161sp23t2asoln

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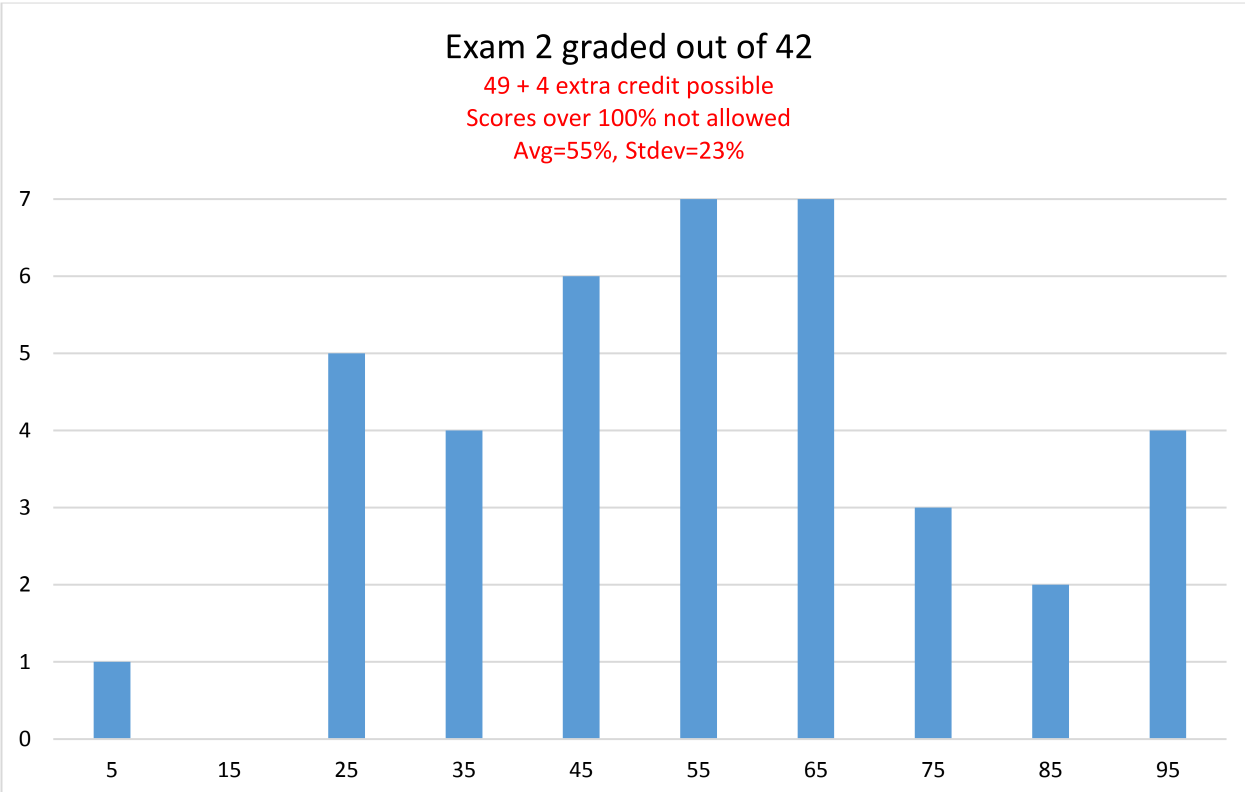


1a) The probe exerts a force on the ship with magnitude *equal to F*.
1b) By Newton's 3rd Law the force exerted by ship on probe must be the same size as the force exerted by the probe on the ship. This is true regardless of the masses.

1c) It is impossible to determine without the masses.
1d) By Newton's 2nd law we know $\vec{a} = \frac{\vec{F}_{NET}}{m}$.
Since the two objects are in deep space, the only force acting on each object comes from the other object.
We know from Newton's 3rd Law the two objects exert equal forces on each other.
Unfortunately, without knowing the masses involved, we cannot determine which object has more acceleration.

Note: on the original version of the exam I accidentally said the two objects were *in orbit* instead of *in deep space*. Being in orbit actually complicates the solution for 1c/d (but not 1a/b).

