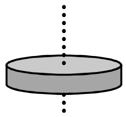
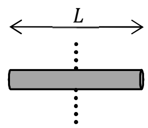
161 Spring 2022 Exam 3C Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

$V_{sphere} = \frac{4}{3}\pi R^3$	$V_{box} = LWH$	$V_{cyl} = \pi R^2 H$	$\rho = \frac{M}{V}$
$A_{sphere} = 4\pi R^2$	$V = (A_{base}) \times (height)$	$A_{circle} = \pi R^2$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$C = 2\pi R$	$A_{rect} = LW$	$A_{CylSide} = 2\pi RH$	
$160\underline{9}$ m = 1 mi	12 in = 1 ft	60 s = 1 min	1000 g = 1 kg
2.54 cm = 1 in	1 cc = 1 cm^3 = 1 mL	60 min = 1 hr	100 cm = 1 m
1 cm = 10 mm	1 yard = 3 ft	3600 s = 1 hr	1 km = 1000 m
1 furlong = 220 yards	$528\underline{0}$ ft = 1 mi	24 hrs = 1 day	1 rev = 2π rad = 360°
$g = 9.8\ \frac{\text{m}}{\text{s}^2}$	$G = 6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	$P_0 = 1.0 \times 10^5$ Pa	1 eV = $1.60\underline{2} \times 10^{-19}$ J
$1\ \text{N} = 1 \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$	$1\ \text{J} = 1\ \text{N}\cdot\text{m}$	$1\ \text{Pa} = 1 \frac{\text{N}}{\text{m}^2}$	
$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$	$v_{fx}^2 = v_{ix}^2 + 2a_x(\Delta x)$	$v_{fx} = v_{ix} + a_x t$	$r = \sqrt{x^2 + y^2}$
$\vec{A}\cdot\vec{B} = AB\cos\theta_{AB}$	$\lVert\vec{A}\times\vec{B}\rVert = AB\sin\theta_{AB}$	$\sin(A \pm B)$ $= \sin A\cos B \pm \cos A \sin B$	$\cos(A \pm B)$ $= \cos A \cos B \mp \sin A \sin B$
$\vec{v}_{ae} + \vec{v}_{eb} = \vec{v}_{ab}$	$\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$	$\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$	
$a_{tan} = r\alpha$	$a_c = \frac{v^2}{r} = r\omega^2$	$\vec{a} = a_r\hat{r} + a_{tan}\hat{\theta}$	$\vec{a} = a_c(-\hat{r}) + a_{tan}\hat{\theta}$
$\Sigma\vec{F} = m\vec{a}$	$f \le \mu n$	$F_G = \frac{GmM}{r^2}(-\hat{r})$	$U_G = -\frac{GmM}{r}$
$TKE = \frac{1}{2}mv^2$	$RKE = \frac{1}{2}I\omega^2$	$U_S = SPE = \frac{1}{2}kx^2$	$U_G = GPE = mgh$
$E_i + W_{\substack{non-con \\ or\ ext}} = E_f$	$\Delta KE = W_{ext.\&\,non-con}$	$W = Fd\cos\theta = F_{\parallel}d$	$W = \int F_x dx$
$\Delta U = -W = -\int_i^f \vec{F}\cdot d\vec{s}$	$F_x = -\frac{d}{dx}U(x)$	$\mathcal{P}_{inst} = \frac{dE}{dt} = \vec{F}\cdot\vec{v}$	$\mathcal{P}_{avg} = \frac{\Delta E}{\Delta t} = \frac{Work}{time}$
$\vec{J} = \Delta\vec{p} = \vec{F}\Delta t$	$\vec{p} = m\vec{v}$	$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$	$x_{\text{CM}} = \frac{\int x\,dm}{\int dm}$
$\vec{\tau} = \vec{r}\times\vec{F}$	$\Sigma\vec{\tau} = I\vec{\alpha}$	$L = I\omega = mvr_{\perp}$	$\mathcal{P}_{inst} = \vec{\tau}\cdot\vec{\omega}$
$s = r\Delta\theta$	$v = r\omega$	$a_{tan} = r\alpha$	$a_c = \frac{v^2}{r} = r\omega^2$
$I_{\parallel axis} = I_{\text{CM}} + md^2$	$I_{zz} = I_{xx} + I_{yy}$	$I = \int r^2 dm$	$\frac{F}{A} = E\frac{\Delta L}{L_0}$
$P = \frac{F}{A}$	$P_{gauge} = P_{abs} - P_{ambient}$	$B = \rho_f V_{disp} g$	$A_1v_1 = A_2v_2$
$P(h) = P_0 + \rho gh$	$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$	$R = \frac{\pi r^4 \Delta P}{8\eta L}$	$F = \eta A\frac{\Delta v_x}{\Delta y}$

