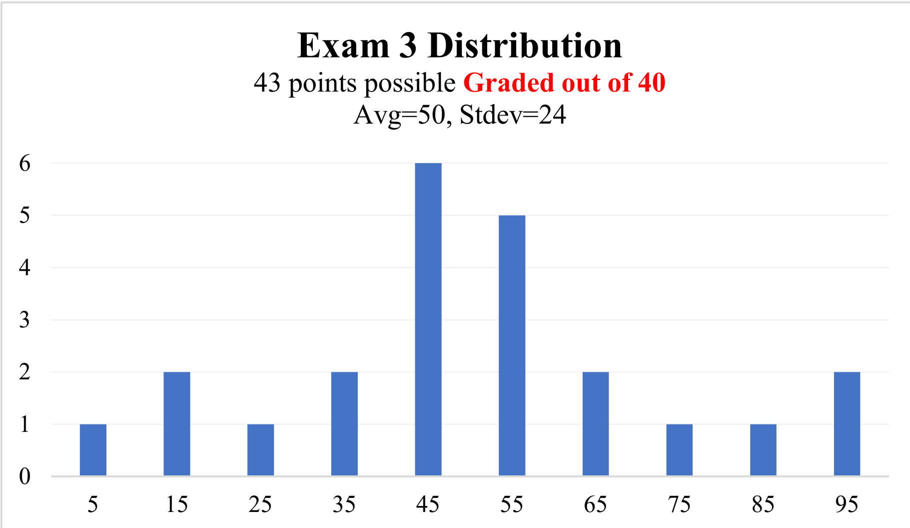
161sp24t3aSoln

Distribution on this page.
Solutions begin on the next page.



*1) We were told both objects have equal mass and identical speed.
This implies both objects have equal momentum magnitudes.

The collision is perfectly inelastic (the two move together as a single object after collision).

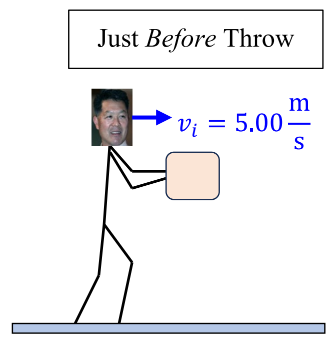
Since Vaida is moving at an angle, her horizontal momentum must be less than Zenobia's.
After they collide and stick together, we know they are moving to the *right*.

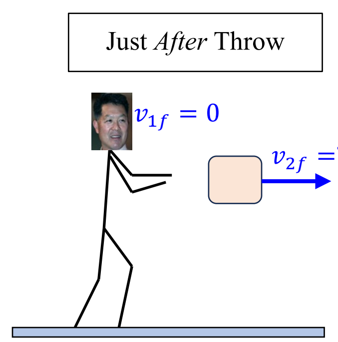
Since Vaida brings downwards momentum to the collision, the two are moving downwards after the collision.

After the collision, we expect the Vaida (and Zenobia) to move into the 4$^{\text{th}}$ quadrant (down and right).

2a) Energy is NOT conserved here.
Len is converting some energy from his cells to change the energy of the Len-mass system.
Linear momentum IS conserved.
Negligible net external force acting on the Len-Mass system.

2b) Consider the before and after pictures shown at right.
Now do conservation of linear momentum.





$$(m_1 + m_2)v_i = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{2f} = \frac{m_1 + m_2}{m_2} v_i$$

$$v_{2f} \approx 50.\underline{68}\,\frac{\mathrm{m}}{\mathrm{s}}$$

2c) Percent change in energy is given by

$$\%\Delta E = \frac{K_f - K_i}{K_i} \times 100\%$$

$$\%\Delta E = \left(\frac{K_f}{K_i} - 1\right) \times 100\%$$

$$\%\Delta E = \left(\frac{\frac{1}{2} m_2 v_{2f}^2}{\frac{1}{2}(m_1 + m_2)v_i^2} - 1\right) \times 100\%$$

