Using Tracker Software to Analyze 1D motion

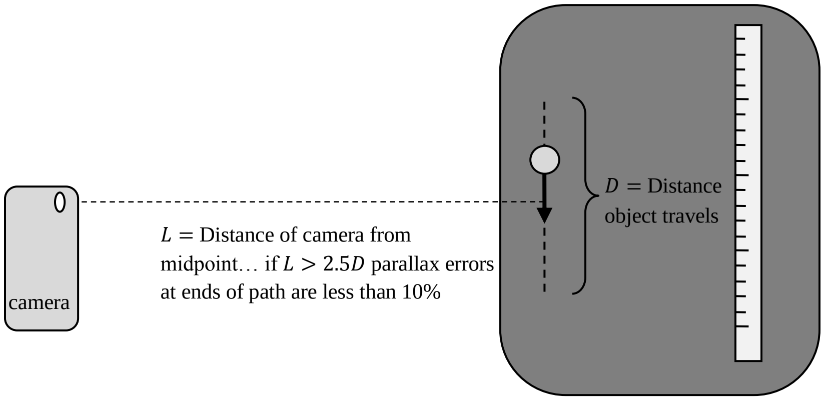
Apparatus: See table below
The goal of this experiment is analyze graphs of motion. In particular, you are to do the following:

1. Choose an option for a moving object
2. Make a video of a moving object (include ruler for scale and clean background)
3. Open the video in the Tracker software program
4. Make plots of position versus time, velocity versus time, and acceleration versus time
5. Analyze the plots to determine if the kinematics of constant acceleration apply
6. Prepare and present a PowerPoint presentation to the class summarizing your work

Each group must pick one of the following options with a different starting number (unless we run out of options).
If you have an idea for a variation (or totally unique idea), discuss it with your instructor as early as possible.
General presentation tips and option specific tips begin on the following pages.

Option	Description
0	Film the fan cart on a track. Use the video to determine if the fan cart is best modeled as operating under constant force or constant power. Questions **5.33** and **7.13e** in the workbook may be useful. Watch out: Make sure the track is level and the fan cart is released from rest.
1a	Film someone running 30 m on a track. Start from rest and try to run as fast as you can. Can you accelerate for the entire 30 m? Place two orange cones on the track 30 m apart for calibration purposes.
1b	Film someone running backwards for 15 m on a track starting from rest. Can you accelerate for the entire 15 m? Place two orange cones on the track 15 m apart for calibration purposes.
2a	Drop a coffee filter from rest. Have it fall for about 2.0 m of distance. Repeat the experiment with 2, 3, 4 & 5 coffee filters nested inside each other. What is terminal velocity? Which model of drag works best?
2b	Fluff up a cotton ball and drop it from rest. Drop it from about 2.0 m using a ladder. Do this inside to avoid wind issues. Experimentally determine the drag coefficient b and compare to my estimate.
3	Obtain a cart which has a spring at one end. Place the cart at the bottom on an inclined track resting against a lead brick. Compress the cart's spring against the brick and release it from rest. Record the motion of the cart as it bounces several times off the brick. Determine the average spring force during collisions and the COR. For times when the spring is not engaged, compare theoretical motion to experimentally observed motion.
4a	Hang a mass on spring and let it come to equilibrium. Pull the mass down a small additional amount. Do your best to ensure the mass oscillates entirely in the vertical direction. Compare theoretical motion to experimentally observed motion.
4b	Cart attached to spring on incline. Start with the spring unstretched with the cart near the top of the ramp. Start video when the cart is released from rest. Record two full oscillations. Compare theoretical motion to experimentally observed motion.
4c	Find the bowling ball pendulum in the lab. Pull the ball a small distance to the right. The string should not exceed an angle of 8° from the vertical. The problem is complicated if vertical motion of the ball is non-negligible. Record two full oscillations. Compare theoretical motion to experimentally observed motion.
5	Throw a tennis ball up in the air. *Include the throwing and catching of the ball* (the times your hand is in contact with the ball) in your video. Do your best to make the motion purely vertical. Try to start and end with your hand in the same position. Determine the average force exerted by the hand while throwing and catching. While the ball is in flight, compare the ball's experimental acceleration to what freefall predicts.
6	Angle a metal track very slightly (approx. 1°) above the horizontal. Place a steel ball on the track at one end and secure a powerful magnet to the other end. Release the ball from rest and film the entire travel including the impact with the magnet. Compare experimental to predicted acceleration while far from magnet. Determine the effective range of magnetic force and the average force exerted by the magnet.
7a/7b	Create simulations of experiment 2b, 4a and 5. Start by completing the suggested tutorials. Create xt-, vt- & at- plots in your simulations. Compare your plots to plots from the experimental groups.