Obviously the masses play a key part in determining the accelerations (so that is still the best answer of the provided choices). That said, while in orbit both objects are already experiencing acceleration towards the center of the orbit (assuming circular orbit). One could imagine scenarios where the additional acceleration caused by the probe launch could be parallel, anti-parallel, perpendicular, or at any other direction relative to the already present orbital acceleration. One should sum the accelerations from orbit and launch force F to get net acceleration. It is still impossible to determine which has more acceleration but for this more complicated reason. If you mentioned this fact I gave you full credit.

2a) Friction acts to the right. Without friction, the block's velocity could not be turned to travel in a circular path!
2b) No work.
2c) Work is given by $W = \vec{F} \cdot \Delta\vec{s}$ where $\Delta\vec{s}$ is displacement.
The frictional force causes no displacement of the block relative to the turntable!
Friction does no work.
Another way to explain this:

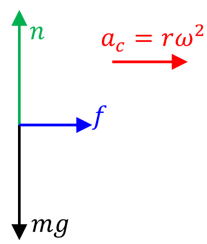
Friction in this scenario acts in a direction perpendicular to displacement.
Forces perpendicular to displacement cause no work.

Another way to explain this:

The speed of the block doesn't change.
If there is no speed change, there is no energy change.
If there is no energy change, there is no work done.
Here normal force and weight together cause no work as those forces balance.
This implies the only remaining force, friction, must also do no work.

2d) The block is not sliding relative to the turntable.
Therefore we know static friction applies ($f \leq \mu_s n$).
Unless we are told the system is on the verge of slipping, it is exceedingly unlikely the block is about to slip.
In real world scenarios, it is almost certainly $f < \mu_s n$.
Usually we use $f = \mu_s n$ condition to get an idea of where things *should* slip in the real world…but we rarely meet this condition *exactly*.

2e) None of the other answers applies.
2f) Here it makes sense to do an FBD and write a force equation.

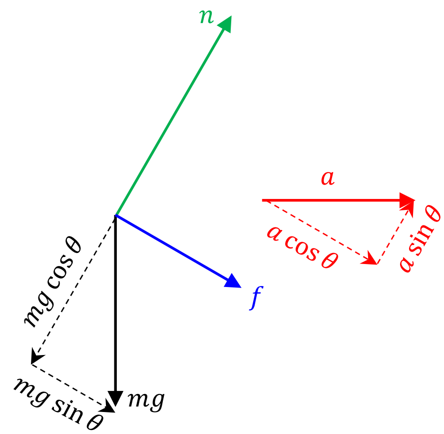


If the rotation rate is cut in half, a_c decreases by factor 4 (not 2).
We expect *four* times less friction.

2g) I ask it *every* year. I tell you I plan to ask it *every* year. It's probably on your final *every* year…

Action:	The *earth* exerts *gravitational* force *downwards* on the *block*.
Reaction:	The *block* exerts *gravitational* force *upwards* on the *earth*.

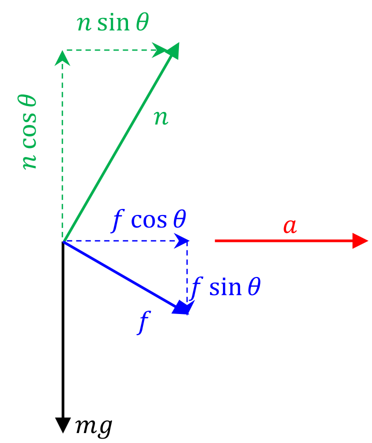
3) In this problem we are interested in learning about the frictional coefficient. It thus makes sense (to me) to choose a coordinate system aligned with friction (upper FBD). The same results can be obtained using the lower FBD (obviously your force equations will look different). Note: I exaggerated the angle to make it easier to label.



$$\text{Upper FBD } \Sigma F_{parallel}: \quad f + mg\sin\theta = ma\cos\theta$$
$$\text{Upper FBD } \Sigma F_{perpendicular}: \quad n - mg\cos\theta = ma\sin\theta$$

Now isolate f & n so you can take a ratio.

$$f = ma\cos\theta - mg\sin\theta$$
$$n = ma\sin\theta + mg\cos\theta$$



Since we are looking for *minimum* coefficient of friction we may use the "on the verge of slipping" condition.

$$\frac{\mu_s n}{n} = \frac{ma\cos\theta - mg\sin\theta}{ma\sin\theta + mg\cos\theta}$$

Notice n cancels on the left side while m cancels on the right side.