$$\%\Delta E = \left(\frac{m_2 v_{2f}^2}{(m_1 + m_2)v_i^2} - 1\right) \times 100\%$$

$$\boldsymbol{\%\Delta E = +914\%}$$

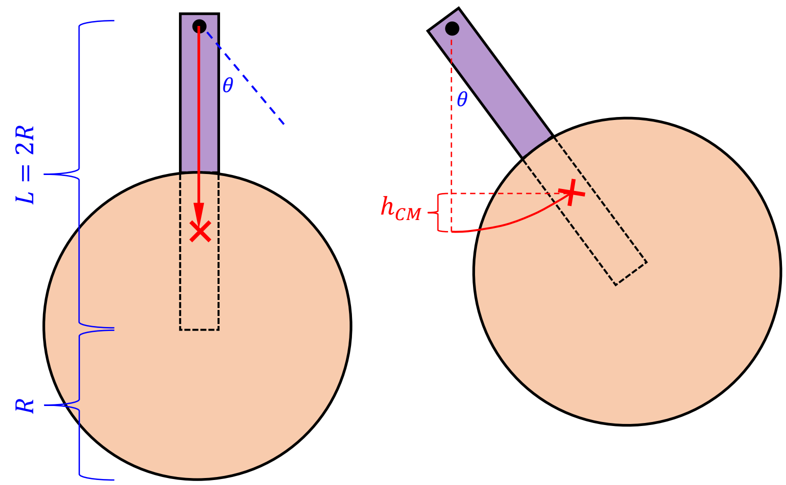
3a) Assume black dot (pivot) is the origin.
Let downwards be the positive direction.

$$r_{CM} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$

In this equation r_i is distance from pivot to center of the i^{th} mass.

$$r_{CM} = \frac{(M)2R + (2M)R}{M + 2M}$$

$$\boldsymbol{r_{CM} = \frac{4}{3}R \approx 1.333R}$$



3b) One must use parallel axis theorem to determine moment of inertia of the spherical shell.

$$I_1 = I_{CM\,1} + m_1 d_1^2$$

$$I_1 = \frac{2}{3}MR^2 + M(2R)^2 = \frac{14}{3}MR^2$$

Get moment of inertia of the rod from eqt'n sheet:

$$I_2 = \frac{1}{3}(2M)(2R)^2 = \frac{8}{3}MR^2$$

Total moment of inertia is thus:

$$\boldsymbol{I_{total} = I_1 + I_2 = \frac{22}{3}MR^2 \approx 7.33MR^2}$$

3c) The center of mass experiences vertical displacement (magnitude):

$$h_{CM} = r_{CM}(1 - \cos\theta) = \frac{4}{3}R(1 - \cos\theta) \approx 0.2667R$$

Do an energy problem. Since the object is *swinging*, model kinetic energy as pure rotational energy. When and object *rolls* we use both translational & rotational kinetic energies…

$$m_{total} g h_{CM\,i} + \frac{1}{2}I\omega_i^2 = m_{total} g h_{CM\,f} + \frac{1}{2}I\omega_f^2$$

Set bottom of swing as $h = 0$. When reaching max angle, pendulum is instantaneously at rest.

$$\frac{1}{2}\left(\frac{22}{3}MR^2\right)\omega_i^2 = (3M)g\left(\frac{4}{3}R(1 - \cos\theta)\right)$$

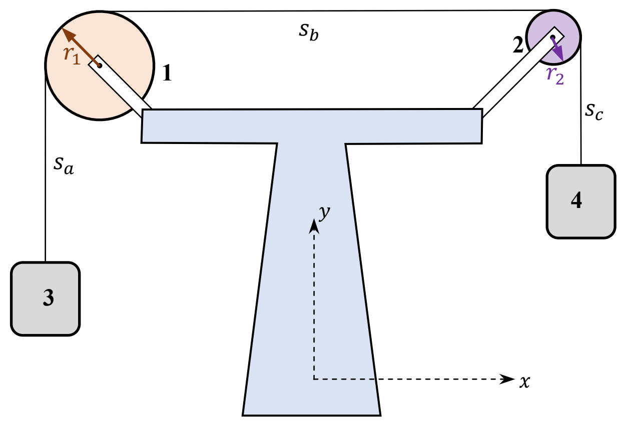
$$\omega_i = \sqrt{\frac{12g}{11R}(1 - \cos\theta)}$$

$$\boldsymbol{\omega_i \approx 0.467\sqrt{\frac{g}{R}}}$$

4a) Expect pulley 1 to rotate CCW.
According to right hand rule:

