

Error Estimation and Using Excel

Apparatus: desktop or laptop for each student with Excel

Goal: You are to practice the following topics relating to significant figures, percent errors, propagation of errors, standard deviation, Excel formulas, and graphing with Excel.

Checklist to Turn in:

- Use engineering paper. Only use one side of the paper (the side without the lines on it).
- Do not cram work into the side or bottom of the page. Write neatly and use plenty of space to make it easy to follow your steps.
- Do not work side to side on the paper. Each step should come below the previous step to make your work easy to follow.
- Put a box around your final answers.
- **Be sure to always include an unrounded answer AND a rounded final answer (with units) for all numerical results.** Also, be sure to show several steps of work for long calculations like the propagation of error and standard deviation.
- Show work and solutions for the numbered questions.
- Completed data table with all sections completed. This should include not only the blank cells filled in but also the two graphs. Erase all red text and check print preview before printing. Try to make everything fit on one page for today.

To determine the number of sig figs in a number and write it in scientific notation:

Rule1: All non-zero digits are significant.

Rule2: Zeros between other significant figures (bounded zeros) are significant.

Rule3: Leading zeros are left of a decimal point and to the left of all non-zero numbers. They are never significant.

Rule4: Trailing zeros are right of all non-zero numbers. Trailing zeros are only significant when at least one of them comes to the right the decimal point!

Examples:

- 1) The number 1234.5 has five sig figs.
- 2) The number 0.0445 has three sig figs. The two zeros are called leading zeros.
- 3) The number 0.0405 also has three sig figs. The first two zero are leading but not the zero between the 4 and the 5. By rule 2 it is significant.
- 4) The number 435.00 has 5 sig figs. The last two zeros are called trailing zeros. Since they are trailing zeros with at least one is right of the decimal (rule 4).
- 5) The number 43500 has three sig figs. Here the two zeros are still trailing zeros but none of them is right of the decimal point.
- 6) The number 4350.0 has five sig figs. The trailing zeros are again significant since at least one of them comes after the decimal.

Exception: The number 4000 appears to have only one sig fig. Often in a physics class this ambiguity is resolved by assuming such a number has at least three sig figs unless otherwise mentioned. It is best to indicate the sig fig using and under bar to clarify this.

For example, the number $4\underline{0}00$ has an underbar indicating the middle zero is significant. This implies the number has three sig figs. The middle zero is significant as noted by the underbar, the left zero is significant by rule 2, and the right zero is not significant by rule 4.

Math with sig figs

- When mult/div the smaller number of sig figs is kept.
- When add/sub the least significant column of sig figs is kept. Watch out: the number of sig figs can change with add/sub! I recommend writing out add/sub problems vertically to keep track of sig figs.
- In scientific notation, all numbers are always significant!
- Usually you keep at least one extra sig fig for all math work and then round to the appropriate number of sig figs in the last step. Trig functions can be a notable exception to this rule. When using trig functions, keep an extra sig fig just in case.
- Keep track of your sig figs using order of operations.

Examples:

Suppose you have $5.68 \times 6.2 = 35.216 = 35$. Notice that 5.68 has three sig figs while 6.2 only had two sig figs. The smaller number of sig figs is two. When multiplying or dividing two numbers, your final answer will use the smaller number of sig figs...in this case two. That's how I knew to round in the ones column for this result.

Suppose you have $4.3 + 7.1$? First write it vertically and put in the sig fig markers like this:

$$\begin{array}{r} 4.\underline{3} \\ +7.\underline{1} \\ \hline 11.\underline{4} \end{array}$$

Notice that the two input numbers each have only two sig figs while the answer actually has three!

Percent Errors

Every measurement has error. This error can be quantified using absolute error or percent error. For example suppose a length is measured with a meterstick to be

$$L = 0.504 \text{ m} = 504 \text{ mm}$$

If you have ever used a meterstick, you know that typically the smallest increment on the meterstick is 1mm. To some this implies an error of at most $\frac{1}{2}$ of the smallest increment. In this lab, to make error calculations a bit faster, we will assume that we never really do any better than the smallest increment.

This implies the absolute error in the measurement is

$$\delta L = 0.001 \text{ m} = 1 \text{ mm}$$

To save space while clearly indicating the error we often write this as

$$L = 0.504 \pm 0.001 \text{ m}$$

Sometimes it is more convenient to express the error as a percent. We say the percent error in L is given by

$$\% \text{ err in } L = \frac{\delta L}{L} = \frac{0.001 \text{ m}}{0.504 \text{ m}} = 0.00198 \approx 0.002 = 0.2\%$$

When the percent error is used we can then say

$$L = 0.504 \text{ m} \pm 0.2\%$$

Propagation of Errors

The back of your lab manual lists a bunch of formulas regarding absolute errors and percent errors. These formulas can be used to determine how errors in initial measurements carry through, or propagate, to your final calculated values. It is simply a more trustworthy way of determining sig figs.

