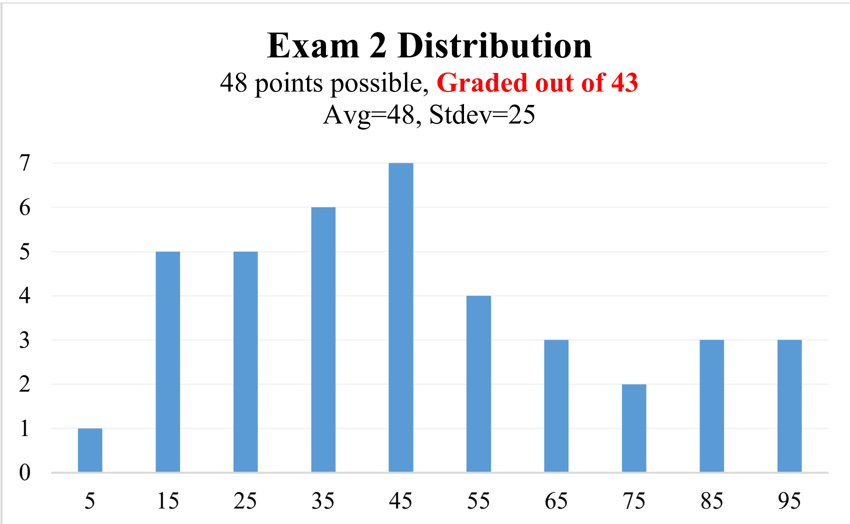
161sp24t2aSoln

Distribution on this page.
Solutions begin on the next page.



1a) Compute initial kinetic energy using

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(8.88\text{ kg})\left(5.00\frac{\text{m}}{\text{s}}\right)^2 = \mathbf{111.0\ J}$$

1b) *Average* power delivered by the *net* force on an object is computed using

$$\mathcal{P}_{avg} = \frac{\Delta K}{\Delta t} = \frac{K_f - K_i}{\Delta t} = \frac{\frac{1}{2}(8.88\text{ kg})\left(12.00\frac{\text{m}}{\text{s}}\right)^2 - 111.0\text{ J}}{3.00\text{ s}} = \mathbf{176.1\ W}$$

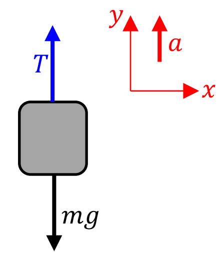
1c) Acceleration (magnitude) of the block is the slope (magnitude) of the block's vt-plot.

$$a = \frac{rise}{run} = \frac{7.00\frac{\text{m}}{\text{s}}}{3.00\text{ s}} = \mathbf{2.3\underline{3}3\frac{m}{s^2}}$$

1d) WATCH OUT!

We were asked about a force.

Hopefully your gut instinct is to immediately consider an FBD to get a feel for the problem.



We see

$$T - mg = ma$$

$$T = m(g + a)$$

Plug in known parameters to find

$$\boldsymbol{T} \approx \mathbf{107.7\ N}$$

1e) *Instantaneous* power delivered by the cable (*tension force only*) at time $t = 3.00$ s is given by

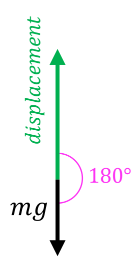
$$\mathcal{P}_{tension} = \vec{T} \cdot \vec{v}$$

$$\mathcal{P}_{tension} \approx (107.7\text{ N}\,\hat{j}) \cdot \left(12.00\,\frac{\text{m}}{\text{s}}\,\hat{j}\right)$$

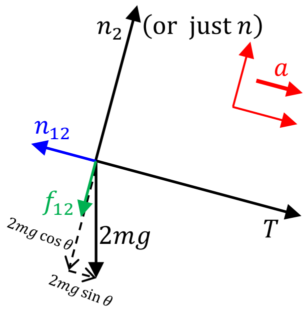
$$\boldsymbol{\mathcal{P}_{tension}} \approx \mathbf{1.293\ kW}$$

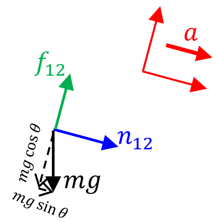
1f) The force of gravity is always anti-parallel to displacement.

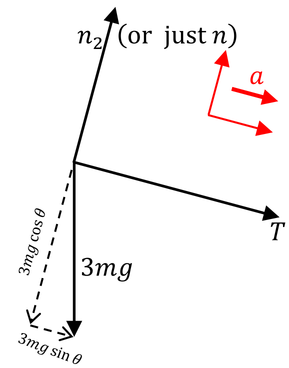
Gravity is doing **negative work** in this case.



