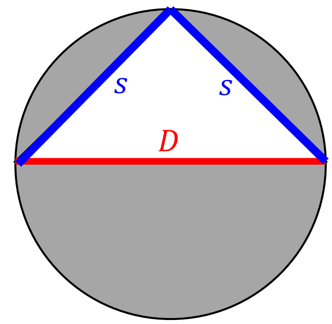
Chapter 1 Pre-test solutions



1) We frequently use

$$\rho = \frac{m_{after\ hole\ cut}}{V_{after\ hole\ cut}}$$

The trick here is to figure out the volume in terms of the given parameters D & t.
From experience, one *probably* knows that for a 45-45-90 triangle

$$\sqrt{2} \times side\ length = hypotenuse$$

If you don't know this, you could always use the Pythagorean Theorem.

$$s^2 + s^2 = D^2$$

$$2s^2 = D^2$$

$$s^2 = \frac{D^2}{2} \quad \textbf{OR} \quad s = \frac{D}{\sqrt{2}}$$

Now square root each side to find:

$$\sqrt{2}s = D$$

We have a circular plate of thickness (or height) t which is essentially a cylinder with a triangular hole.

$$V_{after\ hole\ cut} = V_{cyl} - V_{tri}$$

$$V_{after\ hole\ cut} = \pi r^2 h - (A_{tri}) \times (thickness)$$

$$V_{after\ hole\ cut} = \frac{\pi D^2}{4} t - \frac{1}{2} s^2 t$$

$$V_{after\ hole\ cut} = \frac{\pi D^2}{4} t - \frac{1}{2}\left(\frac{D^2}{2}\right) t$$

$$V_{after\ hole\ cut} = D^2 t \left(\frac{\pi - 1}{4}\right)$$

Now plug this into the density formula

$$\rho = \frac{m_{after\ hole\ cut}}{V_{after\ hole\ cut}} = \frac{m}{D^2 t \left(\frac{\pi - 1}{4}\right)} = \left(\frac{4}{\pi - 1}\right) \frac{m}{D^2 t} \approx \mathbf{1.868} \frac{\boldsymbol{m}}{\boldsymbol{D^2 t}}$$

2) When looking for the units of k we can completely ignore the second term on the right hand side!

$$[F] = \frac{[a]\cdot[t]^3}{[3]\cdot[m]\cdot[k]^2}$$

Since the number 3 has no units, we can ignore that.

$$[k]^2 = \frac{[a]\cdot[t]^3}{[m]\cdot[F]}$$

$$[k] = \sqrt{\frac{\frac{\mathrm{m}}{\mathrm{s}^2}\cdot\mathrm{s}^3}{\mathrm{kg}\cdot\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}}}$$

$$[k] = \sqrt{\frac{\mathrm{m}}{\mathrm{s}^2}\cdot\mathrm{s}^3\cdot\frac{1}{\mathrm{kg}}\cdot\frac{\mathrm{s}^2}{\mathrm{kg}\cdot\mathrm{m}}}$$

$$[\boldsymbol{k}] = \sqrt{\frac{\mathbf{s^3}}{\mathbf{kg^2}}} = \frac{\mathbf{s^{3/2}}}{\mathbf{kg}}$$

Notice we did NOT need to solve this expression for k algebraically.
It is probably good practice to solve for k algebraically so I will now do that as well.

$$F = \frac{at^3}{3mk^2} - \frac{bv^2}{at}$$

$$F + \frac{bv^2}{at} = \frac{at^3}{3mk^2}$$

$$\frac{1}{F + \frac{bv^2}{at}} = \frac{3mk^2}{at^3}$$

$$\frac{at^3}{3m\left(F + \frac{bv^2}{at}\right)} = k^2$$

$$k = \pm\sqrt{\frac{at^3}{3m\left(F + \frac{bv^2}{at}\right)}}$$

Did you remember to include the $\pm$ when taking a square root? That can be crucial in physics problems…
On test days I typically require you to simplify this to s single fraction (no fractions in numerator or denominator).

$$\boldsymbol{k} = \pm\sqrt{\frac{\boldsymbol{a^2t^4}}{\mathbf{3}\boldsymbol{m}(\boldsymbol{atF} + \boldsymbol{bv^2})}}$$

3a) $x_i = 70\underline{0}$ mm
I include the underbar on the significant zero to clarify the ambiguity. Not all people do this but I recommend it.

3b) **3 –** In scientific notation, notice the trailing zeros were to the *right* of a decimal point!

