Mathematical Modeling of 1D Motion

Apparatus: desktop or laptop for each student

Goal: Today's goal is to use 1D motion equations to make plots and analyze the features of those plots. I also hope you will learn about using Excel formulas to do simple tasks such as numerical differentiation. Each task has activities/questions to submit or show to your instructor.

Task 1: Generate a simple data table and plot *y* vs *t* and *v_y* vs *t*.
Open the Excel table from the class website.
Notice in the bottom, left corner of the screen there are several tabs named "Task1", "Task2", etc. Verify you are on the tab for Task1.
The table on the Task1 tab shows a row of constants. These constants represent the initial position, initial velocity, and constant acceleration of an object. Also, you see data columns for elapsed time (t), position (y), and velocity (v_y). From kinematics we know these variables are all related by

$$y = y_i + v_{iy}t + \frac{1}{2}a_y t^2 \tag{1}$$

$$v_y = v_{iy} + a_y t \tag{2}$$

Use these equations to generate theoretical data. Be sure to properly reference the constants using Excel formulas as this will be important later.

- After generating the data, create plots of y vs t and v_y vs t.
- For Task 1, do not worry about axis titles, labels, gridlines, etc.
- You should still add an appropriate trendline to each.
- Also, delete the legends for each graph to economize space.
- Finally, do your best to make sure the time increments of each axis line up as closely as possible.
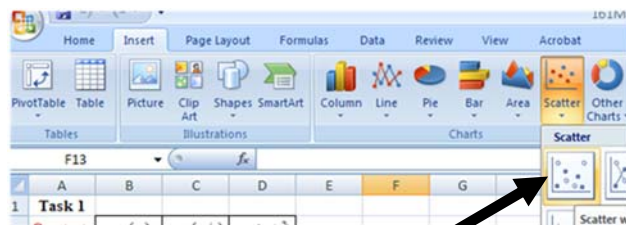

Tip: To make the graph go to the **insert tab** and look for the **scatter** plot (dots only). Never use a line graph in this course.

Once completed, you should have work that looks similar to this:

- Does the slope of the v_y vs t graph make sense?
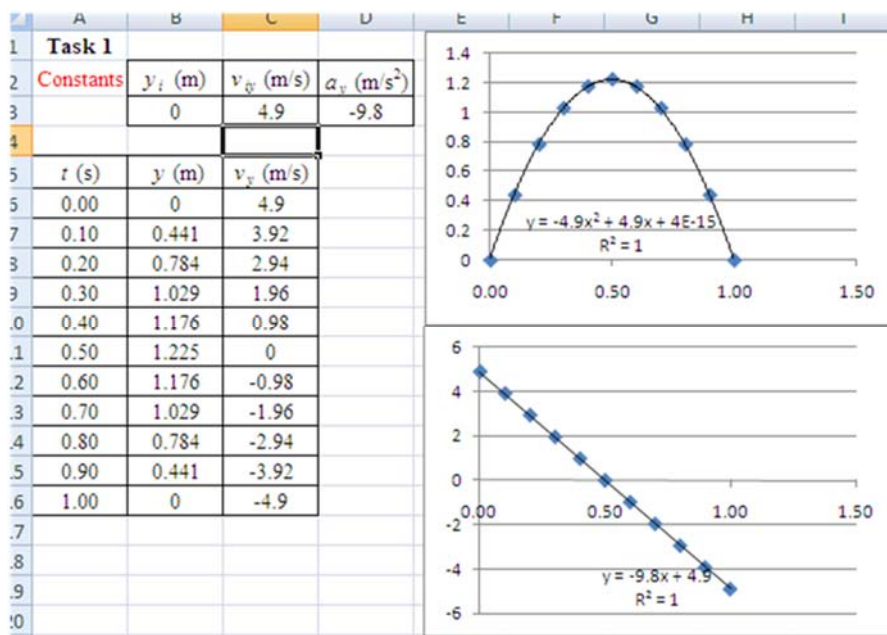- Does the concavity of the y vs t graph make sense?

DOUBLE CHECK: Did you remember to put the appropriate trendlines on each graph AND display the equations? This is needed to answer some questions later.
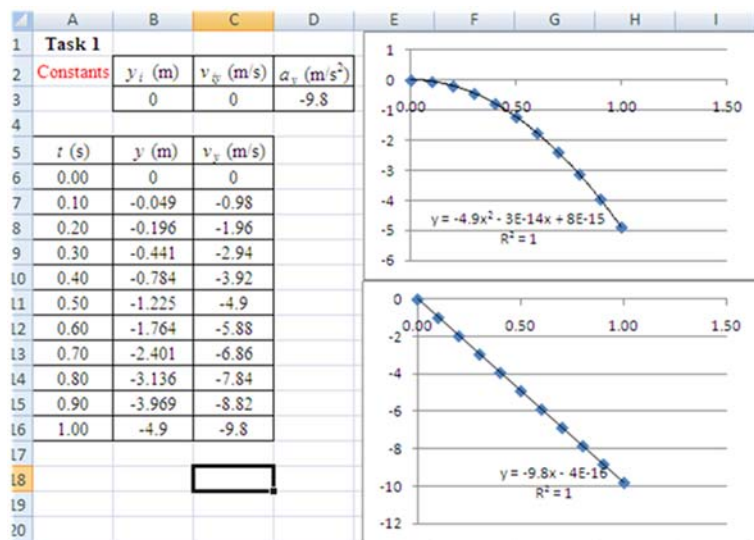
Now comes the fun part. If you've done your work properly, we should be able to change the row of constants and see how the graphs are affected immediately. **Use this page to check if your equations and graphs are working. Do this quickly then move on to the questions for Task 1 on the next page.**

