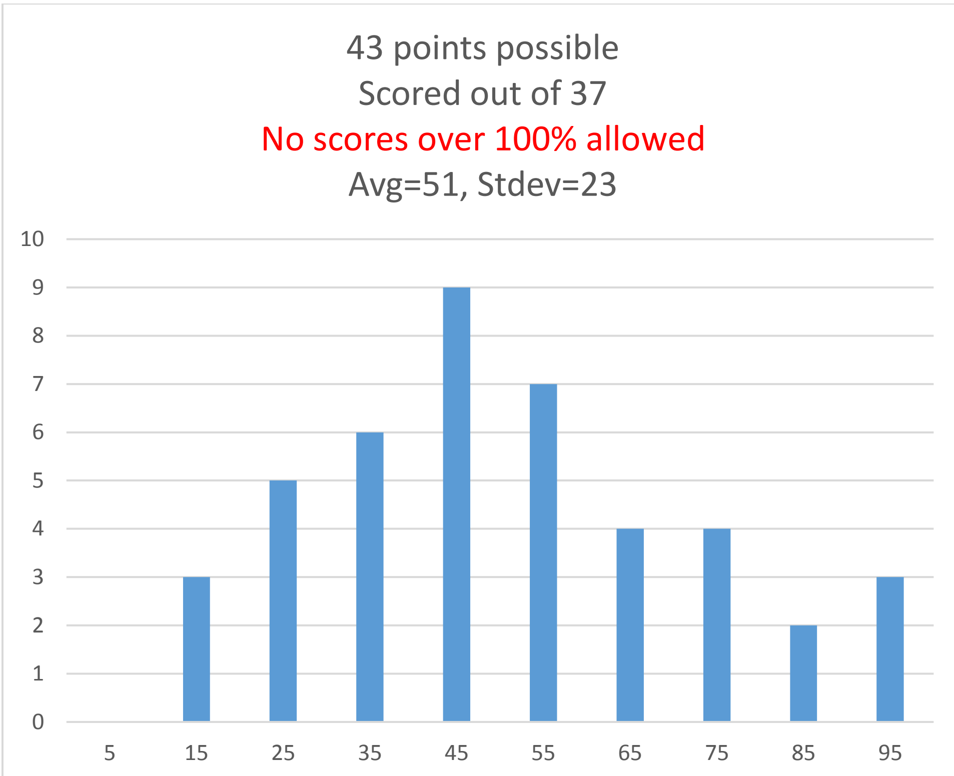
161sp23t1aSoln

Distribution on this page.
Solutions begin on the next page.



1a) Asking "how far" implies distance (not displacement). **Answer:** 16.00 m… did you remember units?
Notice you can get this result by taking the absolute value under the vt-curve during the first 2.00 s.

1b) Moving left implies velocity is negative.
Slowing down implies acceleration and velocity have opposite signs.
Answer: Acceleration is *positive*.

1c) To use kinematics equations one must restart the clock to $t = 0$ the beginning of each stage.
The velocity goes from $-8.00\,\frac{\text{m}}{\text{s}}$ in 4.00 s…not 6.00 s.

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t}$$

$$a = \frac{0 - \left(-8.00\,\frac{\text{m}}{\text{s}}\right)}{4.00\text{ s}}$$

$$\boldsymbol{a = +2.00\,\frac{m}{s^2}}$$

When you are asked for the MAGNITUDE, take the absolute value if necessary. In this case it was positive already.
Did you get the minus signs right? Did you remember units?

1d) To use kinematics equations one must restart the clock to $t = 0$ the beginning of each stage.
The displacement in question occurs over 4.00 s…not 6.00 s.