2a) For two blocks sliding in unison we can use a system FBD. We could also draw two separate FBDs.







Block $2m$	**Block m**	**$m + 2m$ System**
ΣF_x: $T + 2mg\sin\theta - n_{12} = 2ma$	ΣF_x: $n_{12} + mg\sin\theta = ma$	ΣF_x: $T + 3mg\sin\theta = 3ma$
ΣF_y: $n_2 - 2mg\cos\theta - f_{12} = 0$	ΣF_y: $f_{12} - mg\cos\theta = 0$	ΣF_y: $n_2 - 3mg\cos\theta = 0$

We are asked to find the *minimum* thrust required to prevent block m from sliding. Therefore we also know

$$f_{12} = \mu_s n_{12}$$

Using force equations from block m gives

$$f_{12} = mg\cos\theta$$

$$\mu_s n_{12} = mg\cos\theta$$

$$\mu_s(ma - mg\sin\theta) = mg\cos\theta$$

$$a = \frac{g\cos\theta}{\mu_s} + g\sin\theta$$

I'll plug this result into the system FBD to determine T.

$$T + 3mg\sin\theta = 3ma$$

$$T = 3ma - 3mg\sin\theta$$

$$T = 3m(a - g\sin\theta)$$

$$T = 3m\left(\frac{g\cos\theta}{\mu_s} + g\sin\theta - g\sin\theta\right)$$

$$\boldsymbol{T = \frac{3mg\cos\theta}{\mu_s}}$$

As a check, we should get the same result plugging into the force equations for $2m$.

$$T + 2mg\sin\theta - n_{12} = 2ma$$

$$T = 2ma + n_{12} - 2mg\sin\theta$$

$$T = 2ma + (ma - mg\sin\theta) - 2mg\sin\theta$$

$$T = 3ma - 3mg\sin\theta$$

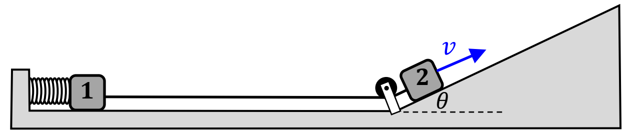
From there I can tell I will get the same result as before…

2b) If $T_{actual} > T_{min}$, block m is no longer on the verge of slipping.
We still have $f_{12} = mg\cos\theta$ but now the condition $f_{12} < \mu_s n_{12}$ applies.
Think if you could somehow increase f_{12}, block m would sliding upwards due to friction…that makes no sense!

3) Notice the question is asking about speed.
We also have a spring changing length (implying forces in the problem are *not* constant).
While not *always* the case, these signs *usually* indicate we should try an energy problem.

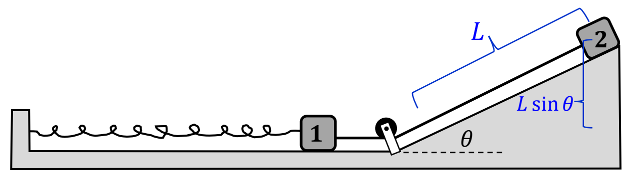
The upper figure at right shows the initial state:

- spring is initially unstretched
- block 2 is moving with unknown initial speed v
- block 1 is tied to block 2, it also moves with speed v



The next figure below shows the final state:

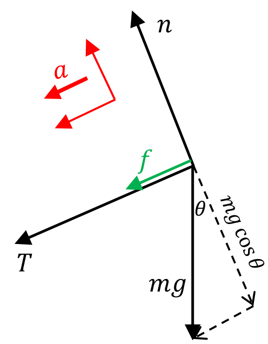
- block 2 has traveled distance L
- block 2 center of mass goes up $L\sin\theta$
- block 2 barely reaches top of ramp; it must be at rest
- block 1 is moving in unison with 2; it is also at rest
- spring had to stretch distance L
- block 1 experienced negligible friction
- friction does non-conservative work on block 2



The next figure below (3rd figure) is the FBD of block 2.
This is useful to determine

$$f = \mu_k n$$

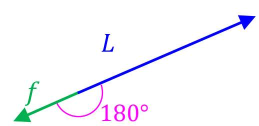
$$f = \mu_k (2m) g \cos\theta$$



The final figure shows displacement and friction.
This is useful for determining

$$W_{friction} = fL\cos 180°$$

$$W_{friction} = -2\mu_k mgL\cos\theta$$



Now do the energy problem:

$$K_i + U_{spring\ i} + U_{grav\ i} + W_{\substack{non-conervative \\ or\ \ external}} = K_f + U_{spring\ f} + U_{grav\ f}$$

$$\frac{1}{2}3mv^2 + 0 + 0 - 2\mu_k mgL\cos\theta = 0 + \frac{1}{2}kL^2 + 2mgL\sin\theta$$