Leave the initial position as 0.0 but change the initial velocity to +4.9. All the data will automatically update as well as the graphs by changing only the one number! You should see the screen change to something that looks like this:
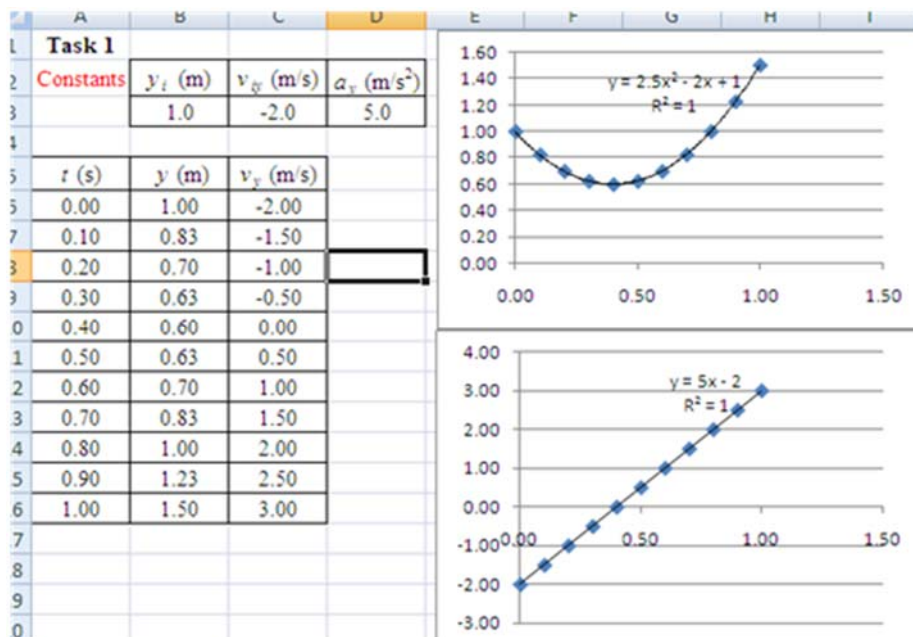
- At what time does the object reach max height?
- What do you notice about the vertical velocity at that same point in time?
- Does the slope of the v_y vs t graph make sense?
- Does the concavity of the y vs t graph make sense?

	A	B	C	D
1	Task 1			
2	Constants	y_i (m)	v_{iy} (m/s)	a_y (m/s²)
3		0	4.9	-9.8
4				
5	t (s)	y (m)	v_y (m/s)	
6	0.00	0	4.9	
7	0.10	0.441	3.92	
8	0.20	0.784	2.94	
9	0.30	1.029	1.96	
10	0.40	1.176	0.98	
11	0.50	1.225	0	
12	0.60	1.176	-0.98	
13	0.70	1.029	-1.96	
14	0.80	0.784	-2.94	
15	0.90	0.441	-3.92	
16	1.00	0	-4.9	

Graph: $y = -4.9x^2 + 4.9x + 4E-15$, $R^2 = 1$

Graph: $y = -9.8x + 4.9$, $R^2 = 1$

Suppose someone is falling down but gets bounced upwards by a trampoline. Perhaps she started 1.0 m above the ground, was initially moving downwards at a rate of 2.0 m/s but had an approximately constant upwards acceleration of 5.0 m/s.

In this case, after properly fixing the constants, you should see:

- Here does she reach a max or a min height?
- Notice that again the velocity is zero when she changes direction.
- Does the slope of the v_y vs t graph make sense?
- Does the concavity of the y vs t graph make sense?

	A	B	C	D
1	Task 1			
2	Constants	y_i (m)	v_{iy} (m/s)	a_y (m/s²)
3		1.0	-2.0	5.0
4				
5	t (s)	y (m)	v_y (m/s)	
6	0.00	1.00	-2.00	
7	0.10	0.83	-1.50	
8	0.20	0.70	-1.00	
9	0.30	0.63	-0.50	
10	0.40	0.60	0.00	
11	0.50	0.63	0.50	
12	0.60	0.70	1.00	
13	0.70	0.83	1.50	
14	0.80	1.00	2.00	
15	0.90	1.23	2.50	
16	1.00	1.50	3.00	

Graph: $y = 2.5x^2 - 2x + 1$, $R^2 = 1$

Graph: $y = 5x - 2$, $R^2 = 1$

Task 1 Questions/Activities:

T1.1: Now we are going to switch from up and down to right and left. You can still use y vs t…just imagine you rotated your coordinate system. Consider a situation where a person is initially moving right and slowing down, but by 1.0 s the person is moving left and speeding up. The initial position is negative, the person reaches a positive position before turning around, then returns to a negative position before 1.0 s has elapsed. Determine an appropriate set of initial constants that causes such a path to occur. Verify these constants work by plugging them into your mathematical model in XL. <u>Write down a set of three constants that work (you will need to show these to your instructor **later** for credit).</u>

T1.2: Consider a situation where an object is always moving left. The object never changes speed but within in the first 1.0 seconds the position is sometimes negative and sometimes positive. <u>Write down a set of constants that model such a situation.</u>

Finally, consider an object on a table 50 cm above the ground with an initial speed is 6.3 m/s.
You are told the object rises in the air then falls back to earth under the influence of gravity.
You are told that air resistance is negligible.

Make sure you have the trendlines on your graph.
Check print preview to ensure your works fits on a single page.
Hint: if you click print preview and then close it out the page margins will appear on your Excel worksheet.
Make your graphs as large as possible while still fitting within the page margins and not obscuring the data table.
<u>Print out the y vs t and v_y vs t plots.</u>
Use the graphs (slopes, areas, values, etc) to answer the following questions.
You shouldn't be using the kinematic equations.

T1.3a: How far above the ground is the particle at 1.0 s? Read it off the graph, don't compute.
T1.3b: Is the particle's displacement after 1.0 s? Use the area under one of the graphs!
T1.3c: What total distance has the particle traveled in 1.0 s? I estimated that $v = 0$ at $t = 0.64$ s.
T1.3d: What max height above the ground was reached by the particle? Read it off the graph, don't compute.
T1.3e: Write down the trendline equation from the position versus time graph. Explain how each of the coefficients relates to the constants of the model (which one is y_i, v_i, …watch out for a_y!).
T1.3f: Write down the trendline from the velocity versus time graph and explain how each of the coefficients relates to the constants of the model.
T1.3g: For this model, does $a_y = g$ or $a_y = -g$? Define g using words and state its value with units for credit.

Task 2: Numerical differentiation

Go to the bottom, left corner of the screen and select the Task2 tab.
The data table should already have a set of t's and y's obtained by a scientist in an experiment. Let's pretend she used video footage of a hummingbird flying back and forth. In the video footage, the film is comprised of still images that each last for $1/20^{th}$ of a seconds (0.050 s). By noting the bird's location in the images, an estimate of position for each time was made.