$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$$

$$\Delta x = \left(-8.00\,\frac{\text{m}}{\text{s}}\right)(4.00\text{ s}) + \frac{1}{2}\left(+2.00\,\frac{\text{m}}{\text{s}^2}\right)(4.00\text{ s})^2$$

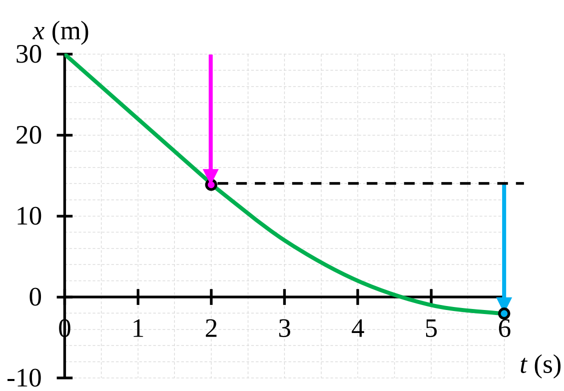
$$\boldsymbol{\Delta x = -16.00\text{ m}}$$

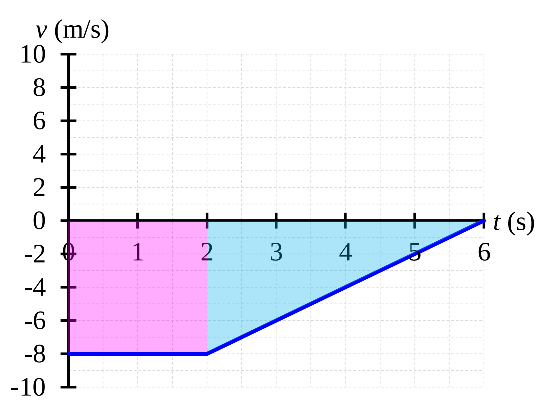
Notice you could also use the area under the vt-curve shown below!

Notice this area corresponds to displacement over the last 4.00 s!

Note: in many engineering classes we round all final results to 3 sig figs.
Exception: some instructors ask you to keep an extra digit if the first digit is a 1.
I'll accept -16.0 m or -16.00 m.
If you want to read more on this, check this link:
https://physics.stackexchange.com/questions/520935/why-is-a-leading-digit-not-counted-as-a-significant-figure-if-it-is-a-1#:~:text=So%20if%20the%20leading%20digit,had%20one%20fewer%20significant%20figure

Please remember sig figs are only an approximate method to estimate precision.
If you are doing serious work where a true understanding of precision is critical, full error analysis is done.





2a) I found

$$q = \pm\sqrt{\frac{ca - U}{\beta}}$$

Note: without more information to clarify the sign, you should include the $\pm$ on the square root!
While not required, sometimes it can be handy to do a unit check.

$$[q] = \sqrt{\frac{(\text{kg}\cdot\text{m})\left(\frac{\text{m}}{\text{s}^2}\right) - \text{kg}\cdot\frac{\text{m}^2}{\text{s}^2}}{\frac{\text{kg}}{\text{s}^2}}} = \sqrt{\frac{\text{kg}\cdot\frac{\text{m}^2}{\text{s}^2}}{\frac{\text{kg}}{\text{s}^2}}} = \sqrt{\text{kg}\cdot\frac{\text{m}^2}{\text{s}^2}\cdot\frac{\text{s}^2}{\text{kg}}} = \text{m}$$

Notice the units of the two terms in the numerator match.
Think: $\frac{\text{mm}}{\text{s}^2}$ could be converted to $\frac{\text{m}}{\text{s}^2}$ so I am not making crap up when I use those units for a.
2b) The problem mentions tracking sig figs for this calculation so I will show all the steps.
Also, beware of the units for $a = 30.0\frac{\text{mm}}{\text{s}^2} = 30.0 \times 10^{-3}\frac{\text{m}}{\text{s}^2} = 0.0300\frac{\text{m}}{\text{s}^2}$.
I am not trying to torture you here…in the real world we often deal with numbers in various forms just like this.

$$q = \pm\sqrt{\frac{(21.25\ \text{kg}\cdot\text{m})\left(0.0300\frac{\text{m}}{\text{s}^2}\right) - 0.555\ \text{kg}\cdot\frac{\text{m}^2}{\text{s}^2}}{6.00\times 10^7\frac{\text{kg}}{\text{s}}}}$$