Probably easiest to first multiply all terms by 2 to eliminate the fractions…

$$3mv^2 - 4\mu_k mgL\cos\theta = kL^2 + 4mgL\sin\theta$$

$$\boldsymbol{v = \sqrt{\frac{kL^2 + 4\mu_k mgL\cos\theta + 4mgL\sin\theta}{3m}}}$$

Maybe it *slightly* easier to check the units if you factor out $\frac{L}{3}$ and cancel some m's:

$$\boldsymbol{v = \sqrt{\frac{L}{3}\left(\frac{kL}{m} + 4\mu_k g\cos\theta + 4g\sin\theta\right)}}$$

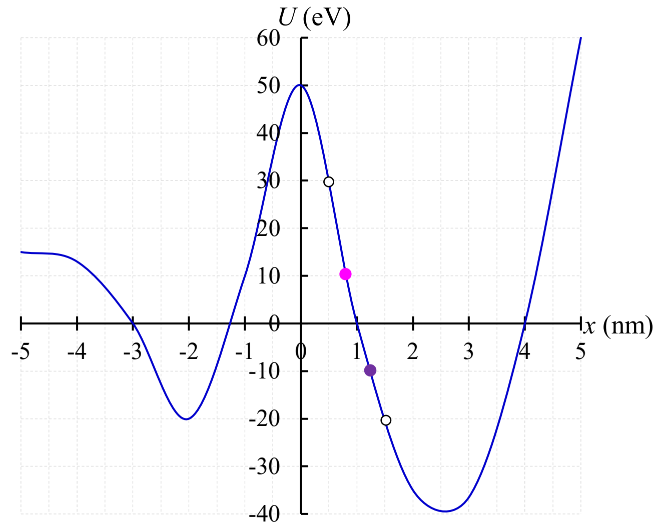
Either of these last two forms seems a reasonable answer.
Units check.
Other checks:

- If you have more friction (larger μ_k), you'd require more initial speed.
- If you have a longer ramp (larger L you'd require more initial speed.
- If the spring was more stiff (larger k), you'd require more initial speed.

4a) Force is given by

$$F_x = -\frac{dU}{dx} = -slope$$

At $x = 1.00$ nm the plot has a *negative* slope.
This implies a *positive* force at $x = 1.00$ nm.
Positive value of F_x implies **force to the right**.



4b) Again:

$$F_x = -slope$$

$$F_x = -\frac{rise}{run}$$

Pick two points fairly close to the point of interest.
Using the pink & purple points (closest to $x = 1.00$ nm).

$$F_x = -\frac{(-10.0\text{ eV}) - (10.0\text{ eV})}{(1.25\text{ nm}) - (0.80\text{ nm})} \approx 44.4\frac{\text{eV}}{\text{nm}}$$

If you chose the two white points you get $F_x \approx 50\frac{\text{eV}}{\text{nm}}$.
Both seem like reasonable estimates.
HOWEVER, you were asked to give the result in NEWTONS…don't forget to convert!
Using the pink and purple points I found

$$F_x \approx 44.4\frac{\text{eV}}{\text{nm}} \times \frac{1.602 \times 10^{-19}\text{ J}}{1\text{ eV}} \times \frac{1\text{ nm}}{1 \times 10^{-9}\text{ m}} \approx \mathbf{7.11 \times 10^{-9}\text{ N}}$$

Using the white points I found

$$F_x \approx 50\frac{\text{eV}}{\text{nm}} \times \frac{1.602 \times 10^{-19}\text{ J}}{1\text{ eV}} \times \frac{1\text{ nm}}{1 \times 10^{-9}\text{ m}} \approx \mathbf{8.01 \times 10^{-9}\text{ N}}$$

If the result had been negative, I'd use an absolute value to get the magnitude.

4c) We want the minimum initial speed required for the particle to travel from $x_i = -5.00$ nm to $x = +4.50$ nm.

Notice the particle will lose the most kinetic energy as it crosses over the potential hill at $x_f = 0$ nm!!!

Do an energy problem using $x_f = 0$ nm with $v_f = 0$!
Remember to convert those eV's!!!

