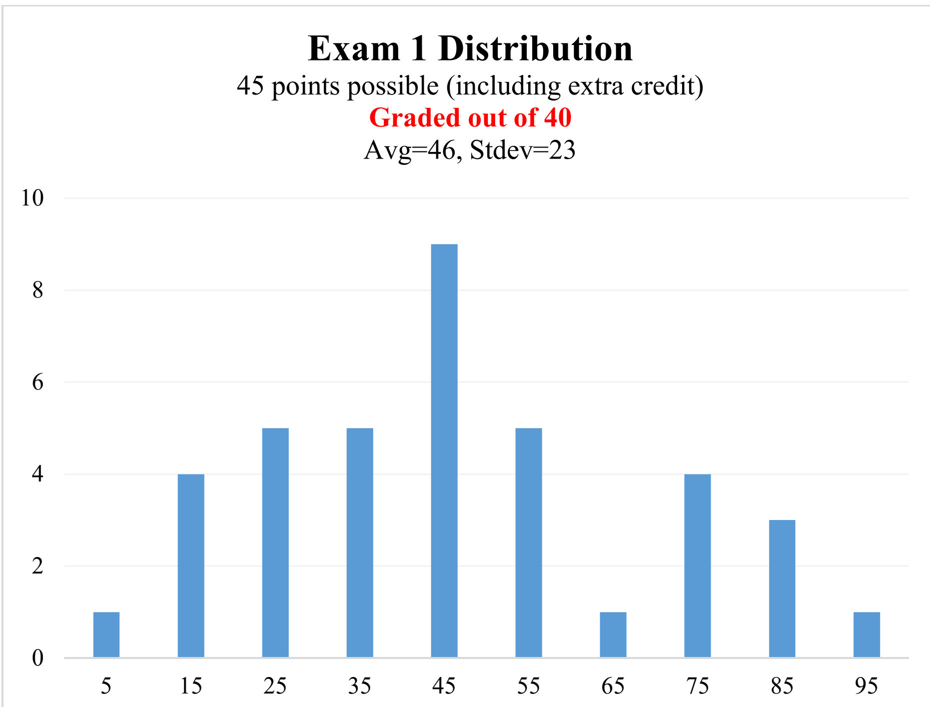
161sp24t1aSoln

Distribution on this page.
Solutions begin on the next page.

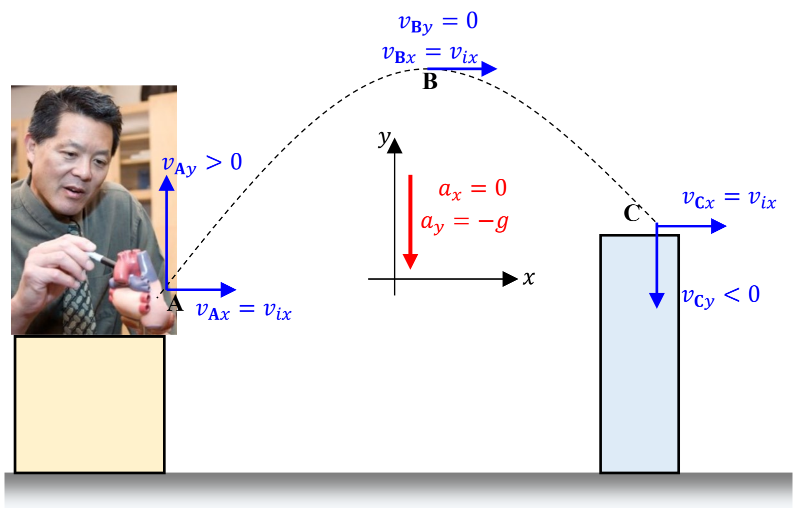
43 points possible (plus 2 extra credit possible).
Top score was 39.
Graded out of 40.



1a) Answer: $v_{\mathbf{B}y} = 0$

If air resistance is negligible, gravity is the only force acting on the heart in flight.
This implies freefall.
There is no acceleration in the x-direction (gravity acts downwardly…not horizontally).

