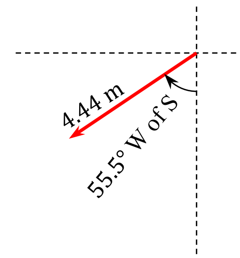
Chapter 3 pre-test solutions



1a) I sketched the vector at right.

$$\vec{z} = \left(4.44 \sin 55.5° \,(-\hat{\imath}) + 4.44 \cos 55.5° \,(-\hat{\jmath})\right)\text{m}$$

$$\vec{z} = \left(-3.6\underline{5}9\hat{\imath} - 2.5\underline{1}5\hat{\jmath}\right)\text{m}$$

$$\boldsymbol{\vec{z} = -(3.66\hat{\imath} + 2.52\hat{\jmath})\text{m}}$$

Either of the last two answers is fine.

REMEMBER: if you need to use one of these answers *in a subsequent part*, it is important to use the *first* form (*un*rounded) to reduce intermediate rounding error.

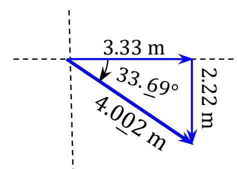
1b) The *unit* vector $\hat{w}$ is given by dividing the vector by the magnitude.

The magnitude is $w = \sqrt{w_x^2 + w_y^2} = \sqrt{(3.3\underline{3})^2 + (-2.2\underline{2})^2} = \sqrt{11.\underline{0}89 + 4.9\underline{2}8} = \sqrt{16.\underline{0}17} = 4.0\underline{0}2$ m.

$$\hat{w} = \frac{\vec{w}}{w} = \frac{(3.33\hat{\imath} - 2.22\hat{\jmath})\text{ m}}{4.0\underline{0}2\text{ m}} = \frac{3.33}{4.0\underline{0}2}\hat{\imath} - \frac{2.22}{4.0\underline{0}2}\hat{\jmath} = 0.83\underline{2}0\hat{\imath} - 0.55\underline{4}7\hat{\jmath}$$

$$\boldsymbol{\hat{w} = 0.832\hat{\imath} - 0.555\hat{\jmath}}$$

WATCH OUT! Notice the units cancel when writing the unit vector!!!



1c) First I make the sketch shown at right.
The magnitude was found in the previous part.
The angle was found using

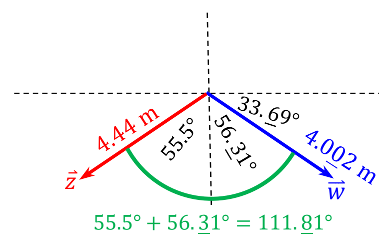
$$\phi = \tan^{-1}\left|\frac{w_y}{w_x}\right| = \tan^{-1}\left|-\frac{2.22\text{ m}}{3.33\text{ m}}\right| = 33.\underline{6}9°$$

The three most common ways to correctly write the answer are:

$$\boldsymbol{\vec{w} = 4.0\underline{0}2\text{ m @ }33.\underline{6}9°\text{ S of E}}$$

$$\boldsymbol{\vec{w} = 4.0\underline{0}2\text{ m @ }56.\underline{3}1°\text{ E of S}}$$

$$\boldsymbol{\vec{w} = 4.0\underline{0}2\text{ m @ }326.\underline{3}1°\text{ from the positive } x \text{ axis}}$$



1d) It is easiest to get the *magnitude* of the cross-product using

$$\|\vec{z} \times \vec{w}\| = zw \sin\theta_{zw}$$

$$\|\vec{z} \times \vec{w}\| = (4.44\text{ m})(4.0\underline{0}2\text{ m})\sin(111.\underline{8}1°)$$

$$\boldsymbol{\|\vec{z} \times \vec{w}\| = 16.\underline{4}97\text{ m}^2}$$

Don't forget the units!!!

