161 Spring 2023 Test 3 A Once the exam has officially started, remove the top sheet. The remaining sheets comprise your exam. It is each student's individual responsibility to ensure the instructor has received her or his completed exam. Any exams not received by the instructor earn zero points. Smart watches, phones, or other devices (except scientific calculators) are not permitted during the exam.

| $V_{\text {sphere }}=\frac{4}{3} \pi R^{3}$ | $V_{\text {box }}=L W H$ | $V_{c y l}=\pi R^{2} H$ | $\rho=\frac{M}{V}$ |
| :---: | :---: | :---: | :---: |
| $A_{\text {sphere }}=4 \pi R^{2}$ | $V=\left(A_{\text {base }}\right) \times($ height $)$ | $A_{\text {circle }}=\pi R^{2}$ | $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$ |
| $C=2 \pi R$ | $A_{\text {rect }}=L W$ | $A_{\text {CylSide }}=2 \pi R H$ |  |
| $160 \underline{9} \mathrm{~m}=1 \mathrm{mi}$ | $12 \mathrm{in}=1 \mathrm{ft}$ | $60 \mathrm{~s}=1 \mathrm{~min}$ | $1000 \mathrm{~g}=1 \mathrm{~kg}$ |
| $2.54 \mathrm{~cm}=1 \mathrm{in}$ | $1 \mathrm{cc}=1 \mathrm{~cm}^{3}=1 \mathrm{~mL}$ | $60 \mathrm{~min}=1 \mathrm{hr}$ | $100 \mathrm{~cm}=1 \mathrm{~m}$ |
| $1 \mathrm{~cm}=10 \mathrm{~mm}$ | 1 yard $=3 \mathrm{ft}$ | $3600 \mathrm{~s}=1 \mathrm{hr}$ | $1 \mathrm{~km}=1000 \mathrm{~m}$ |
| 1 furlong $=220$ yards | $528 \underline{0} \mathrm{ft}=1 \mathrm{mi}$ | $24 \mathrm{hrs}=1$ day | $1 \mathrm{rev}=2 \pi \mathrm{rad}=360^{\circ}$ |
| $g=9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ | $G=6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}}$ | $P_{0}=1.0 \times 10^{5} \mathrm{~Pa}$ | $1 \mathrm{eV}=1.60 \underline{2} \times 10^{-19} \mathrm{~J}$ |
| $1 \mathrm{~N}=1 \frac{\mathrm{~kg} \cdot \mathrm{~m}}{\mathrm{~s}^{2}}$ | $1 \mathrm{~J}=1 \mathrm{~N} \cdot \mathrm{~m}$ | $1 \mathrm{~Pa}=1 \frac{\mathrm{~N}}{\mathrm{~m}^{2}}$ | $m_{e}=9.11 \times 10^{-31} \mathrm{~kg}$ |
| $x_{f}=x_{i}+v_{i x} t+\frac{1}{2} a_{x} t^{2}$ | $v_{f x}^{2}=v_{i x}^{2}+2 a_{x}(\Delta x)$ | $v_{f x}=v_{i x}+a_{x} t$ | $r=\sqrt{x^{2}+y^{2}}$ |
| $\vec{A} \cdot \vec{B}=A B \cos \theta_{A B}$ | $\\|\vec{A} \times \vec{B}\\|=A B \sin \theta_{A B}$ | $\begin{aligned} & \sin (A \pm B) \\ & =\sin A \cos B \pm \cos A \sin B \end{aligned}$ | $\begin{aligned} & \cos (A \pm B) \\ & =\cos A \cos B \mp \sin A \sin B \end{aligned}$ |
| $\vec{v}_{a e}+\vec{v}_{e b}=\vec{v}_{a b}$ | $\hat{r}=\cos \theta \hat{\imath}+\sin \theta \hat{\jmath}$ | $\widehat{\theta}=-\sin \theta \hat{\imath}+\cos \theta \hat{\jmath}$ |  |
| $a_{t a n}=r \alpha$ | $a_{c}=\frac{v^{2}}{r}=r \omega^{2}$ | $\vec{a}=a_{r} \hat{r}+a_{t a n} \hat{\theta}$ | $\vec{a}=a_{c}(-\hat{r})+a_{t a n} \hat{\theta}$ |
| $\Sigma \vec{F}=m \vec{a}$ | $f \leq \mu n$ | $F_{G}=\frac{G m M}{r^{2}}(-\hat{r})$ | $U_{G}=-\frac{G m M}{r}$ |
| $T K E=\frac{1}{2} m v^{2}$ | $R K E=\frac{1}{2} I \omega^{2}$ | $U_{S}=S P E=\frac{1}{2} k x^{2}$ | $U_{G}=G P E=m g h$ |
| $E_{i}+\underset{\substack{\text { non-con } \\ \text { or ext }}}{ }=E_{f}$ | $\Delta K E=W_{\text {ext.\& }}$ non-con | $W=F d \cos \theta=F_{\\|} d$ | $W=\int F_{x} d x$ |
| $\Delta U=-W=-\int_{i}^{f} \vec{F} \cdot d \vec{s}$ | $F_{x}=-\frac{d}{d x} U(x)$ | $\mathcal{P}_{\text {inst }}=\frac{d E}{d t}=\vec{F} \cdot \vec{v}$ | $\mathcal{P}_{\text {avg }}=\frac{\Delta E}{\Delta t}=\frac{\text { Work }}{\text { time }}$ |
| $\vec{J}=\Delta \vec{p}=\vec{F} \Delta t$ | $\vec{p}=m \vec{v}$ | $x_{\mathrm{CM}}=\frac{m_{1} x_{1}+m_{2} x_{2}}{m_{1}+m_{2}}$ | $x_{\mathrm{CM}}=\frac{\int x d m}{\int d m}$ |
| $\vec{\tau}=\vec{r} \times \vec{F}$ | $\Sigma \vec{\tau}=I \vec{\alpha}$ | $L=I \omega=m v r_{\perp}$ | $\mathcal{P}_{\text {inst }}=\vec{\tau} \cdot \vec{\omega}$ |
| $s=r \Delta \theta$ | $v=r \omega$ | $a_{t a n}=r \alpha$ | $\omega=2 \pi f=\frac{2 \pi}{\mathbb{T}}$ |
| $I_{\\| \text {axis }}=I_{\text {CM }}+m d^{2}$ | $I_{z z}=I_{x x}+I_{y y}$ | $I=\int r^{2} d m$ | $\frac{F}{A}=E \frac{\Delta L}{L_{0}}$ |
| $P=\frac{F}{A}$ | $P_{\text {gauge }}=P_{\text {abs }}-P_{\text {ambient }}$ | $B=\rho_{f} V_{\text {disp }} g$ | $A_{1} v_{1}=A_{2} v_{2}$ |
| $P(h)=P_{0}+\rho g h$ | $P+\frac{1}{2} \rho v^{2}+\rho g h=\text { constant }$ | $R=\frac{\pi r^{4} \Delta P}{8 \eta L}$ | $F=\eta A \frac{\Delta v_{x}}{\Delta y}$ |