$$\boldsymbol{\mu_s = \frac{a\cos\theta - g\sin\theta}{a\sin\theta + g\cos\theta}}$$

This answer is acceptable. However, we often divide every term by $g\cos\theta$. This gives a slightly prettier result

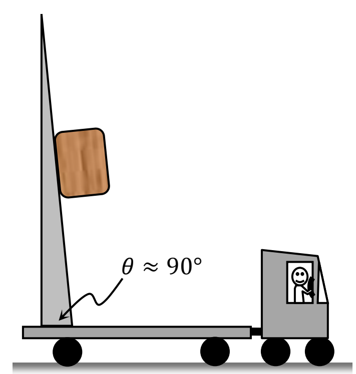
$$\boldsymbol{\mu_s = \frac{\frac{a}{g} - \tan\theta}{\frac{a}{g}\tan\theta + 1}}$$

Notice both results reduce to the result of workbook problem **6.39** when $\theta = 0°$ as we expect it should!

Notice the 1st method of writing the equation produces a non-infinite result when $\theta = 90°$.
Notice the 2nd method of writing the equation initially appears to produce an infinite result when $\theta = 90°$.
However, notice you could use could use l'Hôpital's rule and the two forms do produce the same limit as $\theta = 90°$.

WAIT A MINUTE! Did you notice the result for $\theta = 90°$ is $\mu_s = -\frac{a}{g}$?
Does this makes sense?

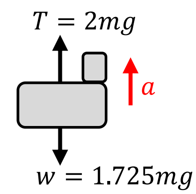
Yes.



Think: if $\theta = 90°$ the situation would look a bit like the figure shown at right.
Of course this problem makes no sense in the $\theta = 90°$ limit!
How could it possibly slide up and over a vertical wall when friction is down the plane!

Most common mistakes on problem 4:

- Saying object 2 has 27.5% less mass implies $m_2 = 0.725m$
- When using the system FBD use $\Sigma F = m_{total}a = (m_1 + m_2)a = 1.725ma$



4a) I think a system FBD will help here.
First I figure out the second mass so I don't screw that up.

$$m_2 = 27.5\%\ less \text{ than } m = (72.5\%)m = 0.725m$$

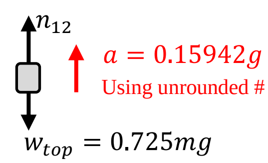
Ignore normal force between the blocks as it is internal to the m_1 & m_2 system.
Since the upward tension is larger than the downward weight, objects must accelerate upwards.

4b) Using the same system FBD one finds force equation

$$T - w = m_{total}a$$

$$2mg - 1.725mg = (1.725m)a$$

$$\boldsymbol{a = 0.1594g}$$



4c) Do an FBD on either of the blocks (not a system FBD).
The upper block has a slightly easier FBD because tension is not directly applied to it.
Right produces force equation

$$n_{12} - w_{top} = m_{top}a$$

$$n_{12} = w_{top} + m_{top}a$$

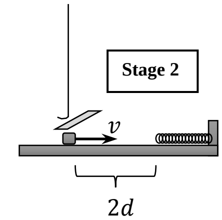
$$n_{12} = 0.725mg + (0.725m)(0.15942g)$$

Here I used the unrounded result for a to avoid intermediate rounding errors.

$$n_{12} = 0.841mg$$

5a) Use energy to compare Stage 2 to Stage 3.
As the block slides across the level surface it experience work done by friction.
In this simple case, I hope you can see $n = mg$ and $f = \mu_k n = \mu_k mg$.
Since the object moves in a straight line without reversing direction we know

$$magnitude\ of\ displacement = distance\ traveled = 2d + d = 3d$$

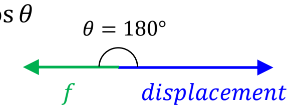


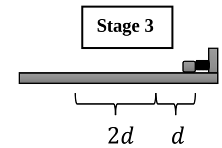
Work done by friction is

$$W_f = f(magnitude\ of\ displacement)\cos\theta$$

$$W_f = (\mu_k mg)(3d)\cos(180°)$$

$$W_f = -3\mu_k mgd$$





Since the block is sliding horizontally, there is no CHANGE to gravitational potential energy.
Said another way, if the block isn't going up or down we can ignore U_{Gi} & U_{Gf}.
Said yet another way, if we set our reference level at the bottom of the swing, $U_{Gi} = U_{Gf} = 0$.

