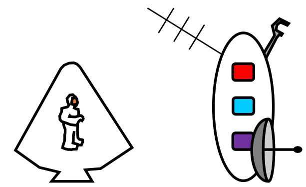
161 Spring 2023 T2A Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

$V_{sphere} = \frac{4}{3}\pi R^3$	$V_{box} = LWH$	$V_{cyl} = \pi R^2 H$	$\rho = \frac{M}{V}$
$A_{sphere} = 4\pi R^2$	$V = (A_{base}) \times (height)$	$A_{circle} = \pi R^2$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$C = 2\pi R$	$A_{rect} = LW$	$A_{CylSide} = 2\pi RH$	
$160\underline{9}\text{ m} = 1\text{ mi}$	$12\text{ in} = 1\text{ ft}$	$60\text{ s} = 1\text{ min}$	$1000\text{ g} = 1\text{ kg}$
$2.54\text{ cm} = 1\text{ in}$	$1\text{ cc} = 1\text{ cm}^3 = 1\text{ mL}$	$60\text{ min} = 1\text{ hr}$	$100\text{ cm} = 1\text{ m}$
$1\text{ cm} = 10\text{ mm}$	$1\text{ yard} = 3\text{ ft}$	$3600\text{ s} = 1\text{ hr}$	$1\text{ km} = 1000\text{ m}$
$1\text{ furlong} = 220\text{ yards}$	$528\underline{0}\text{ ft} = 1\text{ mi}$	$24\text{ hrs} = 1\text{ day}$	$1\text{ rev} = 2\pi\text{ rad} = 360°$
$g = 9.8\ \frac{\text{m}}{\text{s}^2}$	$G = 6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	$P_0 = 1.0 \times 10^5\text{ Pa}$	$1\text{ eV} = 1.60\underline{2} \times 10^{-19}\text{ J}$
$1\text{ N} = 1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$	$1\text{ J} = 1\text{ N}\cdot\text{m}$	$1\text{ Pa} = 1\ \frac{\text{N}}{\text{m}^2}$	
$x_f = x_i + v_{ix}t + \frac{1}{2}a_x t^2$	$v_{fx}^2 = v_{ix}^2 + 2a_x(\Delta x)$	$v_{fx} = v_{ix} + a_x t$	$r = \sqrt{x^2 + y^2}$
$\vec{A}\cdot\vec{B} = AB\cos\theta_{AB}$	$\lVert\vec{A}\times\vec{B}\rVert = AB\sin\theta_{AB}$	$\sin(A \pm B)$ $= \sin A\cos B \pm \cos A\sin B$	$\cos(A \pm B)$ $= \cos A\cos B \mp \sin A\sin B$
$\vec{v}_{ae} + \vec{v}_{eb} = \vec{v}_{ab}$	$\hat{r} = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$	$\hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$	
$a_{tan} = r\alpha$	$a_c = \frac{v^2}{r} = r\omega^2$	$\vec{a} = a_r\hat{r} + a_{tan}\hat{\theta}$	$\vec{a} = a_c(-\hat{r}) + a_{tan}\hat{\theta}$
$\Sigma\vec{F} = m\vec{a}$	$f \le \mu n$	$F_G = \frac{GmM}{r^2}(-\hat{r})$	$U_G = -\frac{GmM}{r}$
$TKE = \frac{1}{2}mv^2$	$RKE = \frac{1}{2}I\omega^2$	$U_S = SPE = \frac{1}{2}kx^2$	$U_G = GPE = mgh$
$E_i + W_{\substack{non-con \\ or\ ext}} = E_f$	$\Delta KE = W_{ext.\&\,non-con}$	$W = Fd\cos\theta = F_{\parallel}d$	$W = \int F_x dx$
$\Delta U = -W = -\int_i^f \vec{F}\cdot d\vec{s}$	$F_x = -\frac{d}{dx}U(x)$	$\mathcal{P}_{inst} = \frac{dE}{dt} = \vec{F}\cdot\vec{v}$	$\mathcal{P}_{avg} = \frac{\Delta E}{\Delta t} = \frac{Work}{time}$
$\vec{J} = \Delta\vec{p} = \vec{F}\Delta t$	$\vec{p} = m\vec{v}$	$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$	$x_{\text{CM}} = \frac{\int x\,dm}{\int dm}$
$\vec{\tau} = \vec{r}\times\vec{F}$	$\Sigma\vec{\tau} = I\vec{\alpha}$	$L = I\omega = mvr_{\perp}$	$\mathcal{P}_{inst} = \vec{\tau}\cdot\vec{\omega}$
$s = r\Delta\theta$	$v = r\omega$	$a_{tan} = r\alpha$	$a_c = \frac{v^2}{r} = r\omega^2$
$I_{\parallel axis} = I_{\text{CM}} + md^2$	$I_{zz} = I_{xx} + I_{yy}$	$I = \int r^2 dm$	$\frac{F}{A} = E\frac{\Delta L}{L_0}$
$P = \frac{F}{A}$	$P_{gauge} = P_{abs} - P_{ambient}$	$B = \rho_f V_{disp} g$	$A_1v_1 = A_2v_2$
$P(h) = P_0 + \rho gh$	$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$	$R = \frac{\pi r^4\Delta P}{8\eta L}$	$F = \eta A\frac{\Delta v_x}{\Delta y}$