1. Curl fingers of right hand in direction of rotation.
2. Thumb indicates direction of rotation.

Rotation of pulley 1 is *out of the page*.
According to the given coordinate system: $\hat{\boldsymbol{\omega}}_{\mathbf{1}} = +\hat{\boldsymbol{k}}$.



4b) When pulleys have mass, tension is *not* the same throughout.
Consider sum of torques on pulley 1:

$$r_1 T_a \sin 90° - r_1 T_b \sin 90° = I_1 \alpha_1$$

Notice we require tension T_a in string s_a must be larger than tension T_b in s_b if $\alpha_1 > 0$!
Similarly, tension in string s_b must be larger than in s_c to cause pulley 2 to rotate out of the page.

4c) If the string is inextensible, and both masses are connected to it, both masses accelerate with same magnitude.
The accelerations can be different if using different strings attached at different radii (or if the string is extensible).

4d) Start with what we know…translational acceleration of both blocks are equal (magnitude).
Then use each pulley radius to convert translational acceleration to angular acceleration.

$$a_3 = a_4$$
$$r_1\alpha_1 = r_2\alpha_2$$
$$\alpha_1 = \frac{r_2}{r_1}\alpha_2$$
$$\alpha_1 = \frac{r_2}{2r_2}\alpha_2$$
$$\boldsymbol{\alpha_1 = \frac{1}{2}\alpha_2}$$

4e) We are told the pulleys have equal density and thickness.

$$m_1 = \rho\pi r_1^2 t \qquad \text{while} \qquad m_2 = \rho\pi r_2^2 t$$

Because m_1 has twice the radius we see it has FOUR times the mass (notice each mass has r^2 in it).

4f) We are told the pulleys have equal density and thickness.

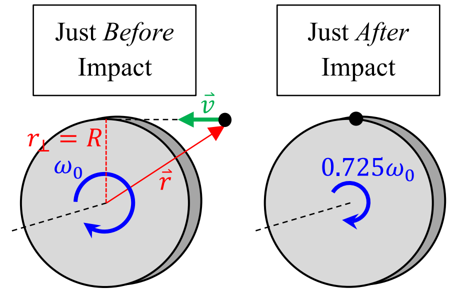
$$I_1 = \frac{1}{2}m_1 r_1^2 = \frac{\pi}{2}\rho t r_1^4 \qquad \text{while} \qquad I_2 = \frac{1}{2}m_2 r_2^2 = \frac{\pi}{2}\rho t r_2^4$$

Because I_1 has twice the radius we see $I_1 = 16I_2$ (notice each mass has r^2 in it).

5a) *Energy* is lost during a perfectly inelastic collision.
Think: the point particle (clay?) had to deform to stick to the disk.

Linear momentum is lost. The axle exerts an external force which prevents the disk from translating to the left just after the collision.

Angular momentum is conserved. The force exerted by the axle generates no torque because it is applied on the axis of rotation.



ω_f = 27.5% *less than* $\omega_0 = 0.725\omega_0$

5b) Do a conservation of angular momentum problem.
Think carefully about the point particle before starting.

$$\vec{L}_{particle\ initial} = \vec{r} \times \vec{p} = mvr_\perp \ \textbf{directed out of the page}$$

Notice angular momentum of the disk is in the opposite direction!!!

$$\vec{L}_i = \vec{L}_f$$

$$I_{disk}\omega_0(-\hat{k}) + mvr_\perp(+\hat{k}) = I_{total}(0.725\omega_0)(-\hat{k})$$

Be careful. The point mass is 8 times lighter than the disk!!!

$$-\left[\frac{1}{2}(8m)R^2\right]\omega_0 + mvR = -\left[\frac{1}{2}(8m)R^2 + mR^2\right](0.725\omega_0)$$

Divide everything by m & R and clean up fractions.

$$-4R\omega_0 + v = -5R(0.725\omega_0)$$

Solve for v.