The energy equation becomes

$$U_{Si} + K_i + W_f = U_{Sf} + K_f$$

Initially the spring is unstretched (also uncompressed). This tells us $U_{Si} = 0$.
We are told the max compression occurs when $x = d$. At max compression $v_f = 0$ which implies $K_f = 0$.

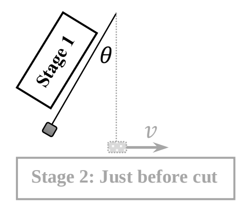
$$0 + \frac{1}{2}mv^2 - 3\mu_k mgd = \frac{1}{2}kd^2$$

$$\boldsymbol{k = \frac{mv^2 - 6\mu_k mgd}{d^2}}$$

5b) Use energy to compare Stage 1 to Stage 2 (Just before the cut).
As shown in the workbook and in lecture (or figure at bottom of page) one finds initial height

$$h = L(1 - \cos\theta)$$

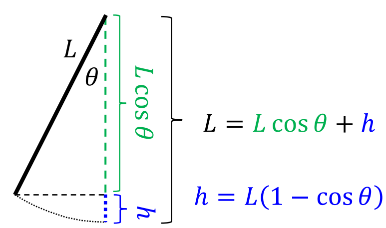
This assume the *bottom* of the swing is our reference level $y = 0$.
No spring potential energy to worry about in this part.
Tension is perpendicular to displacement. Therefore tension does zero work.
Assuming drag is negligible as well. Therefore assume no work.
Using the energy equation one finds



$$U_{Gi} + K_i + W = U_{Gf} + K_f$$

$$mgL(1 - \cos\theta) + 0 + 0 = 0 + \frac{1}{2}mv^2$$

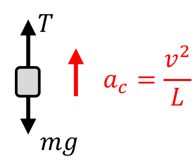
$$\boldsymbol{L = \frac{v^2}{2g(1 - \cos\theta)}}$$



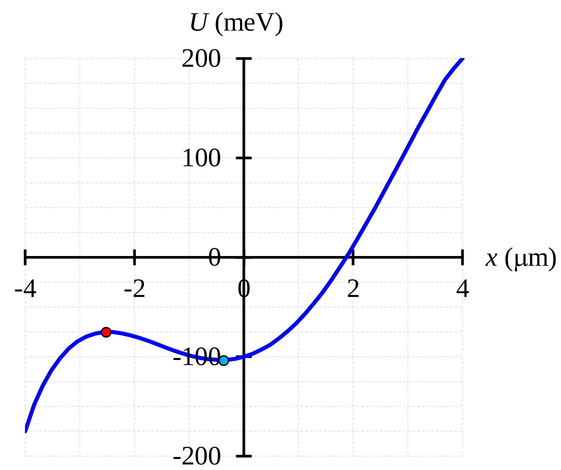
5c) Consider an FBD at the bottom of the swing (the question asked about a force)!

$$T - mg = ma_c = m\frac{v^2}{L} = m\frac{v^2}{\left(\frac{v^2}{2g(1-\cos\theta)}\right)} = m\big(2g(1 - \cos\theta)\big)$$

$$\boldsymbol{T = mg(3 - 2\cos\theta)}$$



6a) Equilibrium positions are occur wherever net force is zero.
Since our problem deals with a single conservative force, we should look for positions where this force is zero.
Hopefully you remember $F_x = -\frac{du}{dx} = -(\text{slope of } U \text{ vs. } x)$



Stable equilibrium implies a concave *up* U vs. x plot.
Unstable equilibrium implies a concave *down* U vs. x plot.
Neutral equilibrium implies a flat U vs. x plot.

x (µm)	Stable/unstable/neutral
-2.5	unstable
0.5	stable

6b) Hopefully you remember $F_x = -\frac{du}{dx} = -(\text{slope of } U \text{ vs. } x)$
Notice the slope is negative at $x = -1.5$ µm.
This implies $F_x > 0$.