Make a video of your situation. Ensure you have a clean backdrop that provides good contrast (usually a black or white background works ok). Also, ensure you have a device of known length (typically a meterstick) clearly visible in your video. Ideally you should position your camera such that it is centered on the middle your object's path of travel. Also, suppose you object of interest travels total distance D. If the distance L from the camera to the midpoint is less than $2.5D$ you introduce significant errors. Try a web search for "parallax error" to learn all kinds of neat stuff related to this type of error. Finally, consider making one student in your group figure out the Tracker software while the rest of you make the video.



Load your video into the tracker software. You can Google "How to use tracker software" and find a YouTube video that explains how to use the software. I liked the one by "Vector Shock" at this link because it is short: https://www.youtube.com/watch?v=D3p-CzWhhY8. This one is also good because it shows you how to rotate the coordinate system at about the 1:50 mark: https://www.youtube.com/watch?v=ibY1ASDOD8Y. If you want to practice while your group is getting your video, open a test video and try fooling around with that.

Determine how you want to align your coordinate system at this point. For example, do you want down the incline to be positive or negative…your call? If you throw a ball downwards, do you want to make downwards the positive direction? I'm cool with that. Is the object in your video moving left instead of right? Perhaps you want to call leftwards positive instead of rightwards? Adjust the coordinate system in Tracker to match your decision.

Get position, velocity, and acceleration versus time data. Use the software to tabulate t, x, v, and a. You may need to hit the button labeled Table on the right side of the screen to ensure you are displaying all four columns of data. Double check the numbers seem reasonable. For instance, to quickly convert m/s to miles/hr use 1 m/s ≈ 2.2 mph. Think: should your accelerations be greater than, less than or roughly equal to 10 m/s^2. Double check the signs on the numbers and make sure that match expectations based on your choice of coordinate system. When satisfied, copy the data from the Tracker software and paste it into an Excel spreadsheet. Note: if your numbers seem abnormally huge (or tiny), check your calibration stick…are you in meters or centimeters?

Make plots of position versus time, velocity versus time, and acceleration versus time IN EXCEL. Tracker will make graphs for you but I want you to get the practice of making the graphs in Excel yourself. Just grab cut and paste the data into Excel.

1D Motion Tips

For everyone:

- Tell people what coordinate system you chose. If non-standard, explain why you chose it that way.
- When you make plots of xt, vt & at, be careful with the time axis. Ensure the time axes are identical on all three graphs. Right click on each time axis and click "format axis" to manually adjust the axis times.
- For formatting on your plots, read the Lab Manual Appendices and Sample Graph Type I or II as appropriate.
- Think! Sometimes trendlines should not be used. For instance, only use a linear trendline if a graph should be modeled by a linear equation! Example: don't put a linear (or polynomial) trendline on data when we expect the physical situation to be modeled by a sine function.

First read the detailed instructions specific to your project on the following pages. *After you make a rough draft of your talk,* come back to this point. The items in the list below are some possible ideas of things to say to fill time. The first talk is usually about 6-8 minutes. You don't have to do *all* of these, but you will probably need to do *some* of these. These ideas match the type of questions I ask about graphs on exam day.

- On your xt graph:
  - Determine the *signs* of v and a for two different instants. Explain how you know the signs of a and v at each instant to the class. Hint: concavity and slope.
  - For those same instants, discuss if the object moving forward/backward/at rest & speeding up/slowing down/constant speed. Hint: compare signs of a and v.
  - Point out any instants where the object is instantaneously at rest. Point out times for which the slope of the xt-graph is zero at those times. These are *usually* instants when the object reverses direction.
  - Get a number with units for the slope at each of your 3-4 different times. Compare each calculation to the appropriate time on the vt graph.
- On your vt graph:
  - Determine the sign of a at 3-4 different times. Hint: slope.
  - For those same times, discuss if the object moving forward/backward/at rest & speeding up/slowing down/constant speed.
  - Determine the area under the vt graph (get a number with units) and compare it to your position versus time graph. It may help to split the area into chunks. Watch out for both positive and negative areas.
  - Point out any spots where the object is instantaneously at rest.
  - Get a number with units for the slope at each point and compare it to the at graph.
- On your at graph:
  - Don't worry if the graph looks extremely noisy. It usually looks terrible since the acceleration data is obtained using multiple steps of approximations. In contrast, the velocity data is obtained using only a single approximation and is much less noisy.
  - Relate the sign of a to the concavity of the xt graph at different times.
  - Relate the sign of a to the slope of the vt graph at different times.
  - Does your experiment exhibit any acceleration trends?
    - Is the acceleration roughly constant?
    - Is the acceleration always increasing or decreasing?
    - Is the acceleration oscillating?
    - Note: it may be easiest to describe these trends using the slope of the vt-graph rather than a very noisy at-graph.
- Watch out for these pitfalls
  - Remember that speed and velocity are different…one includes ± signs while the other does not!
  - If you have a negative velocity that is increasing in magnitude, the speed is increasing
  - When you are speaking about acceleration, be clear if you mean the *magnitude* of the acceleration or the acceleration…one includes ± signs while the other does not!