Suppose we use kinematics to compute g (the magnitude of acceleration of gravity near earth). We use the kinematics formula

$$y = \frac{1}{2}gt^2$$

Someone obtains the following data

y (m)	δy (m)	t (s)	δt (s)
2.00	0.05	0.58	0.05

To find the value of g we should first rearrange the equation algebraically. True, for a single row of data it is faster to plug in the numbers right away. However, in most labs, you'll have a whole bunch of rows and it helps to have a formula so you can have the computer do the calculations for you. Therefore we first obtain

$$g = \frac{2y}{t^2}$$

Example of Propagation of Errors with Sig Fig Rules

$$g = \frac{2y}{t^2} = \frac{2(2.00)}{(0.58)^2} = 11.89 = 12 \text{ m/s}^2$$

Notice that the number 2 from the kinematics formula has no sig figs indicated. It is assumed to be a perfect number with infinite sig figs. Similarly, π in πr^2 is assumed to have infinite sig figs.

Example of Propagation using Absolute Errors

In our example we have the formula $g = \frac{2y}{t^2}$. I look on the back of the lab manual to find a propagation formula that look similar. I see there is a formula for products/quotients with exponents that looks like this:

If q is a product/quotient with powers (say $q = \frac{x^{1/2}y^{-9}}{u^{-5/3}z^4}$) the above formula is modified to:

$$\frac{\delta q}{q} = \sqrt{\left(\frac{1}{2} \frac{\delta x}{x}\right)^2 + \left(9 \frac{\delta y}{y}\right)^2 + \left(\frac{5}{3} \frac{\delta u}{u}\right)^2 + \left(4 \frac{\delta z}{z}\right)^2}$$

Here q is what is being computed so I identify that with g . Notice that we are interested in how much error g has. That means we are looking for δg (or δq in the formula). The number 2 is assumed to have no error since it has infinite sig figs.

This gives me

$$\frac{\delta g}{g} = \sqrt{\left(1 \frac{\delta y}{y}\right)^2 + \left(2 \frac{\delta t}{t}\right)^2}$$

Notice that $\frac{\delta y}{y}$ is simply the percent error in y . Similarly, this formula is computing the percent error in g ! In this case I want to find the absolute error in g so I will first cross multiply by g to make the formula

$$\delta g = g \sqrt{\left(1 \frac{\delta y}{y}\right)^2 + \left(2 \frac{\delta t}{t}\right)^2}$$

WATCH OUT! Many students forget to cross-multiply that g out front! Notice that without doing this step you are finding the percent error in g , not the absolute error in g . Finally, plug in your experimental values (including the experimental value of g). Notice I plug in the unrounded experimental value of g to avoid possible intermediate rounding errors.

$$\delta g = 11.89 \sqrt{\left(1 \frac{0.05}{2.00}\right)^2 + \left(2 \frac{0.05}{0.58}\right)^2} = 2.07 \approx 2 \text{ m}$$

We express this result as

$$g = 12 \pm 2 \frac{\text{m}}{\text{s}^2}$$

Interesting example of sig fig rules giving contradiction

On occasion the sig fig rules give answers that contradict each other based on the order of operation! This serves as a reminder to us that sig fig rules are merely an estimate of precision. By using the sig fig rules we will be in the right ballpark but might not always have the best estimate of precision.

Example: Suppose you are given two spheres with radii 9.6 cm and 5.2 cm respectively. You, a friend, and Susie McPercento are asked to find the combined volume of the two spheres. You are each told to keep track of the sig figs.

