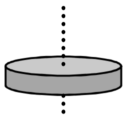
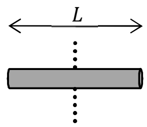
161 Spring 2022 Exam 3A Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

$V_{sphere} = \frac{4}{3}\pi R^3$	$V_{box} = LWH$	$V_{cyl} = \pi R^2 H$	$\rho = \frac{M}{V}$
$A_{sphere} = 4\pi R^2$	$V = (A_{base}) \times (height)$	$A_{circle} = \pi R^2$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$C = 2\pi R$	$A_{rect} = LW$	$A_{CylSide} = 2\pi RH$	
$160\underline{9}$ m = 1 mi	12 in = 1 ft	60 s = 1 min	1000 g = 1 kg
2.54 cm = 1 in	1 cc = 1 cm^3 = 1 mL	60 min = 1 hr	100 cm = 1 m
1 cm = 10 mm	1 yard = 3 ft	3600 s = 1 hr	1 km = 1000 m
1 furlong = 220 yards	$528\underline{0}$ ft = 1 mi	24 hrs = 1 day	1 rev = 2π rad = 360°
$g = 9.8\ \frac{\text{m}}{\text{s}^2}$	$G = 6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	$P_0 = 1.0 \times 10^5$ Pa	1 eV = $1.60\underline{2} \times 10^{-19}$ J
$1\ \text{N} = 1 \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$	$1\ \text{J} = 1\ \text{N}\cdot\text{m}$	$1\ \text{Pa} = 1\ \frac{\text{N}}{\text{m}^2}$	
$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$	$v_{fx}^2 = v_{ix}^2 + 2a_x(\Delta x)$	$v_{fx} = v_{ix} + a_x t$	$r = \sqrt{x^2 + y^2}$
$\vec{A}\cdot\vec{B} = AB\cos\theta_{AB}$	$\lVert\vec{A}\times\vec{B}\rVert = AB\sin\theta_{AB}$	$\sin(A \pm B)$ $= \sin A\cos B \pm \cos A \sin B$	$\cos(A \pm B)$ $= \cos A\cos B \mp \sin A \sin B$
$\vec{v}_{ae} + \vec{v}_{eb} = \vec{v}_{ab}$	$\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$	$\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$	
$a_{tan} = r\alpha$	$a_c = \frac{v^2}{r} = r\omega^2$	$\vec{a} = a_r\hat{r} + a_{tan}\hat{\theta}$	$\vec{a} = a_c(-\hat{r}) + a_{tan}\hat{\theta}$
$\Sigma\vec{F} = m\vec{a}$	$f \le \mu n$	$F_G = \frac{GmM}{r^2}(-\hat{r})$	$U_G = -\frac{GmM}{r}$
$TKE = \frac{1}{2}mv^2$	$RKE = \frac{1}{2}I\omega^2$	$U_S = SPE = \frac{1}{2}kx^2$	$U_G = GPE = mgh$
$E_i + W_{\substack{non-con \\ or\ ext}} = E_f$	$\Delta KE = W_{ext.\&\,non-con}$	$W = Fd\cos\theta = F_{\parallel}d$	$W = \int F_x dx$
$\Delta U = -W = -\int_i^f \vec{F}\cdot d\vec{s}$	$F_x = -\frac{d}{dx}U(x)$	$\mathcal{P}_{inst} = \frac{dE}{dt} = \vec{F}\cdot\vec{v}$	$\mathcal{P}_{avg} = \frac{\Delta E}{\Delta t} = \frac{Work}{time}$
$\vec{J} = \Delta\vec{p} = \vec{F}\Delta t$	$\vec{p} = m\vec{v}$	$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$	$x_{\text{CM}} = \frac{\int x\,dm}{\int dm}$
$\vec{\tau} = \vec{r}\times\vec{F}$	$\Sigma\vec{\tau} = I\vec{\alpha}$	$L = I\omega = mvr_{\perp}$	$\mathcal{P}_{inst} = \vec{\tau}\cdot\vec{\omega}$
$s = r\Delta\theta$	$v = r\omega$	$a_{tan} = r\alpha$	$a_c = \frac{v^2}{r} = r\omega^2$
$I_{\parallel axis} = I_{\text{CM}} + md^2$	$I_{zz} = I_{xx} + I_{yy}$	$I = \int r^2 dm$	$\frac{F}{A} = E\frac{\Delta L}{L_0}$
$P = \frac{F}{A}$	$P_{gauge} = P_{abs} - P_{ambient}$	$B = \rho_f V_{disp} g$	$A_1v_1 = A_2v_2$
$P(h) = P_0 + \rho gh$	$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$	$R = \frac{\pi r^4 \Delta P}{8\eta L}$	$F = \eta A\frac{\Delta v_x}{\Delta y}$