Option 0 Notes (assumes fan cart starts form rest to simplify the math):
If power is constant the velocity as a function of time should obey the equation

$$v = \sqrt{\frac{2\mathcal{P}t}{m}}$$

where $\mathcal{P}$ is the constant power in Watts. Notice we could say

$$v^2 = \frac{2\mathcal{P}}{m}t$$

Using separation of variables one can also show the position as a function of time is given by

$$x = \left(\frac{8\mathcal{P}}{9m}\right)^{1/2} t^{3/2}$$

If power is constant a plot of v^2 vs t should be linear and the power is determined using $slope = \frac{2\mathcal{P}}{m}$. Furthermore, a plot of plot x vs $t^{3/2}$ should be linear with $slope = \left(\frac{8\mathcal{P}}{9m}\right)^{1/2}$.

If force is constant (and thus acceleration) the velocity as a function of time should obey the equation

$$v = at$$

$$v = \frac{F}{m}t$$

The position should obey

$$x = \frac{1}{2}at^2$$

$$x = \frac{F}{2m}t^2$$

If force is constant a plot of v vs t should be linear and the force is determined using $slope = \frac{F}{m}$. A plot of x vs t^2 should be linear with $slope = \frac{F}{2m}$.

Once you make a tracker video of the fan cart, you can make tables in Excel of position, velocity and acceleration versus time. You can use that data to create three additional columns: one column for v^2, one column for $t^{3/2}$, and one column for t^2. Use this augmented data table to make all of the plots discussed above:

1. v^2 vs t
2. x vs $t^{3/2}$
3. v vs t
4. x vs t
5. x vs t^2

Formatting instructions can be found in the lab manual appendices. See "SAMPLE GRAPH TYPE I". Be sure to include a linear trendline showing both the equation and the R^2 coefficient. The plot with the largest R^2 coefficient is said to be the most linear.

If plots 1 & 2 <u>are</u> linear, you know the fan cart is well modeled using a constant *power* model. Use the slope on each of the first two plots to determine the constant power of the fan.
If plots 1 & 2 <u>are</u> non-linear, show this to the class and stress this point.

If plots 3 & 5 <u>are</u> linear, you know the fan cart is well modeled using a constant *force* model. Use the slope on each of the first two plots to determine the constant force exerted by the fan.
If plots 3 & 5 <u>are</u> non-linear, show this to the class and stress this point.

Note: you can also fill times with commentary relating to the items discussed on the previous page.

Options 1a & 1b Notes: **FBD is confusing at first. Discuss with your instructor after acquiring data.**
First take a guess. Can you run with constant acceleration for the entire distance or not? It is tough to get tracker positions accurately. Perhaps try to use the middle of the person's torso. Record multiple people to compare?
Cut and paste your time, position, and velocity data from Tracker to Excel. Make plots of position versus time, velocity versus time, and acceleration versus time. Expect your acceleration versus time plot to be nearly worthless due to excessive noise.
Usually groups split the plot into two stages. Usually we see a runner gradually reaching max speed then running at roughly constant speed for the rest of the time.
During the first part (getting up to max speed):