3c) $x_f = 6.8 \times 10^{-1}$ m (6.8 × 10^2 mm is NOT acceptable)
We should assume the trailing zero is NOT significant since it is to the *left* of the decimal point in 680 mm!
Standard *scientific* notation typically *avoids prefixes* to simplify calculations in calculators & computers.

3d) **2**

3e) $40\underline{0}$ μs
Again, we want to clarify the trailing zeros *are* significant since they came to the *right* of the decimal point.

3f) **3 –** Leading zeros are never significant. These trailing zeros are significant since they were to the right of the decimal point.

3g) First one solves algebraically for a.
This reduces the amount of writing since we don't have to keep writing all those decimal places and units!
Also, it is easier to fix mistakes in algebra compared to a giant wall of numerical work.

$$a = \frac{2\Delta x}{t^2} = \frac{2(x_f - x_i)}{t^2}$$

When you do addition or subtraction while tracking sig figs, it is wise to have all numbers in the same notation.
Ideally, floating point notion is nice if there aren't excessive leading or trailing zeros.
Floating point notation is the "normal" way of writing numbers (without any $\times\, 10^{power}$ stuff).

$$a = \frac{2(0.6\underline{8}\ \mathrm{m} - 0.70\underline{0}\ \mathrm{m})}{(0.00040\underline{0}\ \mathrm{s})^2}$$

$$a = \frac{2(-0.0\underline{2}\ \mathrm{m})}{(0.00040\underline{0}\ \mathrm{s})^2}$$

$$a = -\underline{2}50000\,\frac{\mathrm{m}}{\mathrm{s}^2}$$

Traditional rounding says to round this to $a = -\underline{3}00000\,\frac{\mathrm{m}}{\mathrm{s}^2}$
Many people (banking, chemistry, stats, IEEE, etc.) round numbers whose rightmost digit is 5 to the nearest even.
In my work I tend to leave the next digit but also indicate the rounding digit (in case I need to use this number in a subsequent calculation).
In any event, we were asked to use engineering notation with best choice of prefix.

I would accept any of these answers even though the rightmost column seems unusual to some students at first.		
$\boldsymbol{a = -\underline{2}50\,\frac{\mathbf{km}}{\mathbf{s}^2}}$	$\boldsymbol{a = -\underline{3}00\,\frac{\mathbf{km}}{\mathbf{s}^2}}$	$\boldsymbol{a = -\underline{2}00\,\frac{\mathbf{km}}{\mathbf{s}^2}}$

4)

Number of sig figs implied on A	A in engineering notation with best choice of prefix	Number of sig figs implied on B	B in scientific notation
4	203.0 μs	2	2.4×10^{-7} s *standard scientific notation uses no prefix on the units

5)

$$8.15 \times 10^{-2} \frac{\text{g}}{\text{cm}^3} \times \frac{1\text{ kg}}{1000\text{ g}} \times \frac{1\text{ slug}}{14.5939\text{ kg}} \times \frac{2.54^3\text{ cm}^3}{1^3\text{ in}^3} = \mathbf{9.15 \times 10^{-5}} \frac{\textbf{slug}}{\textbf{in}^3}$$

6a) One finds

$$x = \sqrt{\frac{m}{k}(v^2 - gh)}$$

Before moving on, I check the units.
The term $\frac{m}{k}$ has units of s^2.
Both v^2 & gh have units of $\frac{\text{m}^2}{\text{s}^2}$.
Final units on the right side of the equation will turn out to be m as expected.

6b) First I choose to convert everything to units that match. I converted grams to kg but you could have instead converted kg to grams. Similar with mm to meters.

$v\left(\frac{\text{m}}{\text{s}}\right)$	$k\left(\frac{\text{kg}}{\text{s}^2}\right)$	m (kg)	$g\left(\frac{\text{m}}{\text{s}^2}\right)$	h (m)
7.77×10^{-2}	-800.0	9.99×10^{-3}	9.80	6.78×10^{-4}

I know the final units must be in meters so I choose to leave units off in the calculation.
Plugging in and tracking sig figs gives

$$x = \sqrt{\frac{9.9\underline{9} \times 10^{-3}}{-800.\underline{0}}\left[\left(7.7\underline{7} \times 10^{-2}\right)^2 - \left(9.8\underline{0}\right)\left(6.7\underline{8} \times 10^{-4}\right)\right]}$$

$$x = \sqrt{-1.2\underline{4}88 \times 10^{-5}\left[6.0\underline{3}7 \times 10^{-3} - 6.6\underline{4}4 \times 10^{-3}\right]}$$

$$x = \sqrt{-1.2\underline{4}88 \times 10^{-5}\left[-0.6\underline{0}7 \times 10^{-3}\right]}$$