You know that velocity is the derivative of position. Unfortunately, we don't have a mathematical function so we can't take a derivative. We can, however, get an estimate using the following formula:
$$v_y = \frac{dy}{dt} \approx \frac{\Delta y}{\Delta t}$$

To understand how to use this formula we need to be able to discuss individual elements of the data table. See the table at right. The first number in each column will be denoted with a 0-subscript. To be clear, the first time is t_0 and the first position would thus be y_0. As you go down in rows, the subscripts increase accordingly. Verify this makes sense by considering t_5, v_6, etc in the figure.

This is t_0

This is t_5

This is t_7

This is y_5

This is v_6

This is y_7

	A	B	C	
1	Task2			
2				
3	t (s)	y (m)	v_y (m/s)	
4	0.000	2.95	NA	
5	0.050	2.80	-2.5	
6	0.100	2.70	-1.7	
7	0.150	2.63	0.8	
8	0.200	2.62	-0.2	
9	0.250	2.61	-0.3	
10	0.300	2.59	-1.1	
11	0.350	2.50	-2.5	
12	0.400	2.34	-4.0	
13	0.450	2.10	-3.1	

In order to use $v_y \approx \frac{\Delta y}{\Delta t}$, we want to be clever. Suppose, for instance, you are trying to determine v_6. You might think to try $v_6 \approx \frac{\Delta y}{\Delta t} = \frac{y_6 - y_5}{t_6 - t_5}$…this is not always the best choice. You could similarly try a forward-looking difference formula given by $v_6 \approx \frac{\Delta y}{\Delta t} = \frac{y_7 - y_6}{t_7 - t_6}$. Finally, you could consider the difference between the point just after 6 and just before 6. This results in the equation $v_6 \approx \frac{\Delta y}{\Delta t} = \frac{y_7 - y_5}{t_7 - t_5}$. In more general terms we say:
$$v_n \approx \frac{\Delta y}{\Delta t} = \frac{y_{n+1} - y_{n-1}}{t_{n+1} - t_{n-1}}$$

This is the formula we will use today to determine each value in the v_y-column. To make this work in the Excel spreadsheet in cell C10 we should type "=(B11-B9)/(A11-A9)". WATCH OUT: When using this formula, we are unable to obtain a reasonable value for the first and last v_y. That is discussed later.

Similarly, a set of data can be obtained for the a_y-column using
$$a_n \approx \frac{\Delta v}{\Delta t} = \frac{v_{n+1} - v_{n-1}}{t_{n+1} - t_{n-1}}$$

- Create the values for v_y and a_y using the appropriate XL formulas.
- Make graphs of y vs t, v_y vs t and a_y vs t.
- Notice I tweaked the yt-plot axis so the graphs line up better.
- Delete the legends but don't bother with trendlines.
- Do your best to line up the time increments as well.

You should end up with something similar to the figure shown at right.

t (s)	y (m)	v_y (m/s)	a_y (m/s^2)
0.000	2.95	NA	NA
0.050	2.80	-2.5	NA
0.100	2.70	-1.7	17.0
0.150	2.63	-0.8	15.0
0.200	2.62	-0.2	5.0
0.250	2.61	-0.3	-9.0
0.300	2.59	-1.1	-22.0
0.350	2.50	-2.5	-29.0
0.400	2.34	-4.0	-6.0
0.450	2.10	-3.1	27.0
0.500	2.03	-1.3	28.0
0.550	1.97	-0.3	21.0
0.600	2.00	0.8	13.0
0.650	2.05	1.0	22.0
0.700	2.10	3.0	40.0
0.750	2.35	5.0	20.0
0.800	2.60	5.0	0.0
0.850	2.85	5.0	0.0
0.900	3.10	5.0	0.0
0.950	3.35	5.0	NA
1.000	3.60	NA	NA
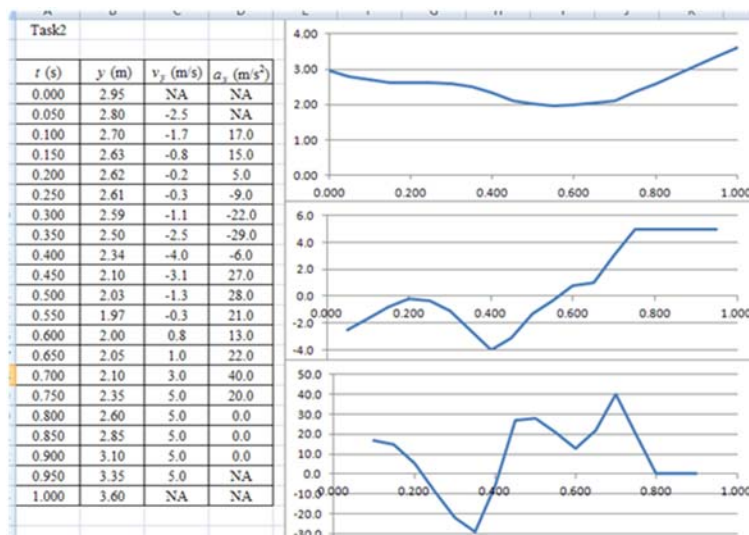
Task 2 Questions/Activities:

T2.1: Explain why it doesn't make sense to compute v_0, a_0, or a_1 with the formula we used.

T2.2: Describe the motion of that particle at time $t = 0.200$s. Is it moving up, down or sitting still? Is it speeding up, slowing down, or traveling with constant speed? Defend your answer by discussing the signs, slopes, and/or concavities of the y vs t and v_y vs t graphs. You can check the signs in the table as well!

T2.3: Describe the motion of that particle at time $t = 0.900$s. Is it moving up, down or sitting still? Is it speeding up, slowing down, or traveling with constant speed? Defend your answer by discussing the signs, slopes, and/or concavities of the y vs t and v_y vs t graphs. You can check the signs in the table as well!