- We can assume the initial acceleration stage has, very roughly, constant acceleration. During the acceleration stage
    - $x = \frac{1}{2}at^2 = \frac{F}{2m}t^2$
    - $v = at = \frac{F}{m}t$
    - In these equations, F is the magnitude of the average force the runner exerts on the ground m is the mass (not weight) of the runner.
    - Recall, by Newton's third law, the force the *runner* exerts on the *ground* is equal in magnitude (and opposite in direction) to the force the *ground* exerts on the *runner*.
    - Remember, using mass in kg should give force in N.
- You might notice the velocity plot is very roughly linear. We can model this stage of the experiment as constant acceleration. The slope of this approximately linear velocity plot gives you the average acceleration of the runner as they get up to speed. Use a trendline to get the slope (see Lab Manual Appendices, Sample Graph Type I). Determine the magnitude of the average acceleration and the average force the runner exerts on the ground.
- The position graph should be quadratic. Use a polynomial order 2 trendline because the theoretical position is $x = \frac{1}{2}at^2$. Show the trendline equation (and R^2 value) on the chart. Think: the coefficient on the squared term in the trendline is *not* acceleration but rather $\frac{1}{2}a$. Again determine the magnitude of the average acceleration and the average force the runner exerts on the ground.

During the second part (@ max speed):

- The velocity should be approximately constant. A constant velocity should look like a horizontal line (slope of zero). Notice the constant velocity section corresponds to zero slope on the vt-plot (zero acceleration). Use a trendline to get the slope (see Lab Manual Appendices, Sample Graph Type I). Verify it is at least close to zero. Discuss with your instructor as this part can be a bit subjective.
- The position graph should be *linear*. Think: after you reach max speed $\Delta x \approx v_{max}t$. Show a *linear* trendline equation (and R^2 value) on the chart. The slope should be v_{max}. Hopefully this compares well with the speed shown on the velocity versus time plot.

Put it all together (both stages on a single plot):

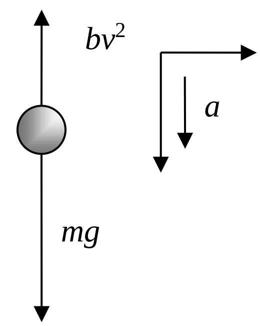
- Show the position versus time plot. Indicate to the class when stage 2 starts. Discuss when the plot should be concave versus linear. Animate in the values you found for a (from slope of vt-plot stage 1) and v_{max} (from slope of xt-plot stage 2) for the appropriate stages.
- Show the velocity versus time plot. Indicate to the class when stage 2 starts. Animate in the value you found for a (from slope of vt-plot stage 1) and the horizontal line $v = v_{max}$ using v_{max} from slope of xt-plot stage 2.
- You should now consider the acceleration plot. You could try to split the acceleration data into the same two stages. It will look pretty noisy, but perhaps you can notice a fain trend. Initially stage the acceleration should be slightly positive change to approximately zero (on average).

Summarize your results for v_{max}, a, and F. Think we ignored air resistance; discuss if this is reasonable. Do a web search for high performance track stars. Compare her or his numbers for a or v_{max} to yours. Perhaps you could find a chart showing max speeds and/or accelerations of various animals, motorcycle, etc and properly cite?

Options 2a & 2b Notes:
Do several practice trials before filming. Ideally you want to have a clean release and watch your object fall straight down. For example, with the coffee filters you might use two hands. For either case you might design an electromagnetic release using a nail, some copper wire, a battery, and a washer. The washer could hold the item of interest onto the electromagnet. Once you disconnect the wire for the magnet the washer will fall and thus allow the item of interest to fall. Spend a little bit of time on this but not too much.

Get time, position, velocity, and acceleration data for your experiment. Cut and paste that date from Tracker to Excel. Create an additional column of data for theoretical velocity. The rest of this page gives you background information on air resistance theory. The next page gives practical information on how to make a theoretical model.

Air resistance theory: Drag being modeled by $R = bv^2$ is usually considered valid for high speeds. Drag being modeled by $R = \beta v$ usually applies for very small objects at very low speeds. While neither model is perfect for our experiments, both models provide a rough approximation exhibiting the main qualitative feature of drag (asymptotic approach to terminal velocity).



A dropped ball of mass m experiences drag given by $R = bv^2$. An FBD showing the forces and coordinate system are shown at right. The equation of motion is determined by

$$mg - bv^2 = ma$$

We know when $a = 0$ the ball has reached terminal velocity given by

$$\boldsymbol{v_T = \sqrt{\frac{mg}{b}}}$$

Separating the variables in the equation of motion gives

$$\frac{b}{m}dt = \frac{dv}{v_T^2 - v^2}$$

Note: we expect $v < v_T$ for all $t > 0$.