| Prefix | Abbreviation | $\mathbf{1 0}^{?}$ |  | Prefix | Abbreviation | $\mathbf{1 0}^{?}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Giga | G | $10^{9}$ |  | milli | m | $10^{-3}$ |
| Mega | M | $10^{6}$ |  | micro | $\mu$ | $10^{-6}$ |
| kilo | k | $10^{3}$ |  | nano | n | $10^{-9}$ |
| centi | c | $10^{-2}$ |  | pico | p | $10^{-12}$ |
|  |  |  |  | femto | f | $10^{-15}$ |



$I_{\text {disk }}=\frac{1}{2} m R^{2}$

$I_{\text {thin }}=\frac{1}{12} m L^{2}$

$I_{\text {disk }}=\frac{1}{4} m R^{2}$


$$
\underset{\text { ring }}{I_{\text {thick }}}=\frac{1}{2} m\left(R_{\text {inner }}^{2}+R_{\text {outer }}^{2}\right)
$$


$\underset{\text { plate }}{I_{\text {thin }}}=\frac{1}{12} m b^{2}$

$\underset{\text { mass }}{I \text { pnt }}=m x^{2}$

$\underset{\substack{\text { solid } \\ \text { sphere }}}{ }=\frac{2}{5} m R^{2}$

$I_{\substack{\text { spherical } \\ \text { shell }}}=\frac{2}{3} m R^{2}$


A basketball collides with a small sphere thrown into the air by a physics crazy reveler high on ketamine. Just before impact, the 99.9 gram sphere is moving upwards with speed $13.0 \mathrm{~m} / \mathrm{s}$. The 0.600 kg basketball is travelling at $2.00 \mathrm{~m} / \mathrm{s}$ to the right. Assume gravitational forces are negligible compared to collision forces during the collision. Assume the collision time is very short. Assume the collision is perfectly inelastic.


1a) During the collision, which object experiences a larger magnitude acceleration?

$$
\begin{array}{|l|l|l|l}
\text { sphere } & \text { basketball } & \text { Same } & \text { Impossible to determine without more info }
\end{array}
$$

1b) During the collision, which object experiences a larger magnitude change in momentum?

$$
\begin{array}{|l|l|l|l|}
\hline \text { sphere } & \text { basketball } & \text { Same } & \text { Impossible to determine without more info } \\
\hline
\end{array}
$$

1c) During the collision, which object experiences a larger magnitude force?

| sphere | basketball | Same | Impossible to determine without more info |
| :--- | :--- | :--- | :--- |

1c) Assume the two objects collide at the origin. Which best the direction of motion after the collision?