Notice the minus signs do cancel under the square root.
Notice the subtraction drops us down to just two sig figs!!!

$$x = 8.71 \times 10^{-5}\text{ m}$$

$$\boldsymbol{x = 87\ \mu\text{m}}$$

7) The standard trick is to know the units of any two terms in an expression must match.
This avoids having to solve algebraically for some nasty expression.
Also, numerical constants like 3, $\frac{1}{2}$, or π have no units.
Also, the minus sign will not affect the units.
In this particular case we know:

$$[v^2] = \frac{[k][a^2]}{[m][x]}$$

$$[k] = \frac{[v^2][m][x]}{[a^2]}$$

$$[k] = \frac{\frac{\mathrm{m}^2}{\mathrm{s}^2} \cdot \mathrm{kg} \cdot \mathrm{m}}{\frac{\mathrm{m}^2}{\mathrm{s}^4}}$$

$$[k] = \frac{\mathrm{m}^2}{\mathrm{s}^2} \cdot \mathrm{kg} \cdot \mathrm{m} \cdot \frac{\mathrm{s}^4}{\mathrm{m}^2}$$

$$\boldsymbol{[k] = \mathbf{kg} \cdot \mathbf{m} \cdot \mathbf{s}^2}$$

Notice we did NOT need to solve this expression for k algebraically.
It is probably good practice to solve for k algebraically so I will now do that as well.

$$v^2 = -\frac{3ka^2}{mx} + \frac{2cma^3}{x^2}$$

$$\frac{3ka^2}{mx} = \frac{2cma^3}{x^2} - v^2$$

$$k = \frac{mx}{3a^2} \cdot \frac{2cma^3}{x^2} - \frac{mx}{3a^2} \cdot v^2$$

On exam days you are expected to simplify that first term!

$$\boldsymbol{k = \frac{2cm^2a}{3x} - \frac{mv^2x}{3a^2}}$$

Let's be honest here, there is often more than one good way to simplify a result.
That said, it is standard to simplify fractions combine like terms as much as possible.
As the class progresses, you should discover we typically try to write final results as a term with correct units times a term with no units.
Do your homework & check your results against the solutions to get the feel for how much simplifying is required.

IMPORTANT Final note: notice I have NOT included units on any *algebraic* results.
Conversely, units are *required* for *numerical* results.
This is standard in every textbook & physics course.
If you *include* units on an *algebraic* result on test day I will take off points!
If you *forget to include* units on a *numerical* result on test day I will take off points!

8) Use the Pythagorean theorem to determine D in terms of x (see figure).
The area shown is given by:

$$A_{total} = \frac{1}{2}A_{circle} + A_{triangle}$$

$$A_{total} = \frac{1}{2}\pi R^2 + \frac{1}{2}(x)\left(\frac{x}{2}\right)$$

$$A_{total} = \frac{1}{2}\pi\left(\frac{D}{2}\right)^2 + \frac{1}{4}x^2$$

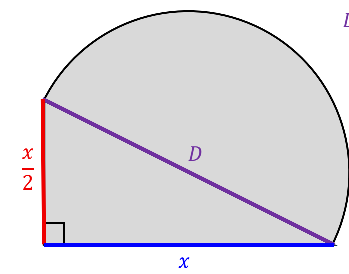
$$A_{total} = \frac{1}{8}\pi D^2 + \frac{1}{4}x^2$$

$$A_{total} = \frac{1}{8}\pi\left(\frac{\sqrt{5}}{2}x\right)^2 + \frac{1}{4}x^2$$

$$A_{total} = \frac{1}{8}\pi \cdot \frac{25}{4}x^2 + \frac{1}{4}x^2$$

$$A_{total} = \left(\frac{25}{32}\pi + \frac{1}{4}\right)x^2$$

$$\boldsymbol{A_{total} = 2.7\underline{0}4x^2 = 2.70x^2}$$



$$D = \sqrt{x^2 + \left(\frac{x}{2}\right)^2}$$

$$D = \sqrt{x^2 + \frac{x^2}{4}}$$

$$D = \sqrt{\frac{5}{4}x^2}$$

$$D = \frac{\sqrt{5}}{2}x$$

Most of you probably solve for radius R algebraically from the figure. That method is also great. In time you may come to appreciate the style I used.

NOTICE: I often include the unrounded number but indicate the rounding column with an underbar $2.7\underline{0}4x^2$.
This is handy when you have a multi-part problem and you don't want to introduce intermediate rounding errors.
As such, on my exams I accept either of the last two forms as correct.
Finally, if the first digit of a final result is 1, many engineers include an *extra* sig fig.
If you care to know why, it has to do with representing percent errors of measurements more accurately.