Prefix	**Abbreviation**	**$10^?$**		**Prefix**	**Abbreviation**	**$10^?$**
Giga	G	10^9		milli	m	10^{-3}
Mega	M	10^6		micro	μ	10^{-6}
kilo	k	10^3		nano	n	10^{-9}
centi	c	10^{-2}		pico	p	10^{-12}
				femto	f	10^{-15}

A spaceship is in orbit around the earth. The ship launches a probe to the right. To launch the probe, the ship exerts constant force magnitude F on the probe during the launch.



1a) *During the launch*, which of the following is true. Circle the best answer.

The probe exerts a force on the ship with magnitude *more than* F.	The probe exerts a force on the ship with magnitude *less than* F.	The probe exerts a force on the ship with magnitude *equal to* F.	It is impossible to determine how the force exerted by the probe (on the ship) compares to F without knowing the exact masses!

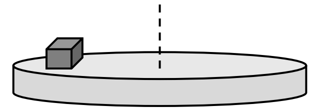
1b) Explain your reasoning for your previous answer with words (not equations).
I'm only expecting a couple lines here.

1c) *During the launch*, which of the following is true. Circle the best answer.

The probe accelerates at a *faster* rate than the ship.	The probe accelerates at a *slower* rate than the ship.	The probe accelerates at the *same* rate as the ship.	It is impossible to determine which accelerates faster without knowing the exact masses!

1d) Explain your reasoning for your previous answer with words (not equations).
I'm only expecting about 3-4 lines here. Be thorough and clear or lose points.

A box is on top of a spinning turntable which rotates at a constant rate ω. The box does not slide relative to the turntable. At some instant, the box is at the leftmost point in the rotation.



2a) For the instant shown, what *direction* does friction act on the block? Circle the best answer.

To the right	To the left	No friction	Impossible to determine without knowing the rotation direction of the turntable.

2b) Does friction do positive or negative work *on the block* as the turntable spins?

Positive	No work	Negative	Impossible to determine without knowing the rotation direction of the turntable.

2c) Explain your reasoning for your previous answer with words (not equations). Full sentences not required.

2d) Which friction condition most likely applies between the block and the turntable? Circle the best answer.

$f = \mu_k n$	$f = \mu_s n$	$f < \mu_s n$	No friction

2e) If the rotation rate was cut in half, what happens to the *magnitude* of the friction force?

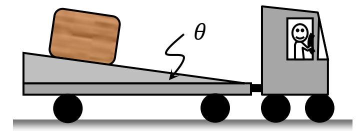
Remains unchanged	Doubles	Cuts in half	None of the other answers applies.

2f) Explain your reasoning for your previous answer with words and an FBD. Full sentences not required.

2g) Let us call weight of the block pulling down on it the "action" force.
What is the "reaction" force associated with this weight force.
Include the following in your description of the reaction force:

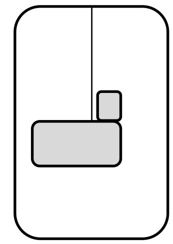
- the object exerting the reaction force
- the type of force & direction of the force
- the object experiencing the reaction force

A block is on an inclined plane of angle θ on the back of a flat-bed truck.
The truck accelerates (to the right) with magnitude a.
******3) Determine the *minimum* coefficient of friction required to prevent the block from sliding up the incline and off the back of the truck.