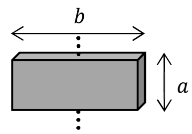
Prefix	**Abbreviation**	**$10^?$**		**Prefix**	**Abbreviation**	**$10^?$**
Giga	G	10^9		milli	m	10^{-3}
Mega	M	10^6		micro	μ	10^{-6}
kilo	k	10^3		nano	n	10^{-9}
centi	c	10^{-2}		pico	p	10^{-12}
				femto	f	10^{-15}



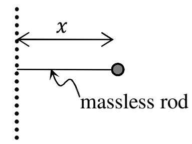
$$I_{disk} = \frac{1}{2} mR^2$$



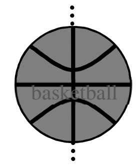
$$I_{\substack{thin\\rod}} = \frac{1}{12} mL^2$$



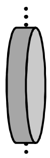
$$I_{\substack{thin\\plate}} = \frac{1}{12} mb^2$$



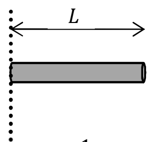
$$I_{\substack{pnt\\mass}} = mx^2$$



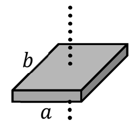
$$I_{\substack{spherical\\shell}} = \frac{2}{3} mR^2$$



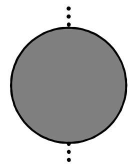
$$I_{disk} = \frac{1}{4} mR^2$$



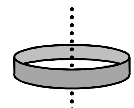
$$I_{\substack{thin\\rod}} = \frac{1}{3} mL^2$$



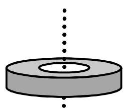
$$I_{\substack{thin\\plate}} = \frac{1}{12} m(b^2 + a^2)$$



$$I_{\substack{solid\\sphere}} = \frac{2}{5} mR^2$$



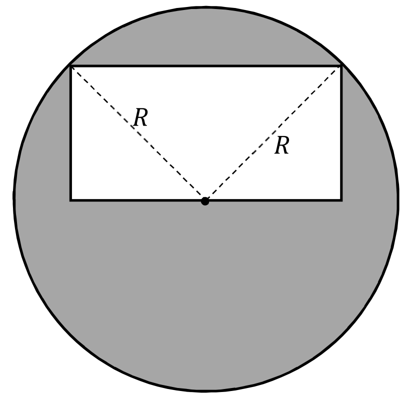
$$I_{\substack{thin\\ring}} = mR^2$$



$$I_{\substack{thick\\ring}} = \frac{1}{2} m(R_{inner}^2 + R_{outer}^2)$$

Name:____

A thin, circular plate has density σ and radius R. A rectangular hole is removed from the plate as shown. The center of the circle is represented by the black dot. Assume the dot is the origin of a standard coordinate system ($\hat{\imath}$ to the right, $\hat{\jmath}$ upwards).



1a) Which *best* describes the location of the center of mass?

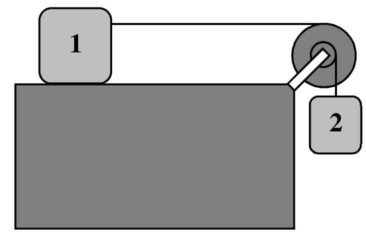
In 2nd quadrant	In 1st quadrant	On $-x$-axis	On $+x$-axis	Impossible to determine without more info
In 3rd quadrant	In 4th quadrant	On $+y$-axis	On $-y$-axis	

*****1b) Determine the y-coordinate of the center of mass.

Extra part I cut out for time 1c) Determine the moment of inertia about the z-axis

1b	

A two mass system is set-up as shown in the figure. The mass on the flat surface is $m_1 = m$, the hanging mass $m_2 = 2m$, while the pulley mass is $m_p = 4m$. Both the horizontal surface and pulley's axel have negligible friction. The system is released from rest and m_2 travels distance h. Notice m_2 connects to the pulley at radius R while m_1 connects to the pulley at radius $3R$ on the pulley. Assume the pulley is a solid disk even though you can connect strings at two different radii.



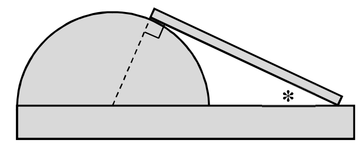
2a) How far does m_1 move when m_2 travels distance h?