Because I already checked the units, I can now rewrite this as

$$q = \pm\sqrt{\frac{(21.25)(0.0300) - 0.555}{6.00\times 10^7}}\ \text{m}$$

$$q = \pm\sqrt{\frac{0.63\underline{7}5 - 0.555}{6.00\times 10^7}}\ \text{m}$$

$$q = \pm\sqrt{\frac{0.08\underline{2}5}{6.00\times 10^7}}\ \text{m}$$

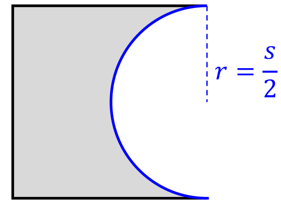
$$q = \pm 3.\underline{7}1\times 10^{-5}\ \text{m}$$

Now use calculator modes to rewrite this in engineering notation and properly round…

$$q = \pm 3\underline{7}.1\times 10^{-6}\ \text{m}$$

$$q = \pm 3\underline{7}\ \mu\text{m}$$

3) The volume of this plate is given by



$$V = Area \times thickeness$$

$$V = \left(A_{square} - \frac{1}{2}A_{circle}\right)t$$

$$V = \left(s^2 - \frac{1}{2}\pi r^2\right)t$$

$$V = \left(s^2 - \frac{\pi}{8}s^2\right)t$$

$$V = \left(1 - \frac{\pi}{8}\right)s^2 t$$

Now use the density equation

$$\rho = \frac{m}{V}$$

$$V = \frac{m}{\rho}$$

$$\left(1 - \frac{\pi}{8}\right)s^2 t = \frac{m}{\rho}$$

$$s^2 = \frac{m}{\rho t\left(1 - \frac{\pi}{8}\right)}$$

$$s = \sqrt{\frac{m}{\rho t\left(1 - \frac{\pi}{8}\right)}}$$

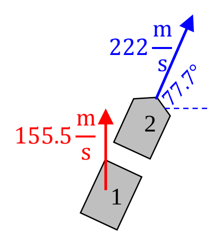
Think: no need for a $\pm$ sign on this particular square root since the side of a square must be a positive quantity. The polite thing to do at this point is to convert the numerical terms into a single number with three sig figs where the number is in the numerator and outside the square root. The problem statement also specifically asked for this.

$$s = \sqrt{\frac{1}{1 - \frac{\pi}{8}}}\sqrt{\frac{m}{\rho t}}$$

$$\boldsymbol{s \approx 1.28\sqrt{\frac{m}{\rho t}}}$$

Here we typically include an extra digit since the first digit is 1…so I also accepted (and prefer) $\boldsymbol{s \approx 1.283\sqrt{\frac{m}{\rho t}}}$.

4) We know the final answer is in $\frac{\text{m}}{\text{s}}$ so I can leave off units on my work and slap those units on the final answer.



$$\vec{v}_{12} = \vec{v}_{1e} + \vec{v}_{e2}$$

$$\vec{v}_{12} = \vec{v}_{1e} - \vec{v}_{2e}$$

$$\vec{v}_{12} = (0\hat{\imath} + 155.5\hat{\jmath}) - (222\cos 77.7°\,\hat{\imath} + 222\sin 77.7°\,\hat{\jmath})$$

$$\vec{v}_{12} = \left(0\hat{\imath} + 155.\underline{5}\hat{\jmath}\right) - \left(47.\underline{2}9\hat{\imath} + 21\underline{6}.90\hat{\jmath}\right)$$

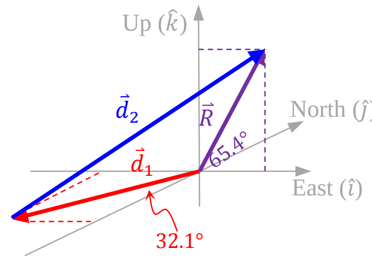
$$\vec{v}_{12} = \left(-47.\underline{2}9\hat{\imath} - 6\underline{1}.40\hat{\jmath}\right)$$

Speed is the magnitude of this velocity vector.

$$\boldsymbol{v_{12} = 7\underline{7}.5\frac{m}{s}}$$

Notice you lose a sig fig if you strictly follow sig fig rules. Since I did not ask for students to follow sig fig rules for this problem, I assumed everyone would revert to the default number of sig figs (3, but include an extra digit if the first digit is 1).