You could do the full wheel of pain method (req'd for 3D vectors), then remember to take the magnitude.

$$\vec{z} \times \vec{w} = \left(-3.6\underline{5}9\hat{\imath} - 2.5\underline{1}5\hat{\jmath}\right)\text{m} \times (3.33\hat{\imath} - 2.22\hat{\jmath})\text{m}$$

Ignore $\hat{\imath} \times \hat{\imath} = 0$ & $\hat{\jmath} \times \hat{\jmath} = 0$ terms.

$$\vec{z} \times \vec{w} = \left(-3.6\underline{5}9\hat{\imath}\right)\text{m} \times (-2.22\hat{\jmath})\text{m} + \left(-2.5\underline{1}5\hat{\jmath}\right)\text{m} \times (3.33\hat{\imath})\text{m}$$

$$\vec{z} \times \vec{w} = 8.1\underline{2}3\text{m}^2\hat{k} + 8.3\underline{7}5\text{m}^2\hat{k} = 16.\underline{4}98\text{m}^2\hat{k}$$

Notice the magnitude is again $16.\underline{4}97\text{ m}^2$.

1e) I asked for the angle between & the NEGATIVE x-axis.
You could use the figure.
The standard method is to use

$$\theta_{-x\,axis} = \cos^{-1}\left(-\frac{w_x}{w}\right) = 14\underline{6}.3°$$

1f) The sketch is shown above in part 1d.
Sometimes on exams, it is easier to do the parts of the question out of order.
For me it was easiest to do part 1f) before part 1d)!

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2) The key sentence in the problem statement is this one:

“After the 3rd displacement, the drone is located 11.11 m above of its starting point.”

Upon close inspection, this sentence tells us about the *final result* after an *unknown 3rd displacement*.

The vectors we are given are as follows (in Cartesian form):

$$\vec{A} = -22.2\underline{2}\hat{\jmath} \text{ m}$$
$$\vec{B} = \left(27.8\underline{4}8\hat{\imath} + 18.3\underline{1}4\hat{\jmath}\right) \text{ m}$$
$$\vec{R} = 11.1\underline{1}\hat{k} \text{ m}$$

The wording of the problem statement tells us

$$\vec{A} + \vec{B} + \vec{C} = \vec{R}$$

Since the third displacement is unknown, I solve this equation for $\vec{C}$ before plugging in numbers.

$$\vec{C} = \vec{R} - \left(\vec{A} + \vec{B}\right)$$
$$\vec{C} = 11.1\underline{1}\hat{k} \text{ m} - \left(-22.2\underline{2}\hat{\jmath} \text{ m} + \left(27.8\underline{4}8\hat{\imath} + 18.3\underline{1}4\hat{\jmath}\right) \text{ m}\right)$$

I now group up like terms then reorder to match standard form ($\hat{\imath}$ term 1st, $\hat{\jmath}$ term 2nd)

$$\vec{C} = \left(-27.8\underline{4}8\hat{\imath} + 3.9\underline{0}6\hat{\jmath} + 11.1\underline{1}\hat{k}\right) \text{ m}$$

This is the required 3rd displacement vector.

The question asked for the *magnitude* of the 3rd displacement vector (and to write the final answer with 3 sig figs).

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2}$$
$$C = \sqrt{\left(-27.8\underline{4}8\right)^2 + \left(3.9\underline{0}6\right)^2 + \left(11.1\underline{1}\right)^2}$$
$$C = \sqrt{775.\underline{5}1 + 15.2\underline{5}7 + 123.\underline{4}3}$$
$$C = \sqrt{914.\underline{2}0}$$
$$C = 30.2\underline{3}6 \text{ m}$$

Notice: if we strictly follow sig fig rules, we expect 4 sig figs on this result.

However, we often add an extra sig fig when the first digit is 1.

As such, the number 11.11 is *written* with 4 sig figs but *behaves* a bit more like a 3 sig fig number.

Also, the default number of sig figs for exam questions is THREE (unless problem specifically asks for sig figs).

If you want to discuss this more, ask in person.