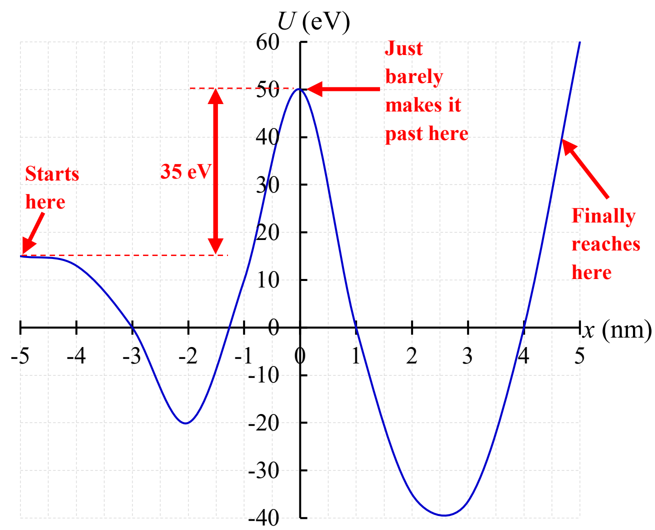
$$35\text{ eV} = 5.607 \times 10^{-18}\text{ J}$$

$$\Delta K = -\Delta U$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\Delta U$$

$$-\frac{1}{2}mv_i^2 = -\Delta U$$

$$v_i = \sqrt{\frac{2\Delta U}{m}}$$



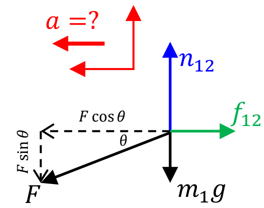
$$\boldsymbol{v_i} = \sqrt{\frac{\mathbf{2(5.607 \times 10^{-18}\text{ J})}}{\mathbf{5.55 \times 10^{-22}\text{ kg}}}} \approx \mathbf{142.1\frac{m}{s}}$$

5a) I drew a set of FBD's for each block to help me visualize the forces.
There is no acceleration in the vertical direction.
This implies upwards forces must balance downwards forces.

$$n_{12} = F\sin\theta + m_1 g$$

$$\boldsymbol{n_{12} > m_1 g}$$

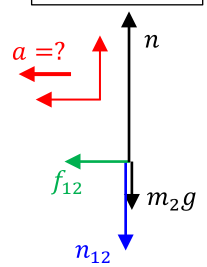
FBD for m_1



5b) It is unclear if the box (m_1) is on the verge of slipping.
In the real world, it is exceedingly unlikely for a person to push with the perfect acceleration such that the box of brains is on the verge of slipping.
In practice, either the box slides (which implies $f_{12} = \mu_k n_{12}$) or friction holding the box in place is less than the max possible amount (which implies $f_{12} < \mu_s n_{12}$).
The wording of *this* problem statement implies the overwhelming likely scenario is:

$$\boldsymbol{f_{12} < \mu_s n_{12}}$$

FBD for m_2



5c) Friction acts to the *left* on the dolly.

5d) Friction does *positive* work *on the dolly* ($\vec{f}_{12}$ points same direction as dolly's displacement).

5e) Normal force between the blocks does *zero* work on the dolly ($\vec{n}_{12}$ points perpendicular to dolly's displacement).

5f) By Newton's 3rd law, Len pushes on the box with equal magnitude to the force exerted by the box on Len.

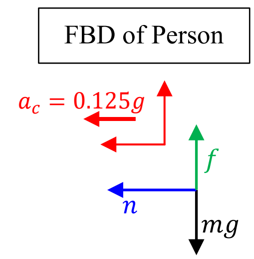
5g)

Action	**The earth** exerts a **gravitational** force on **the dolly** directed **downwards**. Object exerting force / type of force / object experiencing force / direction of force
Reaction	**The dolly** exerts a **gravitational** force on **the earth** directed **upwards**. Object exerting force / type of force / object experiencing force / direction of force

6a) Don't overthink this one. We can use either $a_c = \frac{v^2}{r}$ or $a_c = r\omega^2$.

We were also given the period ($\mathbb{T}$).