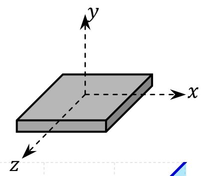
$$\boldsymbol{v = 0.375R\omega_0}$$

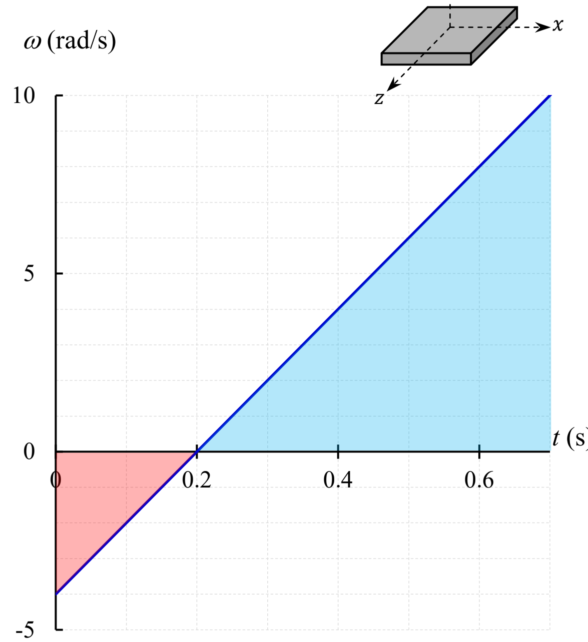
One student wondered if the axle itself was a major contributor to the moment of inertia.

It is not.

Think: the axle runs along the axis of rotation (not perpendicular to it..
All mass of the axle is essentially at the axis itself!
In the equation for moment of inertia ($I = \int r^2\, dm$) we see $r \approx 0$ for all parts of the axle.
Just like in yo-yo problems, the axle produces negligible contribution to the total moment of inertia of the disk!

6a) We are told rotation occurs in the xz-plane.
This is akin to stating the object rotates about the y-axis.
The initial value of ω is *negative*. Rotation direction is $-\hat{\jmath}$.





6b) The initial rotation *speed* is $\left|-4.00\,\frac{\text{rad}}{\text{s}}\right|$. Convert:

$$4.00\,\frac{\text{rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} \approx \mathbf{38.2\ RPM}$$

6c) Initial kinetic energy is given by

$$RKE_i = \frac{1}{2} I \omega_i^2$$

$$RKE_i = \frac{1}{2}\left[\frac{1}{12} m(s^2 + s^2)\right]\omega_i^2$$

$$RKE_i = \frac{1}{2}[0.013450\text{ kg}\cdot\text{m}^2]\left(4.00\,\frac{\text{rad}}{\text{s}}\right)^2$$

$$\boldsymbol{RKE_i} \approx \mathbf{107.6\ mJ}$$

6d) Torque *magnitude* is given by

$$\tau = I|\alpha| = \frac{1}{12} m(s^2 + s^2)\left|\frac{rise}{run}\right| = (0.013450\text{ kg}\cdot\text{m}^2)\left|\frac{4.00\,\frac{\text{rad}}{\text{s}}}{0.200\text{ s}}\right| \approx 269\text{ mN}\cdot\text{m}$$

Notice why the little bullet is important…it clarifies what is a unit versus what is a prefix in this result!