T2.4: Describe the motion of that particle between times $t = 0.450$s and 0.500s. You may assume the acceleration is roughly constant on that time interval. To me, it looks like the acceleration is about $3g$'s (three times the value of g). Is it moving up, down or sitting still? Is it speeding up, slowing down, or traveling with constant speed? Defend your answer by discussing the signs, slopes, and/or concavities of the y vs t and v_y vs t graphs.

T2.5: Notice the acceleration changes wildly between 0.3 and 0.7 s. This seems quite improbable. Meanwhile, both the y vs t and v_y vs t graphs don't seem that unusual. The a_y graph appears to have much more error than the v_y graph even though they are both made from the same data table. This is because the calculation of a_y is an approximation on top of a previous approximation. By using a smaller time increment to record data, the a_y vs t graph would look a lot more sensible. Imagine how bad jerk versus time would look! HAH! You don't have to write anything down but at least I made you read this!

Task3: An acceleration that changes in time (piece-wise functions)

Suppose a car starts from rest and accelerates for 3.00 seconds at a rate of 6.00m/s². Call this stage 1.
In stage 2 the car continues but with the smaller acceleration of 2.00 m/s² for 2.00 additional seconds.
In stage 3 the car stops accelerating but continues on at constant speed for 4.00 s.
It would be easy to calculate by hand the total distance traveled by the car using kinematics formulas.
In today's lab, however, I want you to plot the position, velocity, and acceleration versus time.
We can use the kinematics equations but we need to be a little clever when typing them into XL.
For the first part we simply use:

$$x_{1f} = x_{1i} + v_{1ix}t + \frac{1}{2}a_{1x}t^2 \tag{1}$$

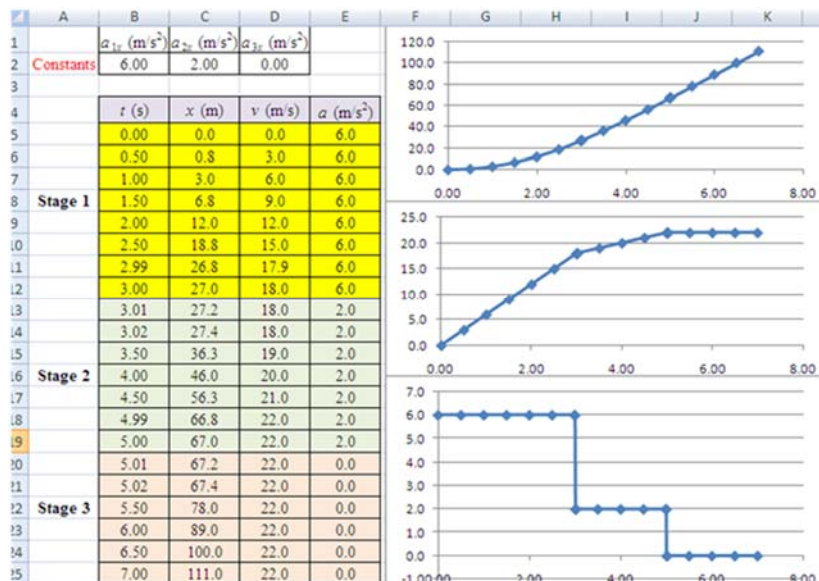$$v_{1fx} = v_{1ix} + a_{1x}t \tag{2}$$

For the next piece of the function, we need to reset the time variable. To do this we simply subtract off the already elapsed 3.00 s from each t in the equations. The initial velocity for the second piece of the function (v_{2ix}) is equal to the final velocity of the first part of the function (v_{1fx}). Similarly $y_{1f} = y_{2i}$. Of course, the acceleration also changes. This gives

$$x_{2f} = x_{2i} + v_{2ix}(t-3) + \frac{1}{2}a_{2x}(t-3)^2 = x_{1f} + v_{1fx}(t-3) + \frac{1}{2}a_{2x}(t-3)^2 \tag{1}$$

$$v_{2fx} = v_{2ix} + a_{2x}(t-3) = v_{1fx} + a_{2x}(t-3) \tag{2}$$

For the next piece of the function, we need to reset the clock by 5 seconds (the initial 3 seconds of stage 1 plus an additional 2 seconds for stage 2). We need to determine the initial conditions for stage 3 (x_{3i} and v_{3xi}) in a manner similar to the previous stage.

- Upon using the formulas, you should be able to make a set of graphs that look like those shown below.
- Delete the legends, ignore axis labels, etc. Add vertical gridlines.
- Use data points connected by a smooth line (no trendlines make sense here as the function changes every few seconds).
- Hit print preview.
- Make the three graphs as large as possible while still fitting on one page.
- Print a copy.

|		a_{1x} (m/s²)	a_{2x} (m/s²)	a_{3x} (m/s²)		
1						
2	Constants	6.00	2.00	0.00		
3						
4		t (s)	x (m)	v (m/s)	a (m/s²)	
5		0.00	0.0	0.0	6.0	
6		0.50	0.8	3.0	6.0	
7		1.00	3.0	6.0	6.0	
8	Stage 1	1.50	6.8	9.0	6.0	
9		2.00	12.0	12.0	6.0	
10		2.50	18.8	15.0	6.0	
11		2.99	26.8	17.9	6.0	
12		3.00	27.0	18.0	6.0	
13		3.01	27.2	18.0	2.0	
14		3.02	27.4	18.0	2.0	
15		3.50	36.3	19.0	2.0	
16	Stage 2	4.00	46.0	20.0	2.0	
17		4.50	56.3	21.0	2.0	
18		4.99	66.8	22.0	2.0	
19		5.00	67.0	22.0	2.0	
20		5.01	67.2	22.0	0.0	
21		5.02	67.4	22.0	0.0	
22	Stage 3	5.50	78.0	22.0	0.0	
23		6.00	89.0	22.0	0.0	
24		6.50	100.0	22.0	0.0	
25		7.00	111.0	22.0	0.0	

Task 3: Questions/Activities

T3.1: Determine the total displacement of the car by looking at the graph. Don't do any calculations.

T3.2: Use a straight edge to split up the *vt* graph into chunks (see the figure). Figure out the area of each chunk. Add up the areas. Think: how should this compare to your previous answer?

T3.3: Notice that distance and displacement can be used interchangeably in this problem. That is not always the case! In fact, Task 2 showed an instance when displacement and distance were not equal. Generally speaking, for which 1D motion problems will distance not equal displacement?