| $+\hat{\imath}$ | $+\hat{\jmath}$ | Into $1^{\text {st }}$ quadrant | Into $3^{\text {rd }}$ quadrant | None of the other answers accurately <br> describes the motion |
| :---: | :---: | :---: | :---: | :---: |
| $-\hat{\imath}$ | $-\hat{\jmath}$ | Into $2^{\text {nd }}$ quadrant | Into $4^{\text {th }}$ quadrant | Impossible to determine if one of the <br> other answers describes the motion |

A rod of length $L$ has non-uniform mass density $\lambda=c x^{2}$ where $c$ is an unknown positive constant. The rod has moment of inertia $I$ when rotated about the $y$-axis.
2a) Which best describes the center of mass position of the rod?


$$
\begin{array}{c|c|c|c}
\text { At the exact } & \text { Closer to left } & \text { Closer to right } & \text { Impossible to determine } \\
\text { middle of rod } & \text { end of rod } & \text { end of rod } & \text { without more info }
\end{array}
$$

***2b) Determine the constant $c$ in terms of $I \& L$.


A thin square plate of side $s$ has a circular hole cut from it as shown.
Assume the radius of the circular hole is exactly $\frac{s}{4}$.
Mass per unit area of the plate is $\sigma$ (uniform density).
The origin of a standard $x y$-coordinate system is at the midpoint of the left side of the plate.
**3a) Determine mass of the plate (after the hole has been cut).
Answer as a 3 sig fig number times $\boldsymbol{\sigma} s^{2}$.
**3b) Determine horizontal center of mass position (after the hole has been cut).
Answer as a 3 sig fig number times $\boldsymbol{s}$.


A uniform rod of length $L$ connects to the ceiling at one end using a frictionless pivot of negligible size.
A string with negligible mass connects to the rod distance $\frac{4}{5} L$ from the pivot.
This string tension has magnitude $T$.
An angle is given in the figure (approximately to scale).
***4a) Determine the mass of the rod.
Answer as a number with 3 sig figs times $\frac{T}{g}$.
***4b) Determine magnitude of the reaction force at the pivot.


## Answer as a number with 3 sig figs times $T$.



A disk of mass 2.00 kg and radius 375 mm rotates about its center as shown.
Initially the disk's rotation rate is $0.925 \frac{\mathrm{rad}}{\mathrm{s}}$.
It slows (at constant rate) to a stop in 0.100 revolutions.
5a) Determine the initial angular velocity of the disk in RPM.
$* * 5 b$ ) Determine the angular acceleration in $\frac{\mathrm{rad}}{\mathrm{s}^{2}}$.
5c) How long does it take for the disk to stop spinning?
**5d) Determine the initial total acceleration (magnitude) of point $\mathbf{P}$ (on the edge of the disk).
5e) Determine the initial kinetic energy of the disk.


| 5 a |  |
| :--- | :--- |
| 5 b |  |
| 5 c |  |
| 5 d |  |
| 5 e |  |
|  |  |

A block of mass $m$ slides to the right with constant speed across a surface with negligible friction. The block collides elastically a second block of unknown mass which is initially at rest.
After the collision, the unknown mass travels to the right with speed $v$ while the block of mass $m$ moves to the left with speed $4 v$.
******6a) Determine the unknown mass. Answer as a number with 3 sig figs times $m$. *6b) Determine the initial speed of mass $m$. Answer as a number with 3 sig figs times $v$.


A metal disk of mass $m$ and radius $R$ is designed to roll without slipping down an inclined track of length $x$ and angle $\theta$ as shown in the figures at right (not to scale). Notice there are two additional disks welded to the main disk with smaller radii $r=\frac{R}{3}$ serving as axles for main disk as it rolls down the track. Figure not to scale. Assume the two additional axle disks have negligible mass compared to $\boldsymbol{m}$. The disk is released from rest and allowed to roll down the track.


7a) True or false: It is appropriate to use constant acceleration kinematics equations for the disk as it rolls down the track.

| TRUE | FALSE | Impossible to determine <br> without more info |
| :---: | :---: | :---: |

7b) Which of the following equations best describes the relationship between angular speed \& translational speed? Circle the best answer.

| $v=\frac{1}{3} R \omega$ | $v=\frac{2}{3} R \omega$ | $v=R \omega$ | $v=\frac{3}{2} R \omega$ | $v=3 R \omega$ |
| :--- | :--- | :--- | :--- | :--- | | None of the other |
| :--- |
| answers is correct |

****7c) Determine angular acceleration (magnitude) at the instant the disk is released from rest.
***7d) Determine the translational speed of the disk as it reaches the bottom of the track (after the disk's center of mass has traveled distance $x$ ).

**Extra Credit: Time consuming for 2 points. No scores over $100 \%$ allowed.
Reconsider the thin square plate of side $s$ has a circular hole cut from it.
Assume the radius of the circular hole is exactly $\frac{s}{4}$.
Mass per unit area of the plate is $\sigma$ (uniform density).
The origin of a standard $x y$-coordinate system is at the midpoint of the left side of the plate.

Determine moment of inertia of the plate about the $y$-axis (after the hole has been cut).


Answer using a simplified expression in term of given variables time a 3 sig fig number in the numerator.


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