The heart reverses direction *vertically* at max height. That implies the *vertical* component of velocity should be zero there.



1b) Answer: $v_{\mathbf{C}y} < v_{\mathbf{B}y} < v_{\mathbf{A}y}$
It takes more time to travel from **A** to **B** than from **B** to **C**.
Velocity should change more when travelling from **A** to **B** than it does when changing from **B** to **C** (gravity acting for more time causes acceleration over longer time interval).

1c) Answer: $v_{\mathbf{C}x} = v_{\mathbf{B}x} = v_{\mathbf{A}x}$
If air resistance is negligible, gravity is the only force acting on the heart in flight.
This implies freefall.
There is no acceleration in the x-direction (gravity acts downwardly…not horizontally).
We expect the horizontal component of velocity should remain unchanged throughout the flight.

1d) Greatest speed at the beginning of the flight.
Since the horizontal component of velocity is constant, heart has greatest speed when vertical component is largest.

1e) Acceleration is constant. All points have the same acceleration.
To be clear, we are only talking about times when the heart is in flight (in freefall).
If one included times when the hand or some other surface was touching the heart, things would be different…

2a) The equation listed in the problem statement only applies if acceleration is constant.
Acceleration is the derivative of velocity.
Taking the derivative of each equation.
Notice particles **B** & **C** have constant acceleration (equation applies…even if $\vec{a} = 0$).
Particles **A** & **D** have accelerations which are functions of time (not constant $\vec{a}$, equation does NOT apply).

2b) We assume $[v] = \frac{\text{m}}{\text{s}}$ & $[t] = \text{s}$.
The equation only makes sense if one assumes $[6] = \frac{\text{m}}{\text{s}^3}$.

3) The problem statement tells us 1 US shipping ton $= 40.00\ \text{ft}^3$.

$$333\ \text{m}^3 \times \frac{1^3\ \text{mi}^3}{1609^3\ \text{m}^3} \times \frac{5280^3\ \text{ft}^3}{1^3\ \text{mi}^3} \times \frac{1\ \text{US shipping ton}}{40.00\ \text{ft}^3} = 29\underline{4}.18\ \text{US shipping ton} = 2.94 \times 10^2\ \text{US shipping ton}$$

4) Here it may help to actually write the numbers out in the same form so we can see the uncertainty columns better.

$$\begin{array}{r} 102\underline{3}000\ \text{s} \\ -\ \ 9987\underline{0}0\ \text{s} \\ \hline 2\underline{4}300\ \text{s} \end{array}$$

We keep the leftmost uncertainty column when adding or subtracting.
As mentioned many times, we see the number of sig figs often changes when doing addition or subtraction.
Now put this in engineering notation using your calculator (or count over in bunches of 3).

24 ks

5) The units of any two terms must match. Compare the units of mg to $\frac{t^3}{d^5c^2}$.
Note: the minus sign has no effect on the units of the last term.

$$[mg] = \left[\frac{t^3}{d^5c^2}\right]$$

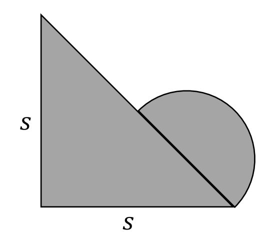
$$[c^2] = \left[\frac{t^3}{d^5mg}\right]$$

$$[c] = \sqrt{\left[\frac{t^3}{d^5mg}\right]}$$

$$[c] = \sqrt{\frac{\text{s}^3}{\text{m}^5 \cdot \text{kg} \cdot \frac{\text{m}}{\text{s}^2}}}$$

$$[\boldsymbol{c}] = \sqrt{\frac{\mathbf{s}^5}{\mathbf{m}^6 \cdot \mathbf{kg}}}$$

6) **Note: we were told the semi-circular plate was made of the same material. It is entirely reasonable to assume equal thickness (unless otherwise specified).**
From memory (or Pythagorean theorem) we know the diagonal has length $\sqrt{2}s$.
The diameter of the semi-circle is half that. The radius is thus $r = \frac{\sqrt{2}s}{4}$.



$$Area = \frac{1}{2}base \times height \ + \ \frac{1}{2}\pi r^2$$

$$Area = \frac{1}{2}s \cdot s \ + \ \frac{1}{2}\pi\left(\frac{\sqrt{2}s}{4}\right)^2$$

$$Area = s^2\left(\frac{1}{2} + \frac{\pi}{16}\right)$$

$$Area = 0.6964s^2$$

Now use $Volume = Area \times thickness$ and $density = \frac{mass}{Volume} \rightarrow mass = density \times Volume$.