A final note regarding piece-wise functions: in reality, the acceleration would not change abruptly from 6.00 m/s^2 to 2.00 m/s^2. The acceleration would gradually change, perhaps over an interval of 0.2 sec or so. This would smooth out all the graphs a little bit. As long as the transition time between accelerations is small compared to the times of the duration of the stages, this piece-wise technique accurately models the situation.
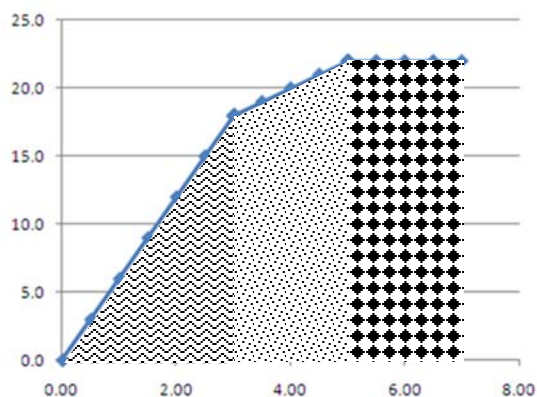
Task4: Plotting position (and velocity) versus time for two objects simultaneously

Suppose a truck (labeled **1** in the figure) drives at a constant speed of 20.0 m/s.
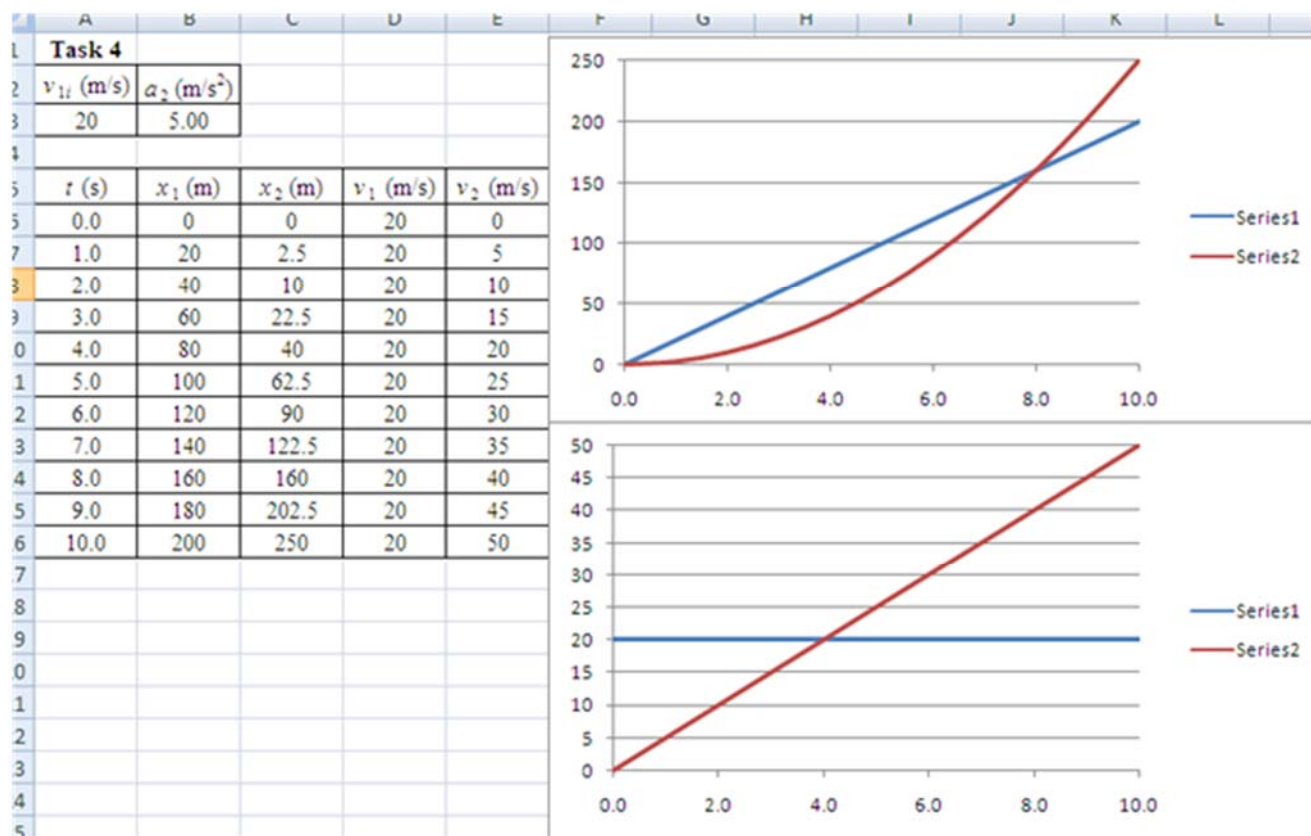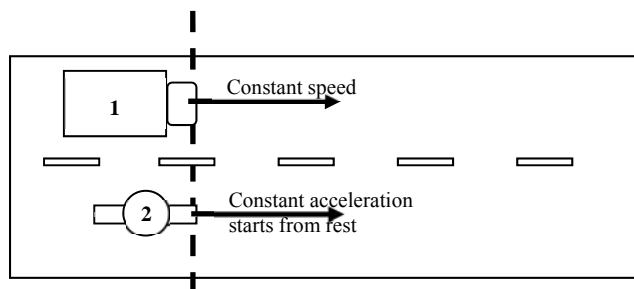At the instant it passes a motorcycle, the motorcycle accelerates from rest with acceleration 5.0 m/s². The motorcycle is labeled **2** in the figure.
The above information allows us to write down the equations of position and velocity as a function of time for each object. For example:

$$x_1(t) = x_{1f} = 20.0t$$

$$v_1(t) = v_{1f} = 20.0$$

- Determine the appropriate equations for $x_2(t)$ and $v_2(t)$.
- Click on Task 4 tab in Excel.
- Use these equations to plot x vs t.
- Include both x_1 and x_2 on the same graph. If you need help getting two data series on the same graph, ask your instructor for help.
- **Do not delete the legend this time! When more than one data set is represented on a graph a legend is appropriate.**
- Plot only smooth lines, no data points.
- Do something similar for v vs. t.
- Your XL sheet should now look like this:

t (s)	x_1 (m)	x_2 (m)	v_1 (m/s)	v_2 (m/s)
0.0	0	0	20	0
1.0	20	2.5	20	5
2.0	40	10	20	10
3.0	60	22.5	20	15
4.0	80	40	20	20
5.0	100	62.5	20	25
6.0	120	90	20	30
7.0	140	122.5	20	35
8.0	160	160	20	40
9.0	180	202.5	20	45
10.0	200	250	20	50

v_{1i} (m/s) = 20, a_2 (m/s²) = 5.00

Task 4: Questions/Activities

T4.1: In the top graph we see that one of the curves is concave up while the other is a straight line. Which line represents the motor cycle? Explain.