6c) We want the particle to travel from $x_i = -4.0$ µm to the origin.
Notice the particle must first get over the potential energy max at -2.5 µm.
Minimum initial speed is found if $v_f = 0$ when $x_f = -2.5$ µm.
The force is conservative: use either $K_i + U_i = K_f + U_f$ or $\Delta K = -\Delta U$.

$$\Delta K = -\Delta U$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -(U_f - U_i)$$

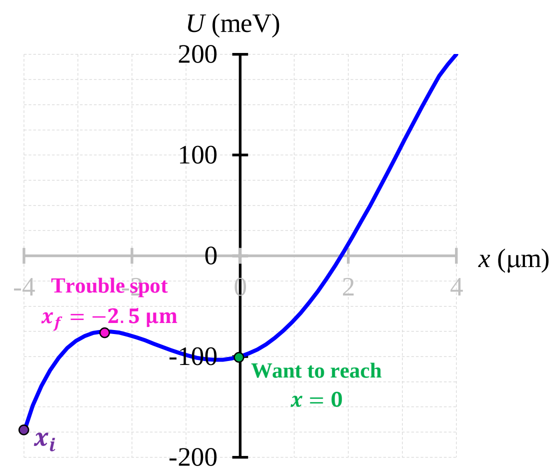
$$0 - \frac{1}{2}mv_{i\,\text{min}}^2 = -(-75\text{ meV} - (-175\text{ meV}))$$

The numbers are read off the vertical coordinate of the plot.
Also, cancel one minus sign out front and do the subtraction.

$$v_{i\,\text{min}} = \sqrt{\frac{2}{m}(100\text{ meV})}$$

WATCH OUT! Convert 100 meV $\rightarrow$ 1.602×10^{-20} J before computing!
If you forget the conversion the speed is not given in units of $\frac{\text{m}}{\text{s}}$!!!

$$\boldsymbol{v_{i\,\text{min}} = 276\frac{\text{m}}{\text{s}} = 2.76 \times 10^2\frac{\text{m}}{\text{s}}}$$



7a) We are told to assume the following force equation is valid

$$F_x \approx -\frac{2kcx}{\sqrt{x^2+c^2}}$$

To determine potential energy use

$$\Delta U = -\int_i^f F_x\, dx$$

$$U_f - U_{x=0} = -\int_0^{x_f} \left(-\frac{2kcx}{\sqrt{x^2+c^2}}\right) dx$$

Note: it is technically bad form to use x in the integrand and the limits.
I will use x_f as the upper limit.
After integration is complete we can then shift $x_f \to x$ to reduce subscript clutter.

$$U_f - U_{x=0} = 2kc \int_0^{x_f} \frac{x}{\sqrt{x^2+c^2}}\, dx$$

An integral in the table reads

$$\int \frac{x\, dx}{(x^2+a^2)^{1/2}} = (x^2+a^2)^{1/2}$$

This matched our integral if we identify $a = c$.

$$U_f - U_{x=0} = 2kc\left(\; x^2 + c^2 \; \right)^{1/2}\Big|_0^{x_f}$$

$$U_f - U_{x=0} = 2kc\left[\left(x_f^2+c^2\right)^{1/2} - (0^2+c^2)^{1/2}\right]$$

Notice the zero limit does NOT drop out!!!! At this point I will change $x_f \to x$ to reduce subscript clutter.

$$U_f - U_{x=0} = 2kc\left[(x^2+c^2)^{1/2} - c\right]$$

$$U_f - U_{x=0} = 2kc(x^2+c^2)^{1/2} - 2kc^2$$

Note we are asked to find $U(x)$. This is essentially the same thing as solving for U_f.

$$U(x) = U_f = U_{x=0} + 2kc(x^2+c^2)^{1/2} - 2kc^2$$

Here one has to actually think.
When $x = 0$ there are two springs of constant k stretched distanced c.

$$U_i = 2\left(\frac{1}{2}kc^2\right) = kc^2$$

Plugging in and simplifying gives

$$\boldsymbol{U(x) = 2kc(x^2+c^2)^{1/2} - kc^2}$$

CHECKS! If you plug in $x = 0$ notice we get back $U_i = kc^2$. Units check. As x increases U increases.

7b) If the two springs are stretched at equilibrium, there is non-zero (POSITIVE) energy at equilibrium.