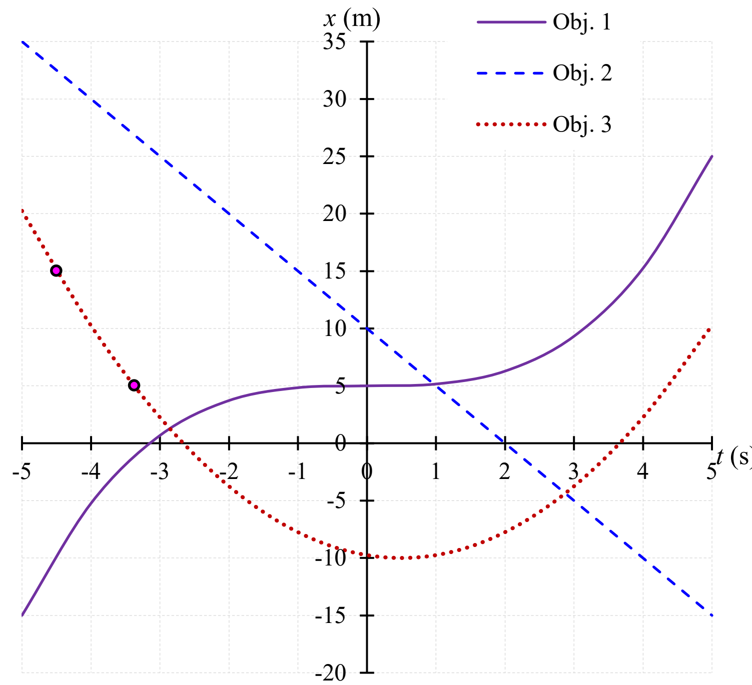
$$Volume = 0.6964s^2t$$

$$\boldsymbol{mass = 0.696\rho s^2 t}$$

7a) Looking at a plot of position versus time.
Velocity is the slope of this plot.
Object 1 is at rest at time $t = 0$.

Note: most of you said it was at rest over the time interval $-1.0 \text{ s} \rightarrow +1.0 \text{ s}$.

I accepted that as well even though the function is not perfectly flat over that entire time interval.



7b) Acceleration manifests itself as concavity on an xt-plot. Notice **Object 2** is a straight line with no concavity (infinite radius of curvature).
Object 2 is NOT accelerating.

7c) Looking at a plot of position versus time.
Velocity is the slope of this plot.
Between times $t = -5.0$ s and $t = 0.0$ s, **object 3** has negative slope. **Moving left entire time.**

7d) Slope is initially large and negative for **object 3**.
Slope gradually becomes small and negative.
The absolute value of the slope is speed.
Slowing down entire time.

Alternatively, slope is negative the entire time for **object 3** (velocity is negative).
Concave up the entire time (acceleration is positive).
Velocity and acceleration are opposite signs.
Slowing down entire time.