You decide to do the problem as follows:

$$\begin{aligned} V_{tot} &= V_1 + V_2 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 \\ &= 4.189 \times 9.6^3 + 4.189 \times 5.2^3 \\ &= 3706 + 589 \\ &= 4295 = 4.3 \times 10^3 \text{ cm}^3 \end{aligned}$$

Your friend decides to do the problem by first factoring out the constants like this:

$$\begin{aligned} V_{tot} &= V_1 + V_2 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi(r_1^3 + r_2^3) \\ &= 4.189(9.6^3 + 5.2^3) \\ &= 4.189(885.7 + 140.6) \\ &= 4.189(1025.3) \\ &= 4295 = 4.30 \times 10^3 \text{ cm}^3 \end{aligned}$$

Finally, Susie McPercento determines an estimate of precision based on percent errors. She first makes the assumption that when someone measures 9.6 cm they are implying 9.6 ± 0.05 cm. Similarly she assumes that the other measurement was 5.2 ± 0.05 cm. This implies the percent error on each measurement is given by:

$$\begin{aligned} \% \text{err in } r_1 &= \frac{0.05}{9.6} \times 100\% = 0.52\% \\ \% \text{err in } r_2 &= \frac{0.05}{5.2} \times 100\% = 0.96\% \end{aligned}$$

Since each radius was cubed, Susie expects the error for each radius to contribute three times. Then Susie further takes the cautious approach and assumes the worst case scenario; Susie assumes that the error in measuring one radius will not cancel out the error in measuring the other radius. For example, Susie is assuming that either both radii were measured too large or too small (not one of each). This gives a total error as follows:

$$\begin{aligned} \% \text{err} &= 3(\% \text{err in } r_1) + 3(\% \text{err in } r_2) \\ &= 3(0.52\%) + 3(0.96\%) \\ &= 1.56\% + 2.88\% \\ &= 4.44\% \approx 4\% \end{aligned}$$

Using this method, Susie decides that the total volume is given by

$$4295 \pm 4.44\% = 4295 \pm 190 \approx 4295 \pm 200$$

Susie infers the 3rd digit isn't significant and the result should be written as $4.3 \times 10^3 \text{ cm}^3$.

Sig Fig Questions

Compute the following using sig fig rules.

Write your final answers with correct sig figs and scientific notation.

1) $30.0 + 80.0 =$

2) $\frac{4.0(30.00)}{0.002} =$

3) $\frac{(2.198 - 2.207)^3}{0.010^2} =$

Practicing with % errors and fractional errors

- 4) A dog gets on a scale at the vet. It keeps wiggling and causes the scale reading to fluctuate. While it is fluctuating, the lowest scale reading is 9.7 kg while the highest is 10.3 kg. What is the best value to record for mass of the dog? Write your answer as $m = \# \pm \# \text{ kg}$
- 5) Abbey measures my height as 74 ± 1 inches. Billy measures my height as 188 ± 1 cm. We know 1 inch = 2.54 cm. What is the % error for each person's measurement?
- 6) Susie measures a current in her circuit as 1.234 A. She states the % error in measuring the current is 0.3%. This means there is a range of possible values she measures for the current. What is this range of values? Hint: use the % error to get a \pm number and use that to find a range of values.

Practicing Propagation of Errors:

Suppose you are given the formula

$$v = \sqrt{\frac{m_3^3}{m_2^2 m_1} g L}$$

You are also given the following information:

m_1 (kg)	m_2 (kg)	m_3 (kg)	g (m/s ²)	L (m)
1.00	2.00	3.00	9.81	1.00

δm_1 (kg)	δm_2 (kg)	δm_3 (kg)	δg (m/s ²)	δL (m)
0.03	0.05	0.07	0.01	0.02

- 7) Use the first row of data to determine an experimental value for v (with units). Be sure to keep track of sig figs. Clearly list your unrounded and rounded final answers with units.
- 8) Use propagation of error formulas to compute the percent error in this experimental value of v . Be sure to show each painful step. Include both an unrounded and rounded final answer.
- 9) Also determine the error in your experimental value of v . Be sure to use the unrounded answer from your previous step to avoid intermediate rounding error.
- 10) If you did the previous step correctly, you will find that $\delta v = 0.4 \text{ m/s}$. This means your experimental value of v should be rounded to the tenth's column. Think about it: errors in the hundredths column are unimportant compared to errors in the tenths column. We shouldn't include any digits beyond the tenths column!
- 11) Write your final result as $v = ?.? \pm 0.4 \text{ m/s}$.