******2b) Determine the magnitude of m_2's *translational* acceleration after the blocks are released.

2a	
2b	

2d) Suppose we instead connected both masses to the smaller radius in the figure. Would the final rotation rate of the pulley increase, decrease, or stay the same? State your answer and explain why for credit.

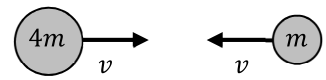
A hemispherical dome has radius R. A rod of mass m and length $2R$ remains at rest when placed on the dome as shown in the figure (not to scale). Negligible friction exists between the dome and the rod. Friction is present between the rod and the horizontal surface. Notice the right angle indicated in the figure.



3a) Determine a numerical value for the angle indicated by the * in the figure.
***3b) Determine the normal force (magnitude) exerted by the dome on the rod. Answer as a number with three sig figs times mg.
***3c) Determine the minimum coefficient of friction (between horizontal surface and rod) required to keep the rod at rest. Answer as a number with 3 sig figs.

3a	
3b	
3c	

Careful: historically students find the algebra on this problem to be time consuming but parts b, c, & d are quick.
For part A: if you set up the equations correctly then do zero algebra (for part a) I'll give you 2/5.
Then come back at the end of the test to do the algebra as time permits?



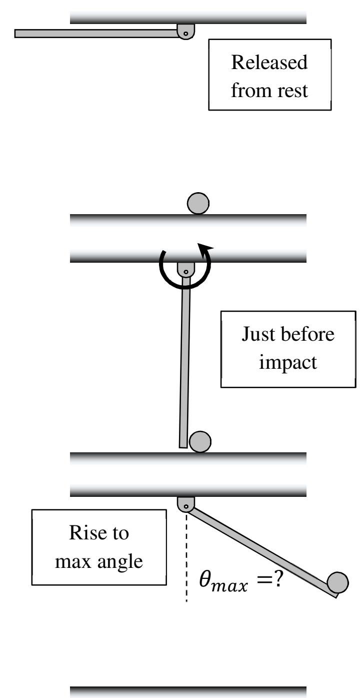
Two masses experience a head-on, elastic collision. Before the collision, both masses move with speed v. The larger mass is $4m$ while the smaller mass is m. You may assume the collision happens so rapidly that all other forces (i.e. gravity, drag) are negligible.
*****4a) Determine the speed of mass $4m$ after the collision.
Answer as a decimal with 3 sig figs times v.
4b) During the collision, which mass experiences a larger (magnitude) change in momentum? Is it m, $4m$, a tie, or impossible to determine without more info…
4c) During the collision, which mass experiences a larger (magnitude) force?
4d) During the collision, which mass experiences a larger (magnitude) acceleration?

4a	
4b	
4c	
4d	

A uniform rod of mass $3m$ and length d is connected to the ceiling using a frictionless pivot. The rod is held parallel to the ground and released. At the lowest point in the swing, the rod impacts a point mass m. The point mass sticks to the rod. The combined rod and point mass system continue to swing as a system after the impact.

*******5) Determine the following:

a) Rod rotation rate *before* impact
b) Center of mass (distance from pivot) *after* impact
c) Moment of inertia about pivot *after* impact
d) Angle of max rise (θ_{max}) *after* impact

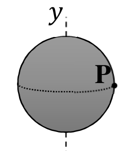


5a	
5b	
5c	
5d	

A thin *spherical shell* has mass $m = 0.200$ kg and radius $r = 0.360$ m.
Assume a point on the shell's surface (indicated **P** in the figure) starts at $\theta_i = 0$.
The initial rotation rate vector is $20.0\,\frac{\text{rad}}{\text{s}}\hat{\jmath}$.
The shell rotates about he y-axis with angular acceleration

$$\vec{\alpha} = \left(-30.0\,\frac{\text{rad}}{\text{s}^4}\right)t^2\hat{\jmath}$$

In this equation values used for t must be in units of seconds.



6a) Determine the initial rotation rate (magnitude) in units of RPM.
6b) Determine $\vec{\omega}(t)$. **Use radians (not rev's) as units for the rest the parts…
It is ok to leave off units on numbers for this part. I will assume they are in radians…
6c) Determine tangential acceleration (magnitude) for point **P** at $t = 2.00$ s.
6d) Determine centripetal acceleration (magnitude) for point **P** at $t = 2.00$ s.
***6e) How many revolutions are completed before the shell first reverses direction?

6a	
6b	
6c	
6d	
6e	

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