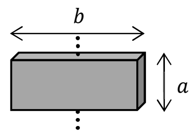
Prefix	Abbreviation	$10^?$		Prefix	Abbreviation	$10^?$
Giga	G	10^9		milli	m	10^{-3}
Mega	M	10^6		micro	μ	10^{-6}
kilo	k	10^3		nano	n	10^{-9}
centi	c	10^{-2}		pico	p	10^{-12}
				femto	f	10^{-15}



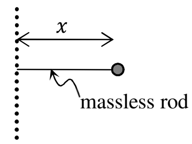
$$I_{disk} = \frac{1}{2}mR^2$$



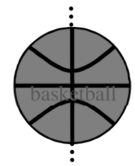
$$I_{\substack{thin\\rod}} = \frac{1}{12}mL^2$$



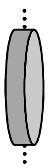
$$I_{\substack{thin\\plate}} = \frac{1}{12}mb^2$$



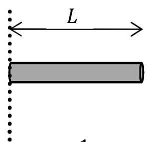
$$I_{\substack{pnt\\mass}} = mx^2$$



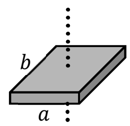
$$I_{\substack{spherical\\shell}} = \frac{2}{3}mR^2$$



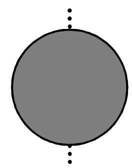
$$I_{disk} = \frac{1}{4}mR^2$$



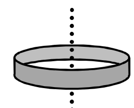
$$I_{\substack{thin\\rod}} = \frac{1}{3}mL^2$$



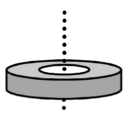
$$I_{\substack{thin\\plate}} = \frac{1}{12}m(b^2 + a^2)$$



$$I_{\substack{solid\\sphere}} = \frac{2}{5}mR^2$$



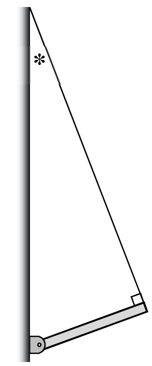
$$I_{\substack{thin\\ring}} = mR^2$$



$$I_{\substack{thick\\ring}} = \frac{1}{2}m(R_{inner}^2 + R_{outer}^2)$$

Name:____

A uniform rod of mass m and length d is connected at one end to a wall using a frictionless pivot (see figure). The other end of the rod is supported by a light, inextensible cable. The cable comes off the rod at a right angle and connects to the wall distance $3d$ above the pivot.

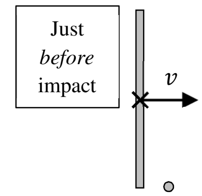


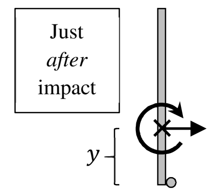
1a) Determine a numerical value for the angle shown by the * in the figure.
***1b) Determine the *magnitude* of tension in the cable.
Answer as decimal number with 3 sig figs times mg.
***1c) Determine the reaction force in Cartesian form. It should look like:

$$\vec{R} = \Big[\ (some\ \#)\hat{\imath}\ + (other\ \#)\hat{\jmath}\ \Big]\ mg$$

1a	
1b	
1c	

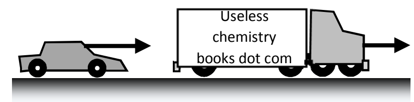
Deep in outer space, a rod of mass $3m$ and length $2d$ moves with speed v.
Again, **the length of the rod is $\boldsymbol{2d}$**…this was used to simplify some algebra.
The rod impacts a stationary piece of clay (point mass) with mass m.
The collision is perfectly inelastic & the collision process happens in a negligible amount of time.
2a) Just *after* impact, determine the distance from the rod's bottom to the rod-clay center of mass (distance y in 2^{nd} figure).
***2b) Just *after* impact, determine the moment of inertia about the center of mass.
2c) Determine <u>translational</u> *speed* of the rod-clay system after the collision.
**2d) Determine <u>rotational</u> *speed* of the rod-clay system after the collision.





2a	
2b	
2c	
2d	

A car of mass 1000 kg moves to the right with speed 30 m/s.
A truck of mass 2000 kg in front of the car moves to the right with speed 20 m/s.
The driver of the car isn't paying attention until it is too late!
Unfortunately, the car impacts the truck while travelling at speed 30 m/s.
The collision time is short enough that we may ignore any sliding of the wheels by either vehicle during impact.



3a) During the collision which vehicle experiences a larger force (magnitude)? Circle the best answer.

Car	Truck	Same force	Impossible to determine without collision time	Impossible to determine even if you did have the collision time

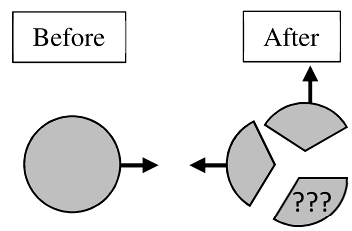
3b) During the collision, which vehicle experiences a larger acceleration (magnitude)?