7e) Note: standard practice for these problems (both in-class and on hmwk) was to assume an implied $\hat{\imath}$.
Determine displacement of **object 1** on an xt-plot using

$$\Delta x = x_f - x_i = 25 \text{ m} - (-15 \text{ m}) = \mathbf{40\ m}$$

7f) Note: standard practice for these problems (both in-class and on hmwk) was to assume an implied $\hat{\imath}$.
Determine velocity of **object 3** at time $t = -4.0$ s by getting the slope.
Remember, select two points relatively close to the point of interest.
The points I chose are shown in the plot above.

$$v = \frac{rise}{run} \approx \frac{5 \text{ m} - 15 \text{ m}}{(-3.3 \text{ s}) - (-4.5 \text{ s})} = \frac{-10 \text{ m}}{1.2 \text{ s}} = \mathbf{-8.3\,\frac{m}{s}}$$

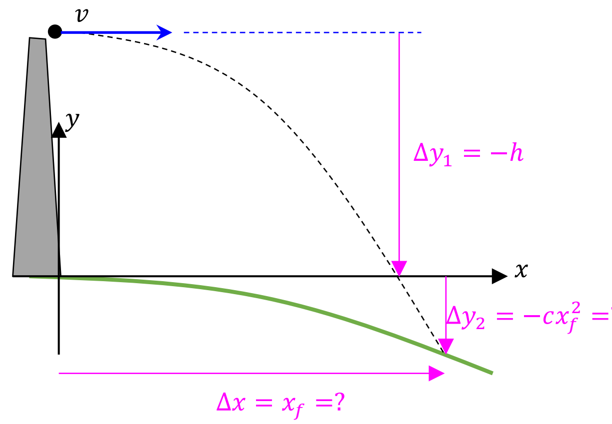
Why did I only use two sig figs here?
When reading the plot, I seriously doubt any of use could read these points to more than 2 sig figs.
Think about the size of the plot on the piece of paper you had…not that big.
I bet most of you will estimate the time interval as 1.0 s and get $-10\,\frac{\text{m}}{\text{s}}$.
That's a reasonable answer given the situation you were given.

8) We were told the ball hits a parabolic curve given by

$$y = -cx^2$$

where c is a positive constant.

We don't know the range of the projectile (how far it travels horizontally), but we do know that range relates to the last little bit of vertical displacement (shown in figure as Δy_2).



Follow the standard procedure as always.

Δx	x_f =?	Δy	$-h-cx_f^2$ =?
v_{ix}	v	v_{iy}	0
v_{fx}	?	v_{fy}	?
a_x	0	a_y	$-g$
t	?		

I'll first use the horizontal displacement equation to determine an expression for x_f.

$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$$

$$x_f = vt$$

Now I'll plug this expression for x_f into the vertical displacement equation.

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$$

$$-h - cx_f^2 = -\frac{g}{2}t^2$$

$$-h - c(vt)^2 = -\frac{g}{2}t^2$$

$$h + c(vt)^2 = \frac{g}{2}t^2$$

Solve for t.

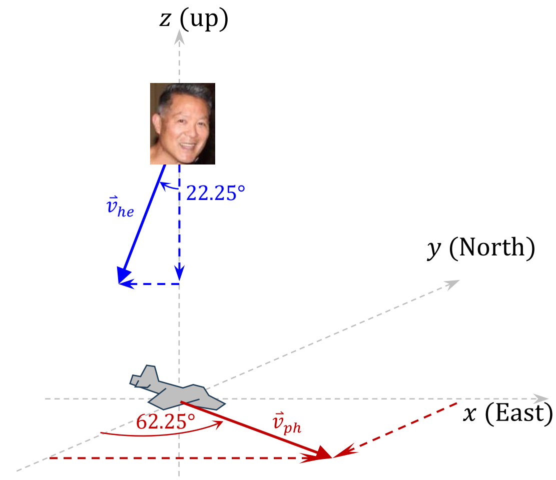
$$h = \frac{g}{2}t^2 - cv^2t^2$$

$$h = t^2\left(\frac{g}{2} - cv^2\right)$$

$$\boldsymbol{t = \sqrt{\frac{h}{\frac{g}{2} - cv^2}}}$$

9a) Pay close attention to following in these types of problems:

- location of the angles (to which axis)
- coordinate system (axis label indicates positive end of each axis)
- the $\pm$ signs & unit vectors appropriate for each vector
- velocity subscripts (relative *to earth*, relative *to plane*, etc)



We want $\vec{v}_{pe}$. It makes sense to write a correct relative velocity equation.