T4.2: In the 2nd graph, one of the lines is upwardly sloping while the other is flat (no slope). Which line represents the motor cycle? Explain.

T4.3: At what time does the motorcycle start to get closer to (rather than farther behind) the truck? Hint: You can read this information off one of the graphs without any computation. Verify with the data table.

T4.4: Between 0 and 8 seconds, what is the maximum distance between the truck and the motorcycle (after the truck passes the motorcycle)?

T4.5: At what time will the motorcyclist finally catch up to the truck? Hint: use one of the graphs and verify with the data table.

T4.6: Determine the speed of the motorcyclist as he overtakes the truck. Hint: use the graph or the data table at the appropriate time. **Answer in miles per hour.**
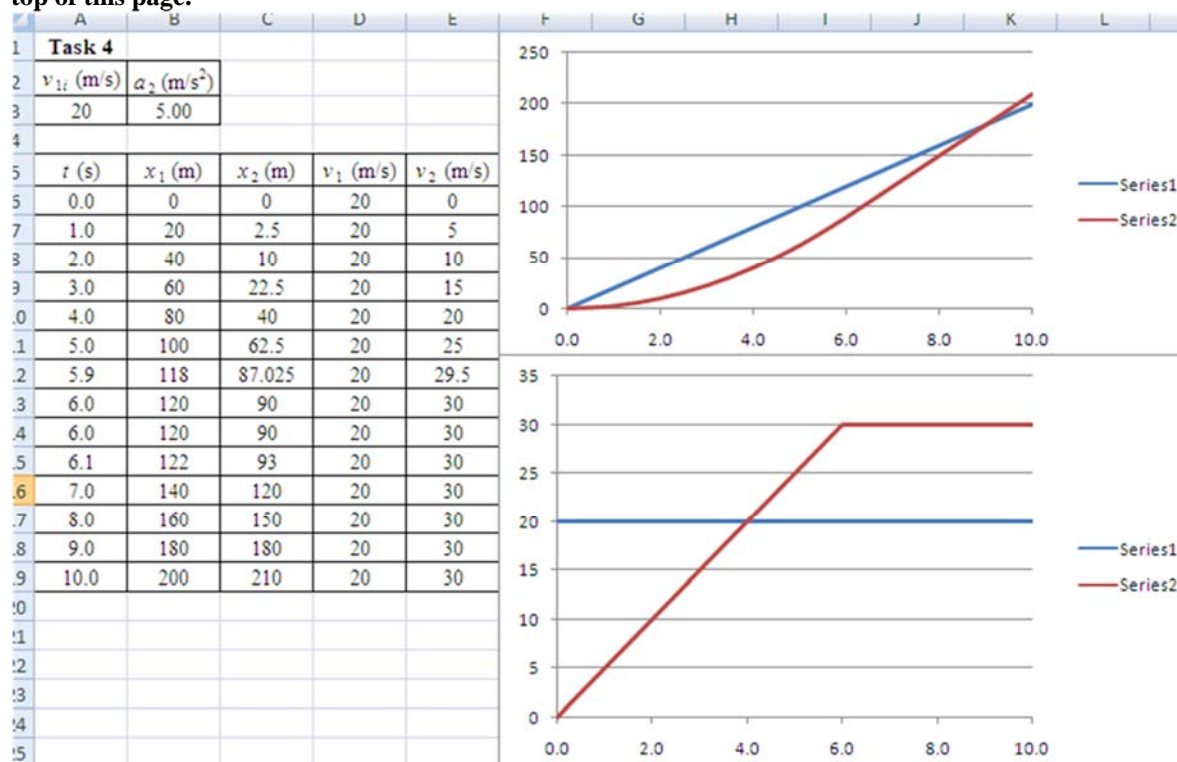
T4.7: How many g's is the motorcyclist experiencing? This is determined by taking the ratio a/g. for example, if $g = 10\frac{m}{s^2}$ and $a = 25\frac{m}{s^2}$ then we could also say $a = 2.5g's$.

T4.8: Based on your previous results, do you think this problem is realistic? Support your answer.

BONUS
A piece-wise function might be more appropriate for the motorcycle. Say it has an acceleration of 5.0 m/s² for the first 6 seconds and then maintains constant speed. The motorcycle now overtakes the truck at 9.0 s instead of 8.0 s. Notice that it is very difficult to distinguish where the parabolic part of the graph ends and the linear part begins for the xt graph of the motorcycle. We see the transition point much more clearly on the vt graph of the motorcycle. This is typical. Notice that the cop only reaches 30 m/s ≈ 67 mph instead of 40 m/s ≈ 89 mph.

Note: the figure *below* is for the BONUS only. The figure on the *previous page* applies to the questions at the top of this page.

	A	B	C	D	E
1	Task 4				
2	v_{1i} (m/s)	a_2 (m/s²)			
3	20	5.00			
4					
5	t (s)	x_1 (m)	x_2 (m)	v_1 (m/s)	v_2 (m/s)
5	0.0	0	0	20	0
7	1.0	20	2.5	20	5
8	2.0	40	10	20	10
9	3.0	60	22.5	20	15
10	4.0	80	40	20	20
11	5.0	100	62.5	20	25
12	5.9	118	87.025	20	29.5
13	6.0	120	90	20	30
14	6.0	120	90	20	30
15	6.1	122	93	20	30
16	7.0	140	120	20	30
17	8.0	160	150	20	30
18	9.0	180	180	20	30
19	10.0	200	210	20	30

Another optional element: Suppose it takes the cop some small reaction time Δt before he starts the acceleration phase. Incorporate this into your model and see how the graphs change as you gradually increase Δt from 0.00 to 1.00 sec.

