not drawn to scale.





m (on level surface)	$3m$ (on incline)
X: $T = (m)\frac{g}{6}$ Y: $n_1 = mg$	X: $3mg\sin\theta - T - f_3 = (3m)\frac{g}{6}$ Y: $n_3 = 3mg\cos\theta$ Block slides relative to incline: $f_3 = \mu_k n_3 = \mu_k(3mg\cos\theta)$

1b) Use the X force equation for m to find

$$T = \frac{mg}{6} = 0.1667mg$$

1c) Rewrite X force equation for m plugging in $T = \frac{mg}{6}$ and $f_3 = 3\mu_k mg\cos\theta$.

$$3mg\sin\theta - \left(\frac{mg}{6}\right) - (3\mu_k mg\cos\theta) = 0.5mg$$

Notice mg cancels in every term!

$$3\sin\theta - \frac{1}{6} - (3\mu_k\cos\theta) = 0.5$$

$$3\mu_k\cos\theta = 3\sin\theta - \frac{1}{6} - 0.5$$

$$\mu_k = \frac{3\sin\theta - \frac{1}{6} - 0.5}{3\cos\theta}$$

Plug in the known angle and crank it out…

$$\mu_k = \frac{3\sin 40.0° - \frac{1}{6} - 0.5}{3\cos 40.0°}$$

$$\boldsymbol{\mu_k = 0.549}$$

2a) **Answer:** $\boldsymbol{T < mg}$
The block is moving up and slowing down.
Because it is moving up we know $v_y > 0$.
For *this* case, slowing down implies $a_y < 0$ (in 1D motion slowing down implies a & v have opposite signs).
We know acceleration should point in the direction of the larger force.
This implies weight has larger magnitude than tension.

2b) Work is *positive* for the tension force pointing in the same direction as displacement.
Acceleration direction has no effect on this statement.

3a) The normal force between the blocks points *to the left* on M.
Block M is being displaced *to the right*.
This implies the normal force between the blocks does *negative* work on M.

3b) The only horizontal force on the system is T (no friction between floor and block).
Doubling the tension must double acceleration.
To double the acceleration of block m, we require twice the normal force between the blocks.

3c) The new frictional force magnitude is unchanged.
Doubling the normal force doubles the *maximum possible* friction between the blocks (given by $f'_{12max} = \mu_s n'_{12}$).
HOWEVER, we only require an upwards frictional force of magnitude mg to blance the weight of block m!
Think: if you pull harder, block m is no longer on the verge of slipping (implying $f'_{12} < \mu_s n'_{12}$)!

3d)
Action: The earth exerts a gravitational force directed downwards on block m.
Reaction: Block m exerts a gravitational force directed upwards on the earth.

4a) $[\alpha] = \frac{\mathrm{N}}{\mathrm{m}^5}$

4b) I choose to start from

$$\Delta U = -\int_{x_i}^{x_f} F_x \, dx$$

$$U_f = U_i - \int_{x_i}^{x_f} F_x \, dx$$

In the problem statement we were told $U_i = 0$ if we use $x_i = 0$.

$$U_f = -\int_0^{x_f} F_x \, dx$$

$$U_f = -\int_0^{x_f} (-\alpha x^5) \, dx$$

$$U_f = \left[\frac{\alpha x^6}{6}\right]_0^{x_f}$$

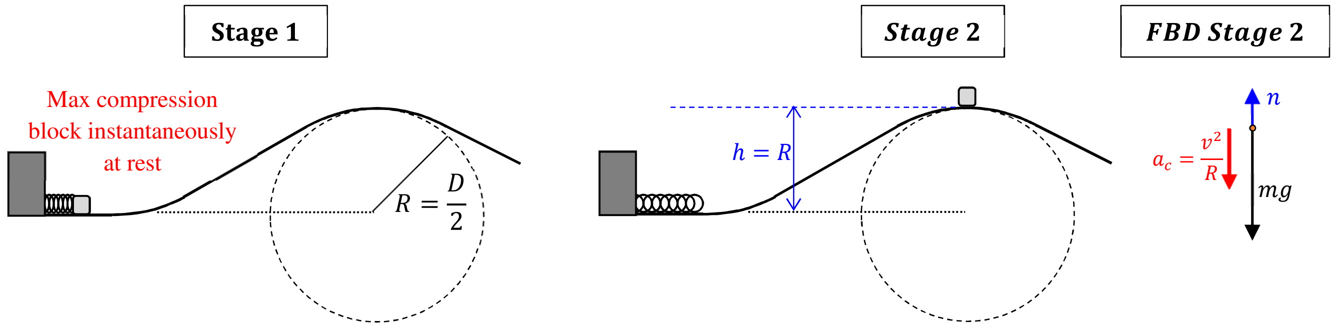
Even though the zero limit drops out in this particular problem, always double check it as a matter of habit.