5a) The graphical vector addition is shown at right.
WATCH OUT!



$$\vec{d}_1 = (-5.75\sin 32.1°\,\hat{\imath} - 5.75\cos 32.1°\,\hat{\jmath})\text{m}$$

$$\boldsymbol{\vec{d}_1 = (-3.0\underline{5}6\hat{\imath} - 4.8\underline{7}1\hat{\jmath})\text{m}}$$

Did you include the units?
Strictly speaking, the answer in the box should be rounded.
However, we know we must use the unrounded result in subsequent calculations.
My compromise is to write the answer with the rounding digit indicated by an underbar. Other teachers might refuse this style…

5b)

$$\hat{R} = \frac{\vec{R}}{R}$$

$$\hat{R} = \frac{(4.25\cos 65.4°\,\hat{\imath} + 0\hat{\jmath} + 4.25\sin 65.4°\,\hat{k})\text{m}}{4.25\text{ m}}$$

$$\boldsymbol{\hat{R} = 0.41\underline{6}3\hat{\imath} + 0\hat{\jmath} + 0.90\underline{9}2\hat{k}}$$

Notice the units cancel out when computing a unit vector!!!
The *direction* of a vector is unitless…the *magnitude* contains all the unit information.
Since we have all the numbers right here:

$$\vec{R} = (1.7692\hat{\imath} + 3.864\hat{k})\text{m}$$

5c) The standard procedure use the dot product. Avoid the cross product so you don't have to do extra thinking.

$$\theta = \cos^{-1}\left(\frac{\vec{d}_1 \cdot \vec{R}}{d_1 R}\right) = \cos^{-1}\left(\frac{d_{1x}R_x + d_{1y}R_y + d_{1z}R_z}{d_1 R}\right)$$

If you are feeling spicy, notice you could also use

$$\theta = \cos^{-1}\left(\frac{\vec{d}_1 \cdot \hat{R}}{d_1}\right)$$

Either way one finds

$$\boldsymbol{\theta = 102.8°}$$

5c) I made this worth 2 points instead of 3.

$$\vec{R} = \vec{d}_1 + \vec{d}_2$$

$$\vec{d}_2 = \vec{R} - \vec{d}_1$$

$$\vec{d}_2 = (1.7692\hat{\imath} + 3.864\hat{k})\text{m} - (-3.0\underline{5}6\hat{\imath} - 4.8\underline{7}1\hat{\jmath})\text{m}$$

$$\vec{d}_2 = (4.8\underline{2}5\hat{\imath} + 4.871\hat{\jmath} + 3.864\hat{k})\text{m}$$

WATCH OUT! I didn't ask for this displacement vector…I asked for the MAGNITUDE of it…

$$d_2 = \sqrt{4.825^2 + 4.871^2 + 3.864^2}\text{ m}$$

$$\boldsymbol{d_2 = 7.87\text{ m}}$$

Remember, our default is to write final answers with 3 sig figs.
This is why I kept at least 4 sig figs on all intermediate answers.
Notice I kept an extra sig fig when the first digit was a 1…

6a) **NON-CONSTANT acceleration**. Fortunately, we're given velocity as a function of *time* (not *position* x).

$$v_1(t) = v_0 - kt^2$$

Taking the derivative (with respect to time) gives **NON-CONSTANT acceleration**.

$$a_1(t) = -2kt$$

6b) Fortunately, we're given velocity as a function of *time* (not *position* x).
This means we can integrate without worrying too much about proper separation of variables technique.
Technically you are still doing separation of variables when you multiply by dt and integrate…

$$\frac{dx}{dt} = v_0 - kt^2$$

$$dx = (v_0 - kt^2)dt$$

$$\int_i^f dx = \int_i^f (v_0 - kt^2)dt$$

$$x_f - x_i = \left[v_0 t - \frac{kt^3}{3}\right]_i^f$$

At this point, AFTER INTEGRATION, we usually set $t_i = 0$ and shift $t_f \to t$.
We solve for x_f which is equivalent to $x(t)$.
Always double check what happens when you plug in $t = 0$…in this case the $t = 0$ limit does drop out.
Remember that day I lost my mind in class? In that case the $t = 0$ limit didn't drop out (wkbk problem **4.25a**)…
Finally, don't forget this particular problem states $x_i = 0$. Always double check to see if x_i is non-zero…
Putting all that together gives

$$\boldsymbol{x_1(t) = v_0 t - \frac{kt^3}{3}}$$

6c) While not *always* true, we generally assume an object reverses direction at times when velocity is zero.
I will explain more about this issue after this solution…
We know $v_1(t_c) = 0$ implies a reversal in this case. When I write t_c I mean "t critical", the special time it reverses.