3	

A block of mass m hangs from an ideal string in an elevator. A second block, with 27.5% less mass, rests on top of the first block. The system is designed such that the second mass does not slide or tip over. The elevator is moving. The tension in the string is exactly twice the weight of the first block.



4a) Which direction is the elevator accelerating?

No acceleration	Acceleration upwards	Acceleration downwards	Impossible to determine without more information

***4b) Determine the acceleration magnitude. If no acceleration, explain why below.
Answer as a number with three sig figs times g.

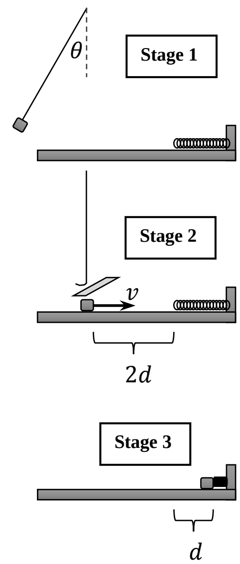
***4c) Determine the normal force (magnitude) between the blocks.
Answer as a number with three sig figs times mg.

4b	
4c	

Stage 1: A block of negligible size and mass m is attached to a massless string. The string is held at angle θ and the block is released from rest.

Stage 2: At the instant the block reaches the lowest point in the swing, a razor blade is used to cut the string. Assume this cutting process requires negligible time and exerts negligible force. The block is just barely above the ground when the cut happens and loses negligible energy as it drops to the surface. As a result of this process, the block is moving with speed v across the level surface.

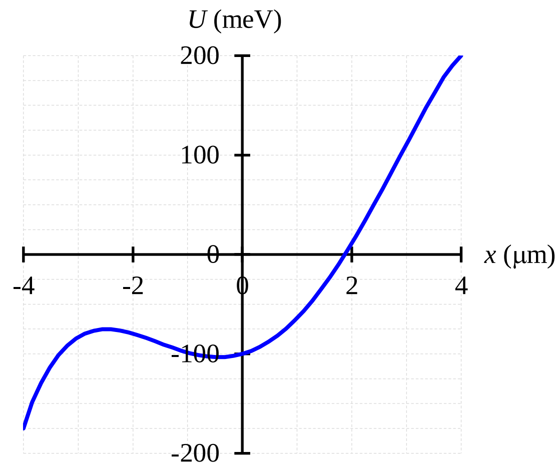
Stage 3: The block then slides across a level surface with frictional coefficients μ_s & μ_k. After travelling distance $2d$, the block impacts a massless spring which experiences maximum compression distance d.



***5a) Determine the spring constant of the spring.
***5b) Determine the length of the string.
**5c) Determine string tension (magnitude) just before the cut at the bottom of the swing.
Simplify all results to ensure credit.

5a	
5b	
5c	

A particle of mass 4.21×10^{-25} kg moves in one dimension subject to a single conservative force. A plot of potential energy versus horizontal position for this particle is shown.



6a) Identify all equilibrium positions by type by filling in the table below. **Note**: you *may* or *may not* require all rows.

Position (in units of μm)	**Stable/unstable/neutral**

6b) Which best describes the *direction* of force acting on the particle at $x = -1.5$ μm? Circle the best answer.

$F_x > 0$	$F_x = 0$	$F_x < 0$	Impossible to determine without more information

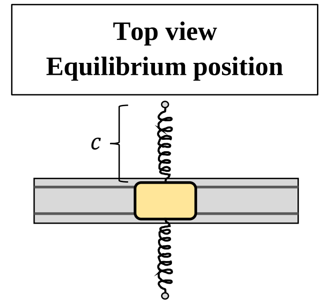
****6c) Assume the particle starts from rest at $x = -4.0$ μm.
Determine the *minimum* initial speed required for the particle to reach the origin.
Answer in units of $\frac{\text{m}}{\text{s}}$ using scientific notation.