Compare sig fig rules to full propagation of error for previous example

- 12) Suppose the accepted value of v for the previous problem is 7.60 m/s. Quantify the percent difference/discrepancy using a formula from the back of your lab manual. Show your work on paper. Do not force the result to be positive.
- 13) Read about target diagrams in your lab manual. There should be information just inside the back cover of the manual. Draw a target diagram based on the % error from the propagation of error calculation done above. Be sure the diagram is drawn to scale, at least $\frac{1}{4}$ page in size, neat, and well-labeled. Hint: to draw your circles to scale, use the gridlines of the engineering paper. Let each block of the paper represent 1%. That means the 5% ring has a five block radius and the 10% has a ten block radius. Hint²: if you have a negative percent difference the bullet is located below the bulls-eye.
- 14) Consider your result in question 4 using sig fig rules only. You should have found that $v = 8.14$ m/s. This implies an error in the hundredths column. We assume then that sig fig rules imply an error of ± 0.01 m/s. If this error was used (instead of the result of question 6), how would the target diagram change? Would the bullet be farther, closer, or the same distance from the center? Would the bullet be larger, smaller, or the same radius?

Comparing Standard Deviation to Propagation of Error

Suppose you have the following data table. Here is a case where, instead of using a single data point, an experiment has been repeated several times. This allows us to compute the error in different ways. Fortunately, most of the time we will use the computer to do the bulk of our computations.

y (m)	δy (m)
1.34	0.01
1.25	0.01
1.37	0.01
1.30	0.01
1.29	0.01

The basic idea: We compute the error in the average value of y in two ways. The first method uses the results derived from our propagation of error formulas. The second method uses the statistical spread of the measured values. One of these methods is usually significantly larger than the other one. We see which error result is larger and use that as our conservative estimate.

Computing the error using method 1:

- 15) Determine the average value of y . Keep at least four sig figs just in case and think about the units!
- 16) Determine the absolute error in the average using the formula $\delta \bar{y} = \frac{\delta y}{\sqrt{N}}$ where N is the number of measurements. Think: does it have units?
- 17) Determine the % error for \bar{y} using your result from the previous step. Think: does this have units?

Verify the final result is $y_{avg} = \bar{y} = 1.310m \pm 0.3\%$.

Computing the error using method 2:

- 18) Write down the average value of y from you found in a previous step.
- 19) Determine the standard deviation of y . The symbol for standard deviation of y is σ_y . Use the formula on the back of the lab manual near the bottom. Show all steps by hand this time.
- 20) The best statistical estimate for error of each individual measurement is σ_y . To get the standard error in the average, we must divide σ_y by \sqrt{N} .
- 21) Determine the percent error based on standard error using $\%err = \frac{\sigma_y/\sqrt{N}}{\bar{y}} \times 100\%$.

Verify the final result is $\bar{y} = 1.30m \pm 1.6\% = 1.30m \pm 2\%$. Note: for errors starting with the number 1, sometimes people keep a second sig fig.

Evaluate which method to use:

- 22) Look at the above results and it is easy to see that the second result is more conservative. If we use the first result, we are underestimating our errors and implying greater precision than appropriate. Clearly state which case which error estimate we should use for this example.

Practice with Excel

Your instructor will provide you the location of an Excel worksheet to use with today's lab. Open this file then follow the instructions below.

Start with Section 1 which has the same data table we used in the previous problem.

- Make a formula that computes all of the percent errors in the appropriate column. Ask your instructor for help if you don't know how to do this.
- Use the "AVERAGE" command to compute the average of the 5 y values.
- Use the "STDEV" command to compute the standard deviation of the 5 y values. WATCH OUT: be sure to only include the 5 data points and not the average.
- Repeat the same procedure on the second data table. Notice the numbers appear to be less random.

Notice when you get essentially the same measurement several times, the error in the average is determined using $\delta y/\sqrt{N}$. When your measurements have a spread greater than the error of the measuring device, the error in the average is determined using σ_y/\sqrt{N} . You can express your average result as $\bar{y} \pm \frac{\delta y}{\sqrt{N}}$ or $\bar{y} \pm \frac{\sigma_y}{\sqrt{N}}$.

Go to Section 2 which has a data table with of v and t .

- Make a plot of v versus t . I am calling this a Type I graph. Look in the table of contents of your lab manual for Sample Graph Type I. Follow those instructions.

Go to Section 3 which has values of F and r_{exp} .

- The top two rows of this section include the constants G , m , and M .
- First use an Excel to compute the theoretical column of data for r_{th} . Use the theoretical function

$$r_{th} = \sqrt{\frac{GmM}{F}}$$

- Use the propagation of error formulas to compute the error for each r_{th} . Notice you are told all errors except δF are negligible.
- Then make a column showing the percent difference between each r_{exp} and r_{th} . Please include any minus signs so I can quickly tell if r_{exp} is too high or too low. The formula to use is on the back cover of your lab manual.
- Make a Type 2 graph that shows r vs F . Be sure to read the table of contents to find the section in your lab manual called Sample Type II Graph.