$$0 = v_0 - kt_c^2$$

$$t_c = \sqrt{\frac{v_o}{k}}$$

Plug this into $x_1(t)$ and simplify.

$$x_1(t_c) = v_0\left(\sqrt{\frac{v_o}{k}}\right) - \frac{k}{3}\left(\sqrt{\frac{v_o}{k}}\right)^3$$

$$x_1(t_c) = v_0\sqrt{\frac{v_0}{k}} - \frac{k}{3}\sqrt{\frac{v_o^3}{k^3}}$$

If you don't simplify, you are being impolite to your audience. You also lose points.

$$x_1(t_c) = \sqrt{\frac{v_0^3}{k}} - \frac{1}{3}\sqrt{\frac{v_0^3}{k}}$$

$$\boldsymbol{x_1(t_c) = \frac{2}{3}\sqrt{\frac{v_0^3}{k}} \approx 0.667\sqrt{\frac{v_0^3}{k}}}$$

Problem continues on next page…

6e) If object 2 catches up to object 1, we know they are at the same position.
Please note, in general this does NOT occur at the instant object 1 reverses direction.
We should NOT use the time t_c found in the previous step.

$$x_1(t) = x_2(t)$$

$$v_0 t - \frac{kt^3}{3} = \frac{2}{5} v_0 t$$

Since we are ignoring the possibility of the initial time $t = 0$ there is no problem in dividing each term by t.

$$v_0 - \frac{kt^2}{3} = \frac{2}{5} v_0$$

$$\frac{3}{5} v_0 = \frac{kt^2}{3}$$

$$t = \sqrt{\frac{9v_0}{5k}} \approx 1.342\sqrt{\frac{v_0}{k}}$$

Notice this is after the object has reversed direction…we expect a NEGATIVE final velocity.
Now plug this time into the equation for $v_1(t)$ and simplify.

$$v_1(t) = v_0 - k\left(\sqrt{\frac{9v_0}{5k}}\right)^2$$

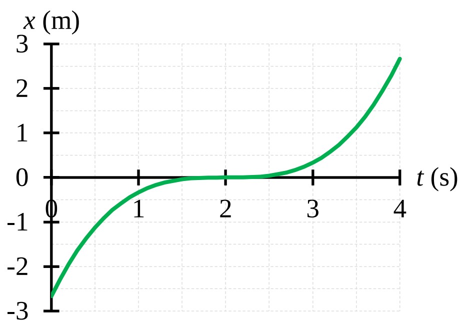
$$v_1(t) = v_0 - k\left(\frac{9v_0}{5k}\right)$$

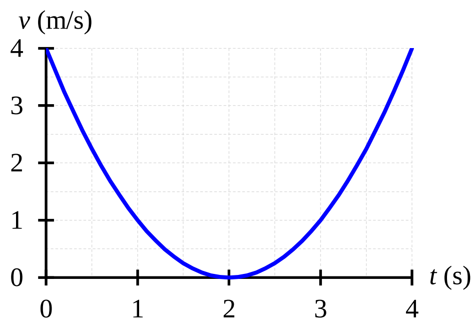
$$\boldsymbol{v_1(t) = -\frac{4v_0}{5} = -0.800v_0}$$

A last comment on setting $v(t) = 0$ to find when the object reverses.
In rare instances this technique will NOT work.
Consider a position function given by

$$x(t) = 0.3333\frac{\text{m}}{\text{s}^3}(t - 2.00\text{ s})^3 \qquad \text{which gives} \qquad v(t) = 1.000\frac{\text{m}}{\text{s}^3}(t - 2.00\text{ s})^2$$

Plots of x versus t and v versus t are shown below.
Notice velocity does go to zero at $t = 2.00$ s but the object does not reverse direction.
Position versus time has zero slope but not at a max or min…just an inflection point!