When it is all said and done, I would accept $\boldsymbol{C = 30.2}$ **m** or $\boldsymbol{C = 30.24}$ **m** if this was your test question.

Don’t forget to include the units!!!!

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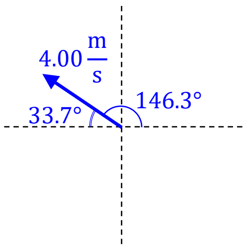
3a) $v = 4.0\underline{0}2\,\frac{\mathrm{m}}{\mathrm{s}}$. The angle should be described in one of the following ways

$33.7°$ **above** the *negative* $x -$ axis

$33.7°$ north of west

$56.3°$ west of north

$146.3°$ **from** the *positive* $x -$ axis



3b) Use $\hat{v} = \frac{\vec{v}}{v}$ where v is the magnitude of vector $\vec{v}$.

$$\hat{v} = -0.832\hat{\imath} + 0.555\hat{\jmath}$$

3c) Use

$$\theta = \cos^{-1}\left(\frac{\vec{r}\cdot\vec{v}}{rv}\right)$$

Notice the only term which survives the dot product comes from the $\hat{\jmath}\cdot\hat{\jmath}$ term since $\hat{k}\cdot\hat{\imath} = 0$ & $\hat{\jmath}\cdot\hat{\imath} = 0$.
Notice the units will cancel out inside the inverse trig function (so I will ignore units for the rest of this part).
Notice we require the magnitude of $\vec{r}$ which is $r = 7.0\underline{7}1$ m.

$$\theta = \cos^{-1}\left(\frac{(4.44)(2.22)}{(7.0\underline{7}1)(4.0\underline{0}2)}\right)$$

$$\boldsymbol{\theta = 69.6°}$$

Note: standard practice dictates we keep an extra sig fig on the final answer when the first digit is a 1.

3d) I like to move all the units to the right end right away so I don't forget to include them on the final answer. When doing the cross-product, I recommend using the wheel of pain. It is usually faster than the determinant method if at least one component in either vector is zero…

$$\vec{L} = m(\vec{r}\times\vec{v})$$

$$\vec{L} = 1.00\left[\left(4.44\hat{\jmath} - 5.55\hat{k}\right)\times(-3.33\hat{\imath} + 2.22\hat{\jmath})\right]\frac{\mathrm{kg\cdot m^2}}{\mathrm{s}}$$

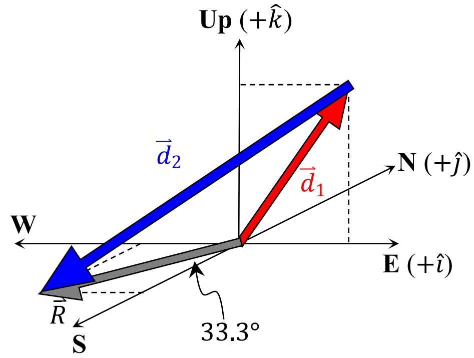
We know the cross-product of any vector with itself is zero (including a unit vector with itself).
Also, the order of the cross-product matters!

$$\vec{L} = \left[(4.44)(-3.33)(\hat{\jmath}\times\hat{\imath}) + (-5.55)(-3.33)\left(\hat{k}\times\hat{\imath}\right) + (-5.55)(2.22)\left(\hat{k}\times\hat{\jmath}\right)\right]\frac{\mathrm{kg\cdot m^2}}{\mathrm{s}}$$

$$\vec{L} = \left[-14.785\left(-\hat{k}\right) + 18.482(\hat{\jmath}) + (-12.321)(-\hat{\imath})\right]\frac{\mathrm{kg\cdot m^2}}{\mathrm{s}}$$

It is standard practice to put the vectors in traditional form ($\hat{\imath}$-term before $\hat{\jmath}$-term before $\hat{k}$-term).
Also, we can now round our intermediate results to 4 sig figs (since each term has first digit of 1).

$$\boldsymbol{\vec{L} = \left[12.32\hat{\imath} + 18.48\hat{\jmath} + 14.79\hat{k}\right]\frac{kg\cdot m^2}{s}}$$



4) We know

$$\vec{d}_1 = (8.88\hat{\imath} + 5.55\hat{k})\text{m}$$

$$\vec{R} = 7.77 \text{ m @ } 33.3° \text{ west of south}$$

We are looking for the vector which connects the tip of $\vec{d}_1$ to the tip of $\vec{R}$!