Task 5: A falling particle with air resistance

Suppose a tiny particle is released from rest from a very tall building. You are told the drag force increases linearly versus time on the falling object. In certain cases the force equation for such an object is given by

$$F_{net} = mg - bv$$

Later in physics we learn that acceleration is given by

$$a = \frac{F_{net}}{m}$$

For our object that falls while experiencing air resistance we find

$$a = g - \frac{b}{m}v \tag{1}$$

As the object falls its speed will increase. As speed increases, notice the acceleration will decrease. This is clearly not a constant acceleration problem. Therefore we cannot use the constant acceleration equations of motion. We can find the equations of $y(t)$ and $v_y(t)$ using separation of variables!

As good practice for an exam, use separation of variables to show

$$y(t) = v_T t - v_T \frac{m}{b}\left(1 - e^{-\frac{b}{m}t}\right)$$

which is also written as

$$y(t) = v_T t - v_T \frac{m}{b}\left(1 - \exp(-\frac{b}{m}t)\right) \tag{2}$$

Hint: in Excel formulas "e" means "×10 to the" while "exp" means "*e* to the"

$$v(t) = v_T \left(1 - \exp(-\frac{b}{m}t)\right) \tag{3}$$

$$where \ v_T = \frac{mg}{b} \tag{4}$$

Note: in the equations above I have assumed that down is the positive direction, not up. I did this to reduce the number of minus signs cluttering up the equations. I also set $y_i = 0$. Furthermore, v_T is called terminal velocity. Open the Task 5 tab in the Excel spreadsheet.
Use the given constants and equations (2) through (4) to make initial *yt* and *vt* plots.
Use equation (1) and your column of calculated *v*'s to make an *at* plot.
The next page shows you what this should look like…

Here is the graph.

Task 5	m (kg)	b (?/?)	g (m/s²)
Constants	6.00	2.00	9.8

t (s)	x (m)	v (m/s)	a (m/s²)
0.0	0	0.0	9.80
1.0	4	8.3	7.02
2.0	16	14.3	5.03
3.0	32	18.6	3.61
4.0	53	21.7	2.58
5.0	75	23.8	1.85
6.0	100	25.4	1.33
7.0	126	26.5	0.95
8.0	153	27.4	0.68
9.0	181	27.9	0.49
10.0	209	28.4	0.35
11.0	237	28.6	0.25
12.0	266	28.9	0.18
13.0	295	29.0	0.13
14.0	324	29.1	0.09
15.0	353	29.2	0.07
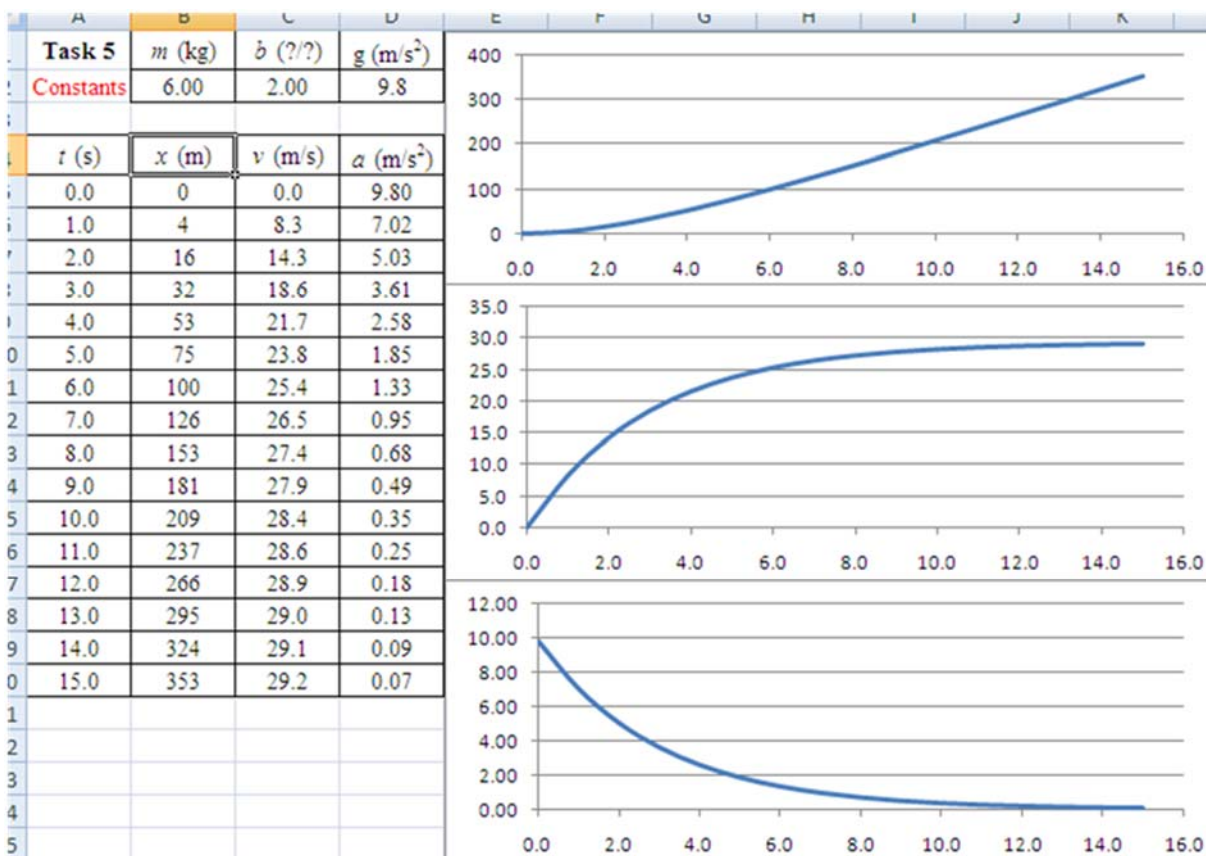
Task 5: Questions/Activities