7a) Remember those brackets or lose points.

$$\alpha = \text{alpha} = a\ number\ with\ units$$

$$[\alpha] = the\ units\ of\ \text{alpha}$$

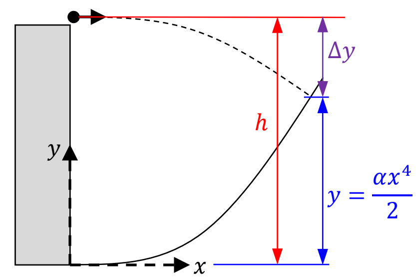
There is a big difference…

$$[y] = [\alpha][x^4]$$

$$[\alpha] = \frac{[y]}{[x]^4}$$

$$[\boldsymbol{\alpha}] = \frac{\mathbf{1}}{\mathbf{m^3}}$$

The symbol a is for acceleration. The symbol α is not the same thing.



7b) Ah yes, my old friend…the horizontal displacement equation…we meet again (and solve you for t).

$$x = v_0 t \rightarrow t = \frac{x}{v_0}$$

Plug into the vertical displacement equation.

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$\frac{\alpha x^4}{2} - h = -\frac{1}{2} g t^2$$

$$\frac{\alpha x^4}{2} - h = -\frac{1}{2} g \left(\frac{x}{v_0}\right)^2$$

$$\frac{\alpha x^4}{2} - h = -\frac{g x^2}{2 v_0^2}$$

$$\frac{\alpha x^4}{2} + \frac{g x^2}{2 v_0^2} - h = 0$$

Quadratic formula…it's not a big deal if you just practice it! Note: I mentioned in class to finish **2.38b**.

Tip: prior to doing all the work that follows, verify units on all terms match (here they are all in meters). Notice that in this instance we are using the quadratic formula to determine x^2 (instead of just x).

$$\text{Let } a = \frac{\alpha}{2}, b = \frac{g}{2 v_0^2}, \text{and } c = -h$$

$$x^2 = \frac{-\left(\frac{g}{2 v_0^2}\right) \pm \sqrt{\left(\frac{g}{2 v_0^2}\right)^2 - 4\left(\frac{\alpha}{2}\right)(-h)}}{2\left(\frac{\alpha}{2}\right)}$$

$$x^2 = \frac{-\frac{g}{2 v_0^2} \pm \sqrt{\left(\frac{g}{2 v_0^2}\right)^2 + 2\alpha h}}{\alpha}$$

Bring the α upstairs by multiplying numerator by $\frac{1}{\alpha}$. WATCH OUT! Must multiply by $\frac{1}{\alpha^2}$ *inside the square root*!

$$x^2 = -\frac{g}{2 v_0^2 \alpha} + \sqrt{\left(\frac{g}{2 v_0^2 \alpha}\right)^2 + \frac{2h}{\alpha}}$$

Since x^2 must be positive, only the positive root makes sense. Notice the term inside is slightly larger than $\frac{g}{2 v_0^2 \alpha}$…

Solution continues on the next page…

Now take the square root.
Again, from the figure shown, we know $x > 0$ so I will only use the positive root.

$$\boldsymbol{x} = \sqrt{\sqrt{\left(\frac{g}{2v_0^2\alpha}\right)^2 + \frac{2h}{\alpha}} - \frac{g}{2v_0^2\alpha}} \quad \text{or} \quad \left\{\left[\left(\frac{g}{2v_0^2\alpha}\right)^2 + \frac{2h}{\alpha}\right]^{1/2} - \frac{g}{2v_0^2\alpha}\right\}^{1/2}$$

I accepted answers that were correct and *approximately* of this form.
Note: It is polite to factor out a term in front which has the correct units times something which then has no units. I suppose the easiest thing would be to factor out $\sqrt{\frac{g}{2v_0^2\alpha}}$ giving

$$\boldsymbol{x} = \sqrt{\frac{g}{2v_0^2\alpha}}\sqrt{\sqrt{1 + \frac{8v_0^4\alpha h}{g^2}} - 1} = \left(\frac{g}{2v_0^2\alpha}\right)^{1/2}\left[\left(1 + \frac{8v_0^4\alpha h}{g^2}\right)^{\frac{1}{2}} - 1\right]^{1/2}$$