I found

$$v(t) = v_T \frac{\left(\frac{v_T + v_i}{v_T - v_i}\right) - e^{-\frac{2bv_T}{m}t}}{\left(\frac{v_T + v_i}{v_T - v_i}\right) + e^{-\frac{2bv_T}{m}t}}$$

For a <u>dropped</u> ball use $\boldsymbol{v_i = 0}$. This simplifies the previous result dramatically:

$$v(t) = v_T \tanh\left(\frac{bv_T}{m}t\right)$$

Note that $v > 0$ for all t; this makes sense as I rotated my coordinates such that down was positive.

Since we are using a tracker video, the first frame of the video might not correspond to $t = 0$. It is sensible to introduce a shift in the time coordinate giving

$$\boldsymbol{v(t) = v_T \tanh\left[\frac{bv_T}{m}(t + \Delta t)\right]}$$

where Δt is the delay time between the release of the ball and the first usable frame of the video.

Note, the term b is theoretically given by

$$b = \frac{1}{2}D\rho A$$

where D is a dimensionless number called the drag coefficient, ρ is the density of the fluid the object is passing through, and A is the cross-sectional area of the object.

For a cotton ball moving through air we expect the following: $\rho = \rho_{air} = 1.2\,\frac{\text{kg}}{\text{m}^3}$, a cotton ball ($r \approx 3\text{cm}$) has approximate cross-sectional area $A = 3 \times 10^{-3}\text{m}^2$. A perfectly smooth sphere has a $D = 0.5$ while rougher surfaces can have numbers as high as 2. I will assume $D = 1.5$ for a cotton ball as surface roughness is significant. This gives

$$b = \frac{1}{2}D\rho A = \frac{1}{2}\left(1.5 \times 1.2\,\frac{\text{kg}}{\text{m}^3} \times 3 \times 10^{-3}\text{m}^2\right) \approx 3 \times 10^{-3}\,\frac{\text{kg}}{\text{m}}$$

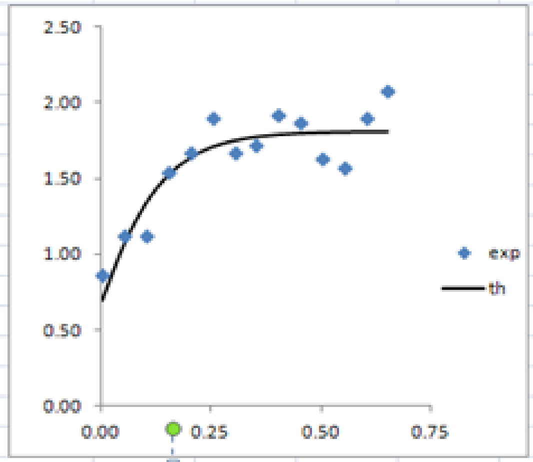
For a single coffee filter a similar analysis can be done. My web research indicated $b \approx 9 \times 10^{-3}\,\frac{\text{kg}}{\text{m}}$.

Theoretical velocity details:

1. Create a table like the one shown at right.
2. Input your best guess for b at this point. See bottom of the previous page for best guess tips.
3. Use an Excel formula to compute cell **D3** from cells **A3**, **B3**, & **C3**. Use the first bold formula on the previous page.
4. Use an Excel formula to determine cell **E3** from **A3** and **B3**.
5. Cell **C6** is computed using the formula shown at the bottom of the figure. This is based on the second bold formula on the previous page.

	A	B	C	D	E	F
1	Mass	Drag coeff.				
2	m (kg)	b (kg/s)	g (m/s^2)	v_T (m/s)	b/m (s^{-1})	Δt (s)
3	0.0010	0.003	9.8	1.8	3.00	0.075

	A	B	C
5	t (s)	v_{exp} (m/s)	v_{th} (m/s)
6	0.00	0.87	0.70
7	0.05	1.14	1.07
8	0.10	1.13	1.34
9	0.15	1.55	1.52
10	0.20	1.68	1.63
11	0.25	1.91	1.70
12	0.30	1.68	1.75
13	0.35	1.73	1.77
14	0.40	1.93	1.79
15	0.45	1.88	1.80
16	0.50	1.64	1.80
17	0.55	1.58	1.80
18	0.60	1.91	1.81
19	0.65	2.08	1.81