T5.0: Figure out the correct units of the constant b and input those units into cell C1.
T5.1: Print out the graphs (preview first, make as large as possible while still fitting on one page).
T5.2: Use equation 4 and a calculator to determine a numerical value for the terminal velocity.
T5.3: Notice that in the vt-plot the graph has what appears to be a horizontal asymptote. Use a straight edge and draw in that asymptote. Compare the value of the horizontal asymptote to v_T obtained in the previous question.
T5.4: When I look at the yt-plot it looks extremely straight after about 8.00 s. What is the approximate acceleration for that time interval? Explain how the straightness of the xt plot and the small acceleration makes sense together.
T5.5: Try adjusting the drag coefficient b and the mass m. Perhaps try out other planets or the moon by adjusting g. Notice how in some cases the particle never seems to reach terminal velocity while in other cases it reaches it extremely quickly. This makes sense; a feather reaches terminal velocity very quickly while a human takes a bit more time…

Task 6: Damped Harmonic Motion

The position as a function of time for a damped harmonic oscillator is given by

$$x(t) = Ae^{-\alpha t}\cos(\omega t)$$

Here A is the initial amplitude of oscillation, α is the decay constant, and ω is called angular frequency. This topic is explored in detail in PHYS 162 but we can use our kinematics and graphing knowledge to take a first look at this exciting and ubiquitous function.

Your goal for task 6 make plots of position, velocity, and acceleration versus time for the damped oscillator. By making these graphs, I hope you will get practice with the following:
- Taking derivatives
- Using Excel to model $x(t)$, $v(t)$, and $a(t)$
- Analyze when the object is speeding up or slowing down
- Get a feel for how the motion is affected as you vary the parameters of A, α, and ω.

Take derivatives to find $v(t)$ and $a(t)$. Show you obtain
$$v(t) = -Ae^{-\alpha t}[?\cos(\omega t) +?\sin(\omega t)]$$

$$a(t) = Ae^{-\alpha t}[(?^2-?^2)\cos(\omega t) + 2??\sin(\omega t)]$$
Here the ?'s indicate a variable I want you to figure out on your own.

T6.0: Determine the appropriate units for the parameters A, α, and ω. *Hint: $x(t)$ is position and has units of…*
T6.1: Use an amplitude of 1.00, a decay constant of 0.100, and an angular frequency of 0.628. Make constant cells for these parameters so we can change them easily later. Use a time step size of 0.10 seconds from 0.00 to 20.00 sec. Make plots of $x(t)$, $v(t)$, and $a(t)$. Compare them to the ones seen on the next page as a check on your work. Tip: to use Excel to calculate $e^{-\alpha t}$ use $\mathbf{exp}(-\boldsymbol{\alpha t})$. Also: verify you are using 0.628 in the constants row to check your data against the plots on the next page.
T6.2: During what time or time interval is the damped oscillator moving as follows (hint: use the graph to estimate the times then look in the data table to make your answers more precise):
  a) Moving forward and speeding up
  b) Moving forward and slowing down
  c) At rest
  d) Moving backward and speeding up
  e) Moving backward and slowing down
T6.3: Now try changing the parameter ω and see how increasing ω affects the graphs. Try out the values of 1.57, 3.14, and 6.28 and see what happens. Notice that increasing the angular frequency cause the position of the oscillator to oscillate more rapidly.
T6.4: Set $\omega = 3.14$. Now try gradually increasing α and seeing how it affects the graph. Try values 0.2, 0.3, 0.5, and 1.0. Notice that as α increases the oscillations dies out more rapidly. The amplitude decays.
T6.5: Set $\omega = 3.14$ and $\alpha = 0.300$. Try increasing the amplitude A and see what happens. Try values of 2, 5, and 10.
T6.6: This function could model a child on a swing after they are no longer pushed (the oscillation dies out). This function might model the motion of a plucked guitar string.
  a) Compare the angular frequencies used for each model (swing vs string). Which model would likely use a larger ω (or would they about the same)? Explain and defend your answer.
  b) Compare the decay constants used for each model (swing vs string). Which model would likely use a larger α (or would they about the same)? Explain and defend your answer.
  c) Compare the amplitudes used for each model (swing vs string). Which model would likely use a larger α (or would they about the same)? Explain and defend your answer.

Interesting Note: Try cranking up ω to about 20. Notice the graph appears distorted. This is occurring because at that high of a frequency our time step is relatively large compared to an oscillation cycle. As such we sometimes don't get any points near the peak of the oscillation causing the graph to look funky…to fix this problem one would need to use a smaller step size (say increment the time by 0.01 sec instead of 0.10). Try it and see if this fixes the problem. This relates to aliasing and sampling rates if you are interested in going further.

A (m)	α (1/s)	ω (rad/s)
1.00	0.100	0.628

t (s)	x (m)	v (m/s)	a (m/s^2)
0.00	1	-0.1	-0.384384
0.10	0.9880982	-0.1378301	-0.3720051
0.20	0.9724773	-0.1743596	-0.3583824
0.30	0.9532736	-0.2094678	-0.3435951
0.40	0.9306349	-0.2430425	-0.3277254
0.50	0.9047197	-0.2749796	-0.3108583
0.60	0.8756968	-0.3051838	-0.293081
0.70	0.8437436	-0.3335684	-0.2744827
0.80	0.8090463	-0.360056	-0.2551542
0.90	0.771798	-0.384578	-0.2351872
1.00	0.7321982	-0.4070752	-0.2146742
1.10	0.6904521	-0.4274977	-0.1937082
1.20	0.6467692	-0.4458049	-0.1723821
1.30	0.6013627	-0.4619652	-0.1507884
1.40	0.5544485	-0.4759567	-0.1290187
1.50	0.5062441	-0.4877662	-0.1071638
1.60	0.4569681	-0.4973896	-0.0853127
1.70	0.4068389	-0.5048317	-0.0635528
1.80	0.356074	-0.510106	-0.0419694
1.90	0.3048892	-0.5132343	-0.0206455
2.00	0.2534977	-0.5142464	0.0003389
2.10	0.2021092	-0.5131804	0.0209063
2.20	0.1509294	-0.5100815	0.0409829
2.30	0.1001589	-0.5050025	0.0604978