$$U_f = \frac{\alpha x_f^6}{6}$$

Finally, let us think about what this function really means.
If you give me a value for x_f, I can compute the potential energy for that value of x_f.
This is exactly what we mean by asking for the potential energy as a function of x.
It thus makes sense to rename things to make it look more standard.

$$U(x) = \frac{\alpha x^6}{6}$$

5a) Consider the before and after pictures shown below. Notice the FBD for Stage 2 at far right.



From the FBD we get the force equation

$$mg - n = ma_c$$

The problem statement tells us $n = 33.3\%\ of\ weight = 0.333mg$.

$$mg - 0.333mg = m\frac{v^2}{R}$$

Mass cancels at this point! Rearranging things a bit gives

$$v = \sqrt{0.667Rg}$$

Now plug in values to find

$$\boldsymbol{v = 1.905\,\frac{m}{s}}$$

5b) Normal force does zero work on the block (normal force always perpendicular to displacement). Friction does zero work (we were told friction was negligible in problem statement).

$$K_i + U_{Gi} + U_{Si} + W_{\underset{n.c.}{ext}} = K_f + U_{Gf} + U_{Sf}$$

$$0 + 0 + \frac{1}{2}kx_1^2 + 0 = \frac{1}{2}mv_2^2 + mgh_2 + 0$$

$$\frac{1}{2}kx_1^2 = \frac{1}{2}m\left(\sqrt{0.667Rg}\right)^2 + mgR$$

$$\frac{1}{2}kx_1^2 = 1.3335mgR$$

$$x_1 = \sqrt{2.667\frac{mgR}{k}}$$

No need to worry about ± sign on this square root since we are solving for a distance (this parameter always +).

$$x_1 \approx 204\ \text{mm}$$

6a) I will assume two sig figs on all numbers since we are reading from a plot.

We can determine force using

$$F_x = -slope$$

$$F_x = -\frac{rise}{run}$$

$$F_x = -\frac{-250\ \text{eV}}{50 \times 10^{-6}\ \text{m}}$$

$$F_x = 5.0 \times 10^{6}\,\frac{\text{eV}}{\text{m}}$$

We were asked to find force *magnitude* in N.

In this case the answer is already a positive quantity so no need for absolute value.

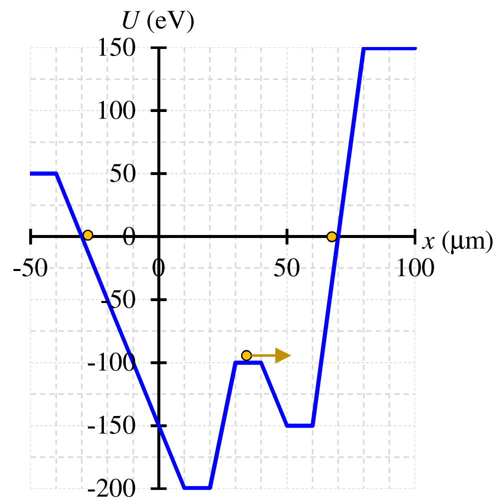
We do need to convert the units!

$$F_x = 5.0 \times 10^{-6}\,\frac{\text{eV}}{\text{m}} \times \frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}}$$

$$\boldsymbol{F_x = 8.01 \times 10^{-13}\ \text{N}}$$

$$\boldsymbol{F_x = 801\ \text{fN}}$$

6b) The particle will oscillate between $x_i = -30\ \mu\text{m}$ and $x = +70\ \mu\text{m}$!