One could use



$$distance_{around\ circle} = rate \times period$$

$$2\pi r = v\mathbb{T}$$

$$v = \frac{2\pi r}{\mathbb{T}}$$

Alternatively, one could use

$$\omega = \frac{2\pi}{\mathbb{T}}$$

Either way, one finds

$$a_c = \frac{4\pi^2 r}{\mathbb{T}^2}$$

Rearrange to determine r…

$$r = \frac{a_c\mathbb{T}^2}{4\pi^2}$$

Now write this as a decimal number with three sig figs times an expression involving the givens.

$$r = \frac{2.34g\mathbb{T}^2}{4\pi^2} \approx \mathbf{0.0593}\boldsymbol{g}\mathbb{T}^2$$

6b) Using the *vertical* force equation form the FBD one finds

$$f = mg$$

The minimum coefficient is found by assuming the rider is on the verge of slipping.

$$\mu_s n = mg$$

Using the *horizontal* force equation from the FBD one finds

$$n = ma_c$$

$$n = 2.34mg$$

One could quickly determine μ_s using a ratio:

$$\frac{\mu_s n}{n} = \frac{mg}{2.34mg}$$

$$\mu_s = \frac{1}{2.34}$$

$$\boldsymbol{\mu_s \approx 0.427}$$

7a) Do energy methods. Work from friction & drag is negligible.
Normal force does zero work (perpendicular to displacement).

$$mgh_i + \frac{1}{2}mv_i^2 = mgh_f + \frac{1}{2}mv_f^2$$

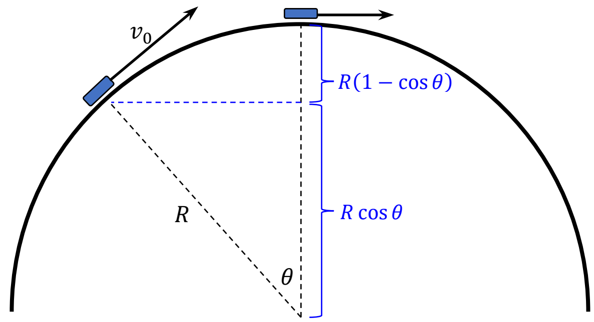
Multiply all by $\frac{2}{m}$ to reduce clutter.

$$2gh_i + v_i^2 = 2gh_f + v_f^2$$

Set lowest height to zero, $h_f = R(1 - \cos\theta)$ and $v_i = v_0$:

$$v_f = \sqrt{v_0^2 - 2gR(1 - \cos\theta)}$$

$$\boldsymbol{v_f \approx 1.779\,\frac{m}{s}}$$



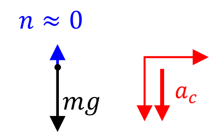
7b) Consider the FBD at right labeled *Ideal Speed*.
Notice we set the normal force to zero.
In this case the force equation becomes:

$$mg = ma_{c\ ideal}$$

$$g = \frac{v_{ideal}^2}{R}$$

$$\boldsymbol{v_{ideal} = \sqrt{Rg} \approx 2.27\,\frac{m}{s}}$$

Ideal Speed FBD at top
@ ideal speed, no normal force req'd



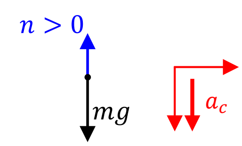
7c) Because *actual* speed is *lower* than *ideal* speed, we expect $n > 0$.
Think: at low speeds you know you need $n_{top} > 0$ to support mg.
The force equation becomes:

$$mg - n_{top} = ma_{c\ actual}$$

$$n_{top} = m\left(g - \frac{v_{actual}^2}{R}\right)$$

$$\boldsymbol{\vec{n}_{top} \approx 1.585\ \mathrm{N}\ \hat{j}}$$

Actual Speed FBD at top
$v < v_{ideal}$, normal force *upwards*



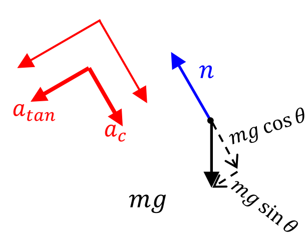
7d) I choose to draw $\vec{n}$ directed radially *outwards*. If I get a negative result, the normal force must be directed radially *inwards*.

$$mg\cos\theta - n_{@\,37.5°} = ma_{c\ @\,37.5°}$$

$$n_{@\,37.5°} = m\left(g\cos\theta - \frac{v_0^2}{R}\right)$$

$$\boldsymbol{\vec{n}_{@\,37.5°} = -0.967\ \mathrm{N}\ \hat{r}}$$

Initial Position FBD
Draw n radially outwards. If you get a negative number, $\vec{n}$ must point radially inwards.



7e) Finally, use the tangential force equation.
I choose to align my tangential coordinate axis in the same direction as tangential acceleration so my result would be positive.
Since we want the *magnitude* of a_{tan}, this works out well.

$$mg\sin\theta = ma_{tan}$$

$$a_{tan} = g\sin\theta$$

$$\boldsymbol{a_{tan} \approx 5.97\,\frac{m}{s^2}}$$