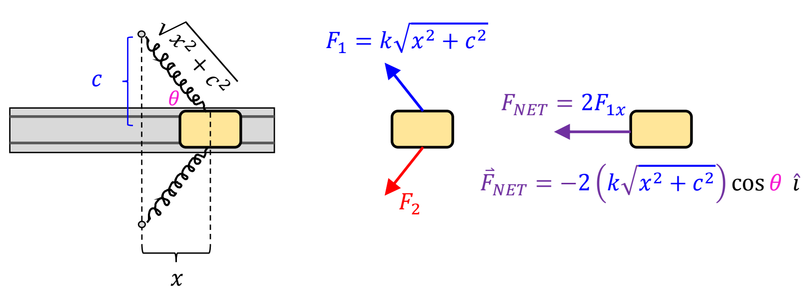
Extra Credit:
Consider the figure at right.
Notice

$$\cos\theta = \frac{x}{\sqrt{x^2+c^2}}$$

Surprisingly, one finds

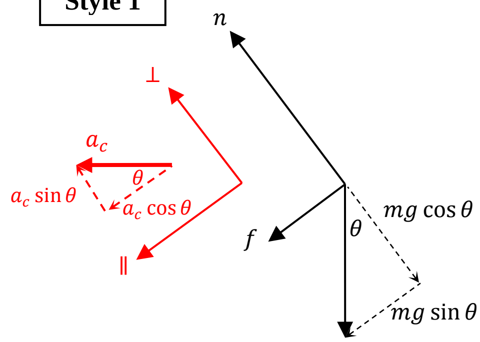
$$\vec{F}_{NET} = -2kx\hat{\imath}$$

$$U(x) = U_0 - \int_0^f F_x\, dx = kc^2 + kx^2$$

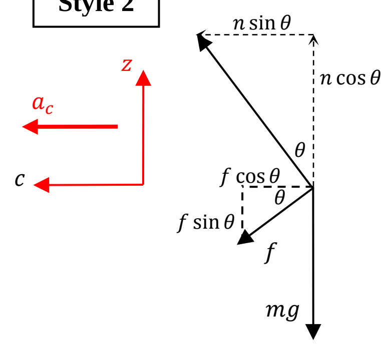


8) If looking for the *minimum* radius of curvature, it is ok to assume the car is on the verge of slipping towards the *outside* of the curve. This implies friction is directed *down* the plane. Two styles of FBDs are shown below. Angle is exaggerated to simplify labeling.

Style 1



Style 2



Style 1	**Style 2**
$\Sigma F_\perp$: $n - mg\cos\theta = ma_c \sin\theta$	ΣF_c: $n\sin\theta + f\cos\theta = ma_c$
$\Sigma F_\parallel$: $f + mg\sin\theta = ma_c\cos\theta$	ΣF_z: $n\cos\theta - f\sin\theta = mg$

Since we are trying to determine r (which appears in a_c), I'd probably use **Style 2** (but either style should work). Since we can assume the car is on the verge of slipping (because looking for *minimum* radius), we can use $f = \mu_s n$. Remember the point where the tires contact the road is not moving relative to the road. Use μ_s not μ_k!

Taking a ratio of the force equations for **Style 2** gives

$$\frac{n\sin\theta + \mu_s n\cos\theta}{n\cos\theta - \mu_s n\sin\theta} = \frac{ma_c}{mg}$$

$$\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{v^2}{rg}$$

$$r = \frac{v^2}{g}\cdot\frac{\cos\theta - \mu_s\sin\theta}{\sin\theta + \mu_s\cos\theta}$$

Divide each term in the fraction by $\cos\theta$ to make it look prettier.

$$r = \frac{v^2}{g}\cdot\frac{1 - \mu_s\tan\theta}{\tan\theta + \mu_s}$$

$$r = \frac{\left(22.2\,\frac{\text{m}}{\text{s}}\right)^2}{9.8\,\frac{\text{m}}{\text{s}^2}}\cdot\frac{1 - (0.888)\tan(12.75°)}{\tan(12.75°) + 0.888}$$

$$\boldsymbol{r = 36.1\ \text{m}}$$

Extra Credit: If the radius of curvature in real life is $r_{actual} = 77.7$ m, the car is *not* on the verge of slipping. I would use the $\Sigma F_\parallel$ equation from **Style 1** to find:

$$f = ma_c\cos\theta - mg\sin\theta = 4.02\ \text{kN}$$