$$\vec{v}_{pe} = \vec{v}_{ph} + \vec{v}_{he}$$

Notice $\vec{v}_{ph}$ lies in the xy-plane but angle was given to the negative y-axis.

$$\vec{v}_{ph} = 80.0\frac{\mathrm{m}}{\mathrm{s}}(\sin 62.5°\,\hat{\imath} - \cos 62.5°\,\hat{\jmath})$$

$$\vec{v}_{ph} = (70.\underline{9}6\hat{\imath} - 36.\underline{9}4\hat{\jmath})\frac{\mathrm{m}}{\mathrm{s}}$$

Notice $\vec{v}_{he}$ lies in the xz-plane but angle was given to the z-axis.

$$\vec{v}_{he} = 47.5\frac{\mathrm{m}}{\mathrm{s}}(-\sin 22.5°\,\hat{\imath} - \cos 22.5°\,\hat{k})$$

$$\vec{v}_{he} = (-18.\underline{1}77\hat{\imath} - 43.\underline{8}8\hat{k})\frac{\mathrm{m}}{\mathrm{s}}$$

Out of habit I typically keep an extra digit along for the ride when the first digit is a 1.

Now do the addition.

$$\vec{v}_{pe} = (70.\underline{9}6\hat{\imath} - 36.\underline{9}4\hat{\jmath})\frac{\mathrm{m}}{\mathrm{s}} + (-18.\underline{1}77\hat{\imath} - 43.\underline{8}8\hat{k})\frac{\mathrm{m}}{\mathrm{s}}$$

$$\vec{v}_{pe} = (52.\underline{7}8\hat{\imath} - 36.94\hat{\jmath} - 43.\underline{8}8\hat{k})\frac{\mathrm{m}}{\mathrm{s}}$$

Speed (of Len Miyahara Head relative to earth) is the magnitude of this vector.

$$v_{pe} = 77.\underline{9}5\frac{\mathrm{m}}{\mathrm{s}}$$

I noticed a lot of students getting 77.94, probably by keeping additional unrounded digits.
Notice sig fig rules are not exactly perfect…just an estimate.

Direction (of Len Miyahara Head relative to earth) is the unit vector $\hat{v}_{pe} = \frac{\vec{v}_{pe}}{v_{pe}}$.

Notice the units drop out when computing a unit vector.
Be sure to use the unrounded result for speed so you can avoid intermediate rounding error.

$$\hat{v}_{pe} = \frac{(52.\underline{7}8\hat{\imath} - 36.94\hat{\jmath} - 43.\underline{8}8\hat{k})\frac{\mathrm{m}}{\mathrm{s}}}{77.\underline{9}5\frac{\mathrm{m}}{\mathrm{s}}}$$

$$\boldsymbol{\hat{v}_{pe} = 0.677\hat{\imath} - 0.474\hat{\jmath} - 0.563\hat{k}}$$

10) Angular momentum of a particle is defined as

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = (2.00\ \hat{\imath})\mathrm{m} \times \left(-3.00\hat{\imath} - 4.00\hat{\jmath} - 5.00\hat{k}\right)\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}$$

Remember $\hat{\imath} \times \hat{\imath} = 0$. Ignore that term. For the rest use the "wheel of pain".

$$\vec{L} = \left[(2.00\ \hat{\imath}) \times (-4.00\hat{\jmath}) + (2.00\ \hat{\imath}) \times \left(-5.00\hat{k}\right)\right] \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}$$

$$\vec{L} = \left[-8.00(\hat{\imath} \times \hat{\jmath}) - 10.00\left(\hat{\imath} \times \hat{k}\right)\right] \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}$$

$$\vec{L} = \left[-8.00\left(+\hat{k}\right) - 10.00(-\hat{\jmath})\right] \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}$$

$$\boldsymbol{\vec{L} = \left[10.00\hat{\jmath} - 8.00\hat{k}\right] \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}}$$