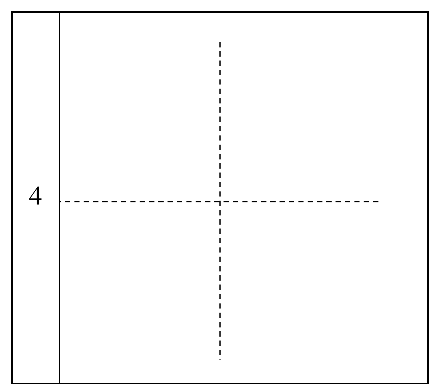
Car	Truck	Same acceleration	Impossible to determine without collision time	Impossible to determine even if you did have the collision time

3c) During the collision, which vehicle experiences a larger change in momentum (magnitude)?

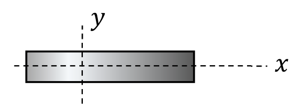
Car	Truck	Same momentum change	Impossible to determine without collision time	Impossible to determine even if you did have the collision time

****4) A bomb is moving horizontal with speed v at the instant it explodes into three equal mass fragments. After the explosion, one fragment moves straight up with speed v. Another fragment moves left with speed v.
Determine the *velocity* of the final fragment. In the box draw an arrow labeled with the speed (# with 3 sig figs times v) & a numerical value of an angle (3 sig figs).





A rod has non-uniform mass density given by $\lambda = kx^4$.
The *left* end of the rod is distance d to the left of the origin.
The rod has length $3d$.



For the following problems, you are expected to simplify your work for full credit.
It is probably easiest to convert all fractions to decimals with three sig figs.
Notice part d at the bottom of the page!

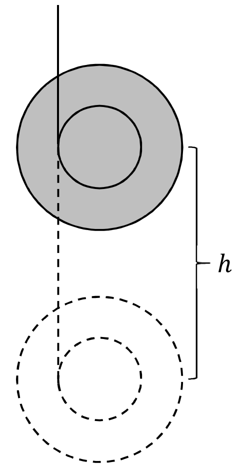
5a	
5b	
5c	

***5a) Determine the horizontal coordinate of the center of mass.
**5b) Determine I_{yy}, the moment of inertia about the y-axis.
**5c) Determine the moment of inertia about an axis parallel to y-axis *running through the center of mass*.

5d) Suppose we instead wanted to determine the moment of inertia about the z-axis (I_{zz}) instead of about the y-axis.
Which of the following best describes the relationship between I_{zz} & I_{yy}? Circle the best answer.

$I_{zz} > I_{yy}$	$I_{zz} = I_{yy}$	$I_{zz} < I_{yy}$	Impossible to determine without more info

A student winds a massless, inextensible string around disk of mass m and radius $2R$. The disk is designed in such a way as to allow the string to connect at radius R instead of going around the outer edge of the disk. Assume the string is wound perfectly such that the disk moves straight down when released from rest. Assume the string unravels without slipping.



6a) Is the disk's *translational acceleration* (magnitude) faster, slower, or the same as g?

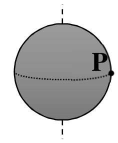
Faster than g	Same as g	Slower than g	Impossible to determine without more info

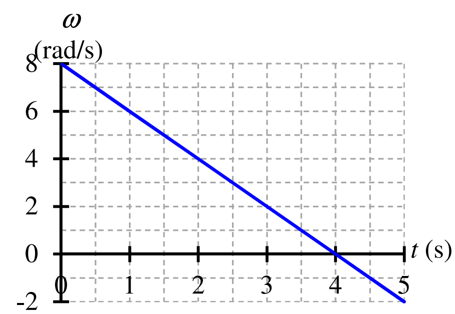
****6b) Determine *translational acceleration* (magnitude) of the disk after being released from rest.
**6c) Determine *rotational speed* of the disk after it has fallen distance h.

6b	
6c	

6d) Initially the disk has *zero* angular momentum. After a short time the disk is rotating and has *non-zero* angular momentum. I thought angular momentum was supposed to be conserved? Explain why this does not contradict the law of conservation of angular momentum.

A plot of ω versus t for a *solid sphere* is shown at right. The axis of rotation is through the center of the sphere in the z-direction. The sphere has mass 1.250 kg and radius 0.375 m. A point **P** on the edge of the sphere is indicated in the figure. You may assume the initial angular position of **P** is zero.





7a) What is the initial rotation rate in RPM?

7b) At $t = 4.50$ s, determine *angular* acceleration.

7c) At $t = 4.50$ s, determine *centripetal* acceleration *magnitude* of the point **P**.

7d) At $t = 4.50$ s, determine *total* acceleration *magnitude* of the point **P.

**7e) How many *revolutions* does the sphere make <u>during the entire time interval</u> shown?

7a	
7b	
7c	
7d	
7e	

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