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=$D$3*tanh($e$3*$d$3*(a6+$f$3))
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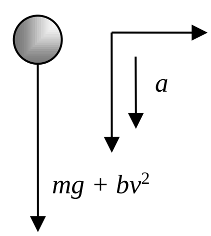
6. Fill down the equation to compute the theoretical column of data (v_{th}).
7. Plot v vs t. Include both v_{exp} and v_{th}. See Lab Manual Appendices, Sample Graph Type II for formatting.
8. Adjust the values of both Δt and b so the graph matches as close as possible.
   - Adjusting Δt should shift the graph left or right. It makes sense that you will need to do this since your object is probably already moving in the first usable frame of your video.
   - Adjusting b changes both the terminal velocity and the rate at which the object approaches terminal velocity. This is essentially adjusting the height of the horizontal asymptote and the sharpness of the bend of the curve.
   - Make an additional column computing the square of the difference between v_{exp} and v_{th}. Sum this column. By minimizing this sum, you are finding the best curve fit (best value for b)!
9. Notice the numerical value of the horizontal asymptote shown on the graph. Also look at the constants listed above the data table. Which constant relates to the asymptote? Does this make sense? Explain.

Going Further: Now that you know a value of b, you could make similar theoretical lines for both $x(t)$ and $a(t)$ using the above equations. Take the derivative of $v(t)$ to obtain $a(t)$. You would need to integrate of $v(t)$ to obtain $x(t)$. See how close theory matches the experiments on each of those graphs by repeating the above procedure. It might be challenging as well. You might need tables, Mathematica or Maple to do the math.

Last note: For a ball *thrown upwards* with $R = bv^2$ you do NOT obtain the same solution for $v(t)$. The equation of motion is determined by

$$mg + bv^2 = ma$$

Since one of the signs has changed it no longer gives the same result upon integration! This sign change gives a tangent function instead of hyperbolic tangent. Also, this situation will not make sense if $v_i = 0$. Try it out and see what you get! The result is easily found with an internet search. I used "ball thrown vertically with air resistance" as my search term. Furthermore, solutions to these types of problems are even messier if the initial velocity is actually *greater* than terminal velocity…



Alternative Coffee Filter Style (avoids the tanh function but requires 5 tracker vids)
Do one coffee filter experiment as described previously. Plot position versus time, velocity versus time, and acceleration versus time**.**

Instead of making the theoretical model described on the previous page, repeat the coffee filter experiment several times. First repeat the experiment using two nested coffee filters. Repeat the experiment 3 more times using 3, 4, and 5 nested coffee filters.

<u>Do not make plots for each extra trial.</u> Instead determine terminal velocity (v_T) for each extra trial.
Plot v_T versus m.
Also plot and v_T^2 versus m.

If drag is modeled by $\boldsymbol{R = bv^2}$ the force equation gives

$$mg - bv^2 = ma$$

We know when $a = 0$ the ball has reached terminal velocity given by

$$v_T = \sqrt{\frac{mg}{b}}$$

$$\boldsymbol{v_T^2 = \frac{mg}{b}}$$

If the plot of $\boldsymbol{v_T^2}$ versus $\boldsymbol{m}$ is linear we know $R = bv^2$ is a good model for drag. The slope is $\frac{m}{b}$ and $b = \frac{m}{slope}$.
Compare the experimentally determined value of b to my prediction of $b \approx 9 \times 10^{-3}\,\frac{\text{kg}}{\text{m}}$ from web research using a percent difference.