6e) Angular *displacement* is determined by adding up total area under the curve on the ωt-plot.
Areas are always drawn to the time axis with areas below the time axis being negative.
To get angular *distance* one would instead sum the absolute value of these areas…

$$\Delta\theta = Area_1 + Area_2$$

$$\Delta\theta = \frac{1}{2}\left(-4.00\,\frac{\text{rad}}{\text{s}}\right)(0.200\text{ s}) + \frac{1}{2}\left(10.00\,\frac{\text{rad}}{\text{s}}\right)(0.500\text{ s})$$

$$\Delta\theta = 2.100\text{ rad}$$

Don't forget to convert to revolutions as requested!!!!

$$\boldsymbol{\Delta\theta} \approx \mathbf{0.334\ rev}$$

6f) The corner of the plate is radius $r = \sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2} = \frac{s}{\sqrt{2}} \approx 0.2440\text{ m}$ from the axis of rotation.
Initial total acceleration (magnitude) is thus

$$a_{total} = \sqrt{a_{tan}^2 + a_c^2}$$

$$a_{total} = \sqrt{(r\alpha)^2 + (r\omega_i^2)^2}$$

$$a_{total} = r\sqrt{\alpha^2 + \omega_i^4}$$

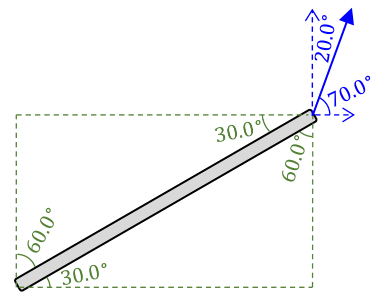
$$\boldsymbol{a_{total}} \approx \mathbf{6.25\,\frac{m}{s^2}}$$

7a) I used the upper figure to figure out some angles.
The lower figure shows the FBD.

The question is asking about linear mass density.
It makes sense to rewrite mass using

$$m = \lambda L$$

where λ is linear mass density.
Note: in general we assume $m = \int \lambda \, dx$, but $m = \lambda L$ is valid for *uniform* density.



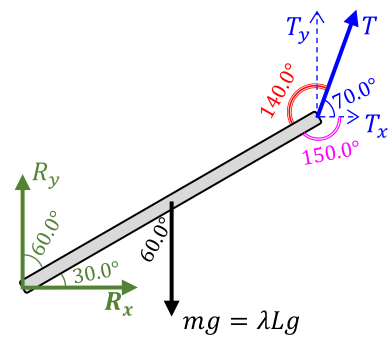
DO TORQUES FIRST!!! I will use $\Sigma\vec{\tau} = 0$ about the pivot point.
Assuming torques *out of the page* are *positive*, sum of torques gives:

$$-\frac{L}{2}(\lambda L)g \sin 60.0° + LT \sin 140.0° = 0$$

$$\frac{\lambda L g}{2} \sin 60.0° = T \sin 140°$$

$$\lambda = \frac{2T \sin 140.0°}{LG \sin 60.0°}$$

$$\boldsymbol{\lambda \approx 45.\underline{4}9 \, \frac{kg}{m}}$$



7b) For reaction forces, I want to split up all forces (including tension) on a standard xy-coordinate system.
We do it this way since almost everyone splits the pivot reaction force into R_x & R_y with standard xy-coordinates.
Do *horizontal* sum of forces:

$$\Sigma F_x: \quad T \cos 70.0° + R_x = 0$$

$$R_x = -T \cos 70.0°$$

$$\boldsymbol{R_x = -34.\underline{2}0 \text{ N}}$$

Notice the result is *negative*.
This implies the *actual* direction of the horizontal reaction force component is *opposite the direction drawn*.
Said another way, we know R_x is *actually* directed to the left.

Now do *vertical* sum of forces (using the same standard coordinate system).

$$\Sigma F_y: \quad T \sin 70.0° + R_y - \lambda L g = 0$$

$$R_y = \lambda L g - T \sin 70.0°$$

$$\boldsymbol{R_y = -54.\underline{4}8 \text{ N}}$$

Notice the result is *negative*.
This implies the *actual* direction of the vertical reaction force component is *opposite the direction drawn*.
Said another way, we know R_y is *actually* directed downwards.

Now get the magnitude as requested:

$$\boldsymbol{R = \sqrt{R_x^2 + R_y^2} \approx 64.3 \text{ N}}$$