In equation form, we know

$$\vec{d}_1 + \vec{d}_2 = \vec{R}$$

$$\vec{d}_2 = \vec{R} - \vec{d}_1$$

Strategy is to convert all vectors to Cartesian, do the math, then compute the magnitude.

We notice the angle of 33.3° is adjacent to the negative y-axis…the y-component should get a cosine!
Also, both x- and y-components will be negative.
Stick the units at the right end so you don't forget them!

$$\vec{R} = (-7.77 \sin 33.3° \,\hat{\imath} - 7.77 \cos 33.3° \,\hat{\jmath}\,)\text{m}$$

$$\vec{R} = (-4.2\underline{6}6\hat{\imath} - 6.494\hat{\jmath}\,)\text{m}$$

Before moving on, THINK.
This vector is angled closer to the y-axis than it is to the x-axis.
We expect the y-component should be the larger number. I notice this matches my math then move on.
We have already discussed the signs (both should be negative from the image).

$$\vec{d}_2 = (-4.2\underline{6}6\hat{\imath} - 6.4\underline{9}4\hat{\jmath}\,)\text{m} - (8.88\hat{\imath} + 5.55\hat{k})\text{m}$$

$$\vec{d}_2 = (-13.146\hat{\imath} - 6.494\hat{\jmath} - 5.55\hat{k})\text{m}$$

THINK: Do these signs agree with the signs we expect from the picture?
YES; in the figure $\vec{d}_2$ point left, out of the page, and down…
Move on to get the magnitude.

$$d_2 = \|\vec{d}_2\| = \sqrt{(-13.146)^2 + (-6.494)^2 + (5.55)^2}$$

$$\boldsymbol{d_2 = 15.68\text{ m}}$$

Notice I waited until the after I computed the last step to round.
Here I rounded to 4 sig figs since the first digit of the result was a 1…

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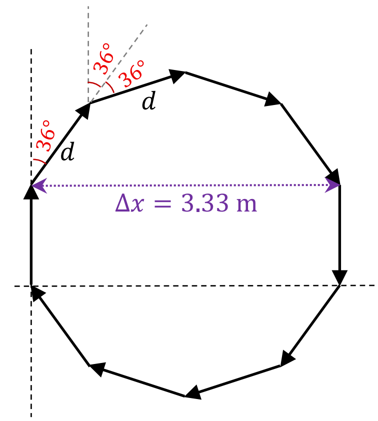

PHYS 110 Extra credit Q:
While other methods might be possible, the following was my thought process.
The exterior angle of any regular n-gon is given by $\frac{360°}{n}$.
That was the tricky part (knowing that fact during an exam with no internet).
From there it is straightforward.
We are trying to find the length of a single displacement. Call that parameter d.
From the figure you should find

$$\Delta x = 2(d \sin 36° + d \sin 72°)$$

$$d = \frac{\Delta x}{2(\sin 36° + \sin 72°)}$$

$$\boldsymbol{d = 1.082 \text{ m}}$$

A comment on sig figs: Time and again you see me mentioning the default number of sig figs for an exam result is three. However, some engineers include an extra sig fig when the first digit of a result is 1. On my exams I'll accept either style.

Challenge: figure out the required individual displacement magnitude for a regular n-gon (arbitrary value of n).
Your answer should be a formula with x_{max} and n in it.
Verify your formula works for both even and odd values of n.
It may be practical to write your solution with series notation!
Then think about the shape as $n \to \infty$…

Again, I was hoping this problem might give you an idea of how this stuff might be useful…