If drag is modeled by $\boldsymbol{R = \beta v}$ the force equation gives

$$mg - \beta v = ma$$

We know when $a = 0$ the ball has reached terminal velocity given by

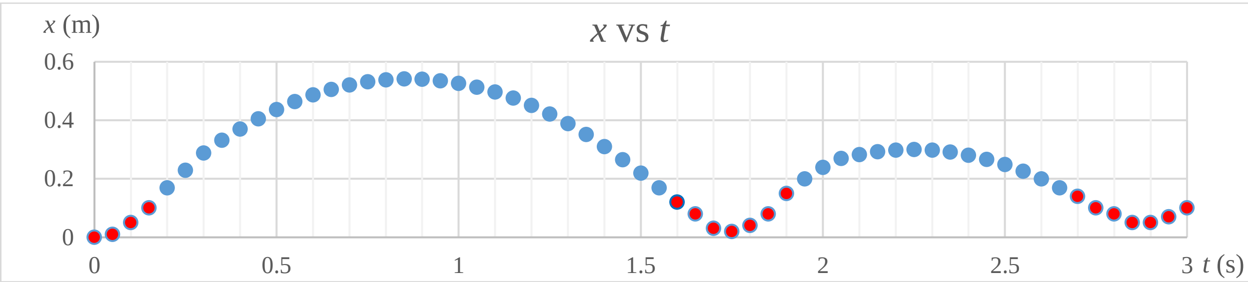
$$\boldsymbol{v_T = \frac{mg}{\beta}}$$

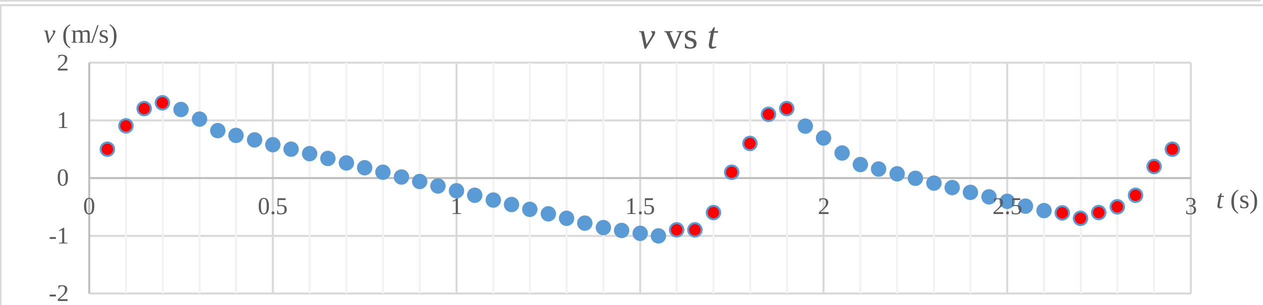
If the plot of $\boldsymbol{v_T}$ versus $\boldsymbol{m}$ is linear we know $R = \beta v$ is a good model for drag. The slope is $\frac{m}{\beta}$ and $\beta = \frac{m}{slope}$.
Determine the value of β.

Show the class both plots and tell us which model is better. Be sure to include a trendline. Tip: the R^2 value on a trendline indicates which plot is more linear. The closer R^2 is to one, the better the fit. Also, you could use the LINEST function in Excel to get an estimate of statistical error for your value of b or β. The Lab Manual Appendices have a help file on how to use the LINEST function. I'm pretty stoked to see the results as I've never done this one!

Option 3 Notes:
The cart should bounce several times. Note: I like to start with the spring compressed heading up the track. If you do the experiment this way you know the spring will never bottom out.

The shapes of the position versus time and velocity versus time plots should look similar to the ones shown below. Note: I made up numbers randomly so expect yours to be very different. The red dots indicate times when the spring is in contact with the cart.





Once you have your real data from tracker, cut and paste time, position, velocity, and acceleration data into Excel. Plot position vs time, velocity versus time, and acceleration versus time.

Now make an extra copy of the entire data set on a second sheet. Cut up the extra set of data into stages where the spring is or is not in contact with the cart. Perhaps go frame by frame in your tracker video to determine the best times to split the plot into stages? Or mouse over the data points in your plot?

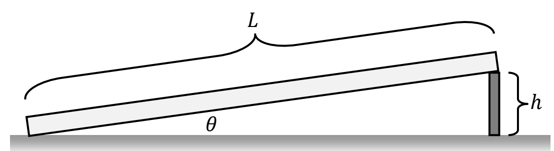
For each stage make a vt-plot showing a linear trendline equation with R^2 coefficient. See Lab Manual Appendices, Sample Graph Type I for formatting tips. Think about what each number in the trendline means. Do the numbers seem reasonable? When you put your plots up in front of the class, that is what I will be thinking…

For each stage when the spring is NOT in contact make an xt-plot, show a quadratic trendline equation (polynomial, order 2) with R^2 coefficient. Think about what each number in the trendline means. Do the numbers seem reasonable?