11a) $[a] = [k] \cdot [v] \quad \rightarrow \quad [k] = \frac{[a]}{[v]} = \frac{\frac{\mathrm{m}}{\mathrm{s}^2}}{\frac{\mathrm{m}}{\mathrm{s}}} = \frac{\mathrm{m}}{\mathrm{s}^2} \cdot \frac{\mathrm{s}}{\mathrm{m}} = \boldsymbol{\frac{1}{\mathrm{s}}}$

11b) Use separation of variables.

$$\frac{dv}{dt} = -kv$$

$$\frac{dv}{v} = -kt\ dt$$

$$\int_{v_0}^{v_f} \frac{dv}{v} = \int_{t_0}^{t_f} -kt\ dt$$

$$\ln \frac{v_f}{v_0} = -kt_f - kt_0$$

When we now shift $t_0 \rightarrow 0$ and $t_f \rightarrow t$ we know $v_f \rightarrow v(t)$!

$$\ln \frac{v(t)}{v_0} = -kt$$

Exponentiate both sides to isolate $v(t)$.

$$\frac{v(t)}{v_0} = e^{-kt}$$

$$\boldsymbol{v(t) = v_0 e^{-kt}}$$

12) This is effectively a three-stage problem (one stage for Angelica, two stages for Billie).
I chose to set Angelica to stage 3 to clarify the stage numbering, but I started the problem with stage 3.

Angelica at constant speed (Stage 3):

$$\Delta x_3 = d \qquad a_3 = 0 \qquad v_{3i} = v$$

$$d = vt_3$$

$$t_3 = \frac{d}{v}$$

Billie accelerates from rest (Stage 1):

We know Billie accelerates until reaching "30% greater" speed.

$$v_{1f} = v_{1i} + a_1 t_1$$

$$1.3v = at_1$$

$$t_1 = 1.3\frac{v}{a}$$

Now use this time in the Δx eqt'n.

$$\Delta x_1 = \frac{1}{2}a_1 t_1^2 + v_{1i}t_1$$

$$\Delta x_1 = \frac{1}{2}a\left(\frac{1.3v}{a}\right)^2$$

$$\Delta x_1 = 0.845\frac{v^2}{a}$$

Billie at constant speed (Stage 2):

We know total distance traveled is

$$\Delta x_1 + \Delta x_2 = d$$

$$\Delta x_2 = d - \Delta x_1$$

$$\Delta x_2 = d - 0.845\frac{v^2}{a}$$

We can write a second expression for Δx_2 using kinematics with $v_{2i} = v_{1f} = 1.3v$ & $a_2 = 0$ (constant speed).

$$\Delta x_2 = v_{2i}t_2 + \frac{1}{2}a_2 t_2^2$$

$$\Delta x_2 = 1.3vt_2$$

Setting these equations equal gives a result for t_2.

$$1.3vt_2 = d - 0.845\frac{v^2}{a}$$

$$t_2 = \frac{d}{1.3v} - \frac{0.845}{1.3}\cdot\frac{v}{a}$$

It is customary to have all numerical factors in the numerator.

$$t_2 = 0.7692\frac{d}{v} - 0.6500\frac{v}{a}$$

To get time delay, relate all of Biliie's times to total time of Angelica.

$$t_{delay} + t_1 + t_2 = t_3$$

$$t_{delay} = t_3 - t_1 - t_2$$

$$t_{delay} = \left(\frac{d}{v}\right) - \left(1.3\frac{v}{a}\right) - \left(0.7692\frac{d}{v} - 0.6500\frac{v}{a}\right)$$

$$\boldsymbol{t_{delay} = 1.950\frac{v}{a} - 0.231\frac{d}{v}}$$

Extra Credit:

mT	(in italics)	Italicized implies variables (not units). Expecting $mass \times Tension$ or $mass \times Period$.
Mt	(in italics)	Italicized implies variables (not units). Expecting $Mass \times time$.
mT	(no italics)	Un-italicized implies units (not variable). Expecting millitesla.
Mt	(no italics)	Un-italicized implies units (not variable). Expecting megatonne. Here in the US, a tonne is more often referred to as a metric ton $= 1000$ kg $= 1$ Mg