The ball on a track analogy works well here.

$$U_i + K_i = U_f + K_f$$

Note: unless told otherwise we assume $W_{\underset{n.c.}{ext}} = 0$ for these types of problems.

Because the ball is released from rest we can set $K_i \to 0$.

$$U_i = U_f + K_f$$

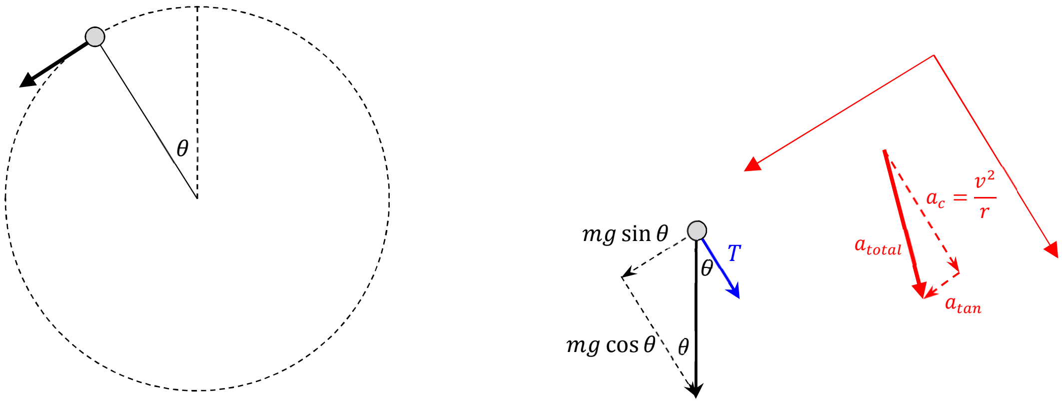
As the ball rolls along the track, it has enough energy to make it over the small hill around $x \approx 35\ \mu\text{m}$.

Notice when the ball reaches $x = 70\ \mu\text{m}$ we have $U_i = U_f$ and thus $K_f = 0$.

At this point the force on the ball is to the left.

It reverses direction and oscillates between $x_i = -30\ \mu\text{m}$ and $x = +70\ \mu\text{m}$.

7a) Figure not drawn to scale. Original figure on the left with an FBD on the right. Notice $a_{tan} \neq 0$!!!



Force equation in from forces towards the center gives

$$T + mg\cos\theta = ma_c$$

$$\frac{T}{m} + g\cos\theta = a_c$$

$$\frac{T}{m} + g\cos\theta = \frac{v^2}{r}$$

$$r = \frac{v^2}{\frac{T}{m} + g\cos\theta}$$

Plug in numbers (watch out for $m = 222\text{ g} = 0.222\text{ kg}$).

$$\boldsymbol{r} = \mathbf{0.57\underline{9}1\ m = 579\ mm}$$

*For extra practice you can show $\boldsymbol{a_c} = \mathbf{53.2}\,\mathbf{\frac{m^2}{s}}$.

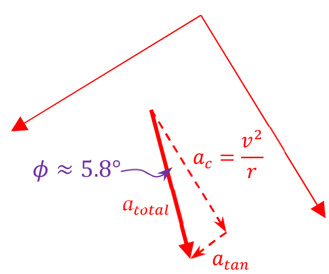
7b) Force equation in the tangential direction gives

$$mg\sin\theta = ma_{tan}$$

$$a_{tan} = g\sin\theta$$

$$a_{tan} = 0.549g$$

$$\boldsymbol{a_{tan}} = \mathbf{5.38\,\frac{m^2}{s}}$$



*For extra practice I found

$$\boldsymbol{a_{total}} = \sqrt{\boldsymbol{a_c^2} + \boldsymbol{a_{tan}^2}} = \mathbf{53.5\,\frac{m^2}{s}}$$

*I also determined the angle between a_c and a_{tot}.

Please ensure you understand these extra practice parts (lines with *'s) in case I put them on a future test.

7c) The force of tension is always perpendicular to displacement.

As such it does zero work!