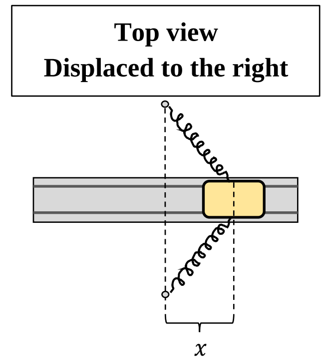
6c	

A cart on a level track is attached to two identical ideal springs as shown (**not to scale**).
You may assume each spring has negligible unstretched length and spring constant k.
In the equilibrium position, notice each spring is stretched distance c.
Notice x is the cart's horizontal position relative to the equilibrium.
A physicist claims the net horizontal force on the cart is *approximately* given by

$$F_x \approx -\frac{2kcx}{\sqrt{x^2+c^2}}$$

Assume all other horizontal forces acting on the spring are negligible.
****7a) Determine potential energy as a function of x using this force function.
Potentially useful derivatives and integrals are shown below.





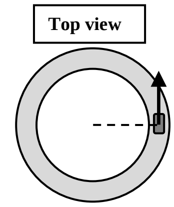
$\frac{d}{dx}\left(\frac{x}{x^2+a^2}\right)=\frac{a^2-x^2}{(x^2+a^2)^2}$	$\int \frac{dx}{(x^2+a^2)^{1/2}}=\ln\lvert x+(x^2+a^2)^{1/2}\rvert$
$\frac{d}{dx}\left(\frac{1}{(x^2+a^2)^{1/2}}\right)=\frac{-x}{(x^2+a^2)^{3/2}}$	$\int \frac{x\,dx}{(x^2+a^2)^{3/2}}=\frac{-1}{(x^2+a^2)^{1/2}}$
$\frac{d}{dx}\left(\frac{x}{(x^2+a^2)^{3/2}}\right)=\frac{1}{(x^2+a^2)^{3/2}}-\frac{3x^2}{(x^2+a^2)^{5/2}}$	$\int \frac{x\,dx}{(x^2+a^2)^{1/2}}=(x^2+a^2)^{1/2}$
$\frac{d}{dx}\left(\frac{x}{(x^2+a^2)^{3/2}}\right)=\frac{1}{(x^2+a^2)^{1/2}}-\frac{x^2}{(x^2+a^2)^{3/2}}$	$\int \frac{x\,dx}{x^2+a^2}=\frac{1}{2}\ln\lvert x^2+a^2\rvert$

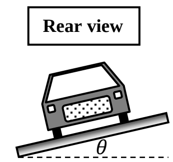
7a	

7b) Which best describes the potential energy in the equilibrium position? Circle the best answer.

Positive	Zero	Negative	Impossible to determine without more information

A car of mass 999 kg drives at $22.2\,\frac{\mathrm{m}}{\mathrm{s}}$ on a curve banked at angle $\theta = 12.75°$. The coefficients of sliding friction between the tires and the road are $\mu_s = 0.888$ & $\mu_k = 0.666$. Assume rolling friction is negligible for this problem. Assume the car tires are not slipping.





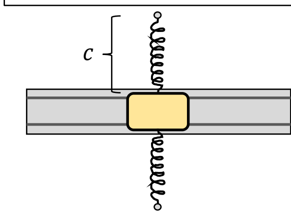
******8) Determine the *minimum* radius of curvature for the car to circumnavigate the turn.
**ExtraCredit) Determine frictional force (magnitude) acting on the car if the *actual* turn radius is 77.7 m. Note: this is not worth much so please do not attempt until the rest of the test is done. There is a second extra credit problem on the next page. Scores over 100% are not possible.

8	
EC	

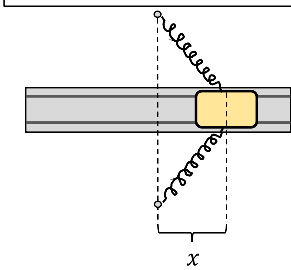
****Extra Credit:** Reconsider problem 7. Determine an *exact* (not approximate) result for the horizontal force when the cart is in position x. Then derive potential energy as a function of position for the exact force equation. Again, assume identical springs with negligible unstretched length and spring constant k.

Extra credit is not worth much so focus on regular credit.
Do this only if you finish the test early.
Scores over 100% are not possible.

Top view
Equilibrium position



Top view
Displaced to the right



$F_x(x) =$

$U(x) =$

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