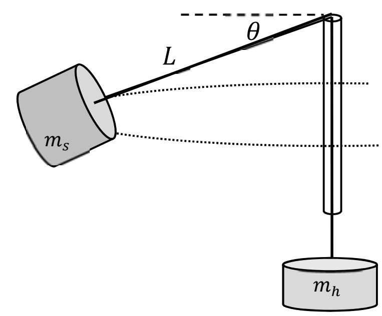
Circular Motion



Apparatus: hanging mass sets, braided string, scissors, uniform circular motion apparatus, stopwatches, goggles.

Purpose: Analyze the forces acting on objects undergoing circular motion. Confirm objects in circular motion accelerate toward the center of the circle and the magnitude $a_c = \frac{v^2}{r}$. We will do this by comparing the theoretical velocity (determined using FBDs and force equations) to experimental velocity (determined using period and radius of the circular motion).

Theory: In your calculations section it is expected that you will draw the FBD's for each object, determine the force equations for each object, and solve algebraically for v_{th}. This can be done by starting with Newton's second law. Newton's 2nd law states that $\Sigma\vec{F} = m\vec{a}$. In this case, the tension in the string is constraining the stopper to move in a circle in a horizontal plane (see figure). Newton's 2nd law for the rotating stopper (in the centripetal direction) gives

$$T\cos\theta = m_s a_c \text{ using radius } r = L\cos\theta$$

If we make the assumption that $\theta < 15°$ we find that $\cos\theta > 0.95$. If we accept errors on the order of 5%, we may set the angle to 0° giving

$$T \approx m_s a_c \text{ using radius } r \approx L$$

Again, it is worth emphasizing these assumptions require a data set with angle smaller than $\theta < 15°$.

We can learn about the string tension by using forces on the hanging mass. When you spin the stopper such that the hanging mass is motionless *in the vertical direction* we know $T = m_h g$.

When the vertical motion of the hanging mass is zero, AND while $\theta < 15°$ these equations can be combined to solve for the theoretical velocity. You should find that

$$v_{th} = \sqrt{\frac{?\,Lg}{?}}$$

where the ?'s are terms I expect you to figure out (they relate to the masses).

Lastly, one can measure the time t it takes the object to complete 10 revolutions. Knowing the length of the string L that extends **from the tube to the center of the stopper** one can determine the distance traveled in 10 revolutions. When I count revolutions it helps me to say the number "zero" aloud as I start the count. Students who start counting with the number 1 often stop the clock after only 9 revolutions…

Recall that one revolution is equivalent to the circumference of a circle with $r = L$. The experimental velocity can then be obtained by

$$v_{exp} = \frac{Total\ Distance}{Total\ Time}$$

Procedure:

- Before taking any measurements you will want to get used to the feel of the experiment. Try to find a length of string that works for both the heaviest and lightest masses before taking data.
- Start with your lightest mass. Take the tube with the string and attach the hanging mass (m_h) to one end and a 20-50-gram stopper (m_s) to the other.
- Rotate the stopper in a circle over your head at a speed fast enough so the hanging mass is suspended at a fixed location. Notice what happens to the speed of the stopper with a long radius versus a short radius (you will be asked about this later).
- Now try to do the same tests with a 500 g hanging mass. Can you keep the 500 g mass from slipping with a radius of about 100 cm? If not, ask your instructor for help.
- In order to maintain a constant radius, you can try marking the string with a pen. Practice spinning the mass and observing the mark on the string. Select a target radius between 90-120 cm. Try to use only this radius for the rest of the day.
- Have your lab partner measure the time t for 10 revolutions.
- Measure the length (L) of the string from **the center of the stopper to where it enters the tube**. Don't forget, if you marked the string and tried to keep a constant radius that should help you determine L.
- **Repeat the experiment for five different m_h's. Maintain fixed values of m_s and L for all five experiments.** Though you cannot do this perfectly, try to keep your radii within a few cm of your target value of the radius.
- You should end up with 5 different values for the theoretical velocity (from the FBD's and force equations) and 5 different values for the experimental velocity (from the measurements of t and L).
- Tabulate your data. Be sure to calculate the percent difference and precision.
- Make a graph of both v_{exp} and v_{th} versus m_h. If you need help plotting, try the lab training vid playlist.
- You can also plot graphing v_{exp}^2 versus m_h (data only). Think: this data is linear…it is ok to add a linear trendline on *this* plot but not the previous one!
- Make an FBD. Get an equation from your FBD and solve it for v^2.
- Compare the equation of a line ($y = slope \cdot x + intercept$) to your equation relating m_h and v^2. What variable is x? What variable is y? What expression is equal to the slope?
- From that slope, and knowing your values of L and m_s, figure out an experimental value of g. Hint: compare the equation of a line ($y = slope \cdot x + intercept$) to your trendline from the plot of v_{exp}^2 versus m_h.
- Compare this experimental value (from the trendline) to the known value of g with a percent difference.
- Use the LINEST function to determine the percent error associated with your slope. There is a training vid in the playlist on how to use LINEST. This video also explains how to get a statistical error in your slope! Recall, the percent error in a measurement is simply the reading error over the measurement itself times 100%.
- Assume the % precision associated with all terms involving θ is about 5%. Estimate the remaining % errors for your measurements. Determine the precision of your experiment by using the error analysis appendix.

Conclusions:

1. Did v_{th} match v_{exp}? Defend your conclusions by comparing the %differences to the precision.
2. Did the experimental determination of g (for v_{exp}^2 vs m_h plot) match up to the accepted value of g?
   Defend your conclusions by comparing the %differences to the precision.
3. In this experiment we kept m_s and L constant.
   As m_h increases, what *should* happen to both period and velocity?
   You should already have an algebraic expression for v_{th} which can give some insight.
   You can also rearrange the algebraic equation to instead solve for *period* in terms of m_s, m_h, g, and L.
   Then let this new version of the equation guide your reasoning for how the period should change…
   Does the result seem to make sense?
4. What if our experiment allowed us to keep *speed* constant as we changed m_h (m_s still held constant).
   Think: this would only be possible if the radius of motion was free to vary.
   What should happen to the radius of circular motion as m_h increases?
   Hint: first rearrange your equation to solve for…
   Does the result seem to make sense?
5. Finally, suppose we instead held *period* constant as we changed m_h (m_s still held constant).
   What should happen to the radius of circular motion as m_h increases?
   Use your results to sketch a plausible plot of L versus m_h data.