More on the next page…

There are several major things you should be able to learn from your plots:

1) Determine the *average* force exerted by the spring while it is in contact with the cart.
    a. Sketch an FBD while spring in contact. Force up the ramp (average spring force) minus force down the ramp ($mg \sin\theta$) should equal ma. (Ask your instructor for help here.)
    b. Rearrange equation to solve for average spring force.
    c. Get mass of cart from a balance and acceleration from slope of vt-plot. Be sure to use a stage where the spring is in contact with the cart!
    d. Plug in numbers to get spring force.
    e. Expect average spring force to decrease with each bounce as spring compresses slightly less after each stage due to energy losses in system.
2) Compare acceleration of cart not in contact with spring to theoretical prediction
    a. Sketch FBD while not in contact. Ignoring friction and drag, the force down the ramp ($mg \sin\theta$) should equal ma. This gives $a_{th} = g \sin\theta$.
    b. Get the angle of your ramp by measuring the length of the ramp and the height of whatever block you used to lift it. You'll want to double check this with an angle indicator or an app on your phone.



    c. Compare a_{th} to the slope of your vt-plots (for stages not in contact with spring).
    d. Think: should this be the same after each bounce or different after each bounce? If it should be the same, get an average of your experimental a's and compare to your theory value with a percent difference.
3) Going Further: Determine the Coefficient of Restitution
    a. Coefficient of restitution (COR) is given by

$$COR = \sqrt{\frac{\text{energy } after \text{ collsion}}{\text{energy } before \text{ collsion}}} = \sqrt{\frac{mgh_2}{mgh_1}} = \sqrt{\frac{h_2}{h_1}}$$

    In this equation h_1 is the max height of the cart *before* and h_2 is max height *after* each collision (when the spring on the cart is compressed).
    b. Look at the position versus time plot for the entire time (not split into stages)
    c. Determine the COR between each stage using the bold formula above.
    d. Get an average value of the COR.
    e. Do a web search for "ball bouncing in slow motion". Show no more than two videos. Keep the total time for videos as short as possible…less than a minute in total including these videos and any tracker videos you show.

If you need more crap to fill time: Compare the area under the vt-plot to displacement on the xt-plot. Calculate the area under each vt-plot triangle with units. Estimate the displacement from these areas. Be sure to include the signs. Then, show how these displacement numbers match up to your xt-plot.

Since this talk has so many neat things it is crucial to finish your talk with a clear summary. Think carefully key takeaways you wish to emphasize in your summary slide. Afterwards, revise your initial goals to match well with your summary slide. If desired you can ask for an extra minute of time if you are doing a lot of cool stuff.

Option 4a-4c notes: **I USUALLY ALLOW THESE GROUPS AN EXTRA 2 MINUTEs TO TALK.**
FBDs are optional for 4a-c, I'd rather you focus on the stuff mentioned in "1D Motion Tips" on about pg 39.
First part is for 4a and 4b only: Record the length of the unstretched spring before anything is attached to it. Next, attach your object (hanging mass or cart on ramp) and allow the system to reach equilibrium. Record the distance the spring *stretches* between equilibrium. WATCH OUT: the *length* of the stretched spring is not exactly the same as the amount the spring *stretches*. The amount the spring stretches should be the difference between the stretched length and the unstretched length. The forces balance at equilibrium. This information helps you determine the theoretical period in the table shown below. Deriving these equations is not my intention for this lab. If you want to see the derivations, discuss with your instructor after you have acquired data.

For 4a	**For 4b**	**For 4c**
$mg = k\Delta x_{eq}$ which implies $k = \frac{mg}{\Delta x_{eq}}$ Use this value of k to determine the theoretical period $\boldsymbol{T_{th} = 2\pi\sqrt{\frac{m}{k}}}$	$mg\sin\theta = k\Delta x_{eq}$ which implies $k = \frac{mg\sin\theta}{\Delta x_{eq}}$ Use this value of k to determine the theoretical period $\boldsymbol{T_{th} = 2\pi\sqrt{\frac{m}{k}}}$ Notice $\sin\theta$ is NOT in the period equation!	$x_{eq} = 0$ Theoretical period is given by $\boldsymbol{T_{th} = 2\pi\sqrt{\frac{L}{g}}}$ where L is the length of your pendulum (to the *center of the ball*).

Make a video that includes *at least* one full oscillation but *preferably* two full oscillations.
To simplify things, put the origin of your Tracker coordinate system at the <u>equilibrium position</u> of the ball.
Copy your time, position, velocity, and acceleration data from Tracker and paste it into Excel.
Make an extra columns of *theoretical* data for position, velocity, and acceleration. Practical instructions for this are found on the next page. You should then be able to make a second set of xt, vt, and at plots showing both theoretical and experimental data at the same time for easy comparison. For formatting tips see Lab Manual Appendices, Sample Graph Type II.

Suggested talking points are listed below. You do not need to use *every* talking point.

- Define of amplitude, period and angular frequency. Show how to determine A and T from an xt-plot.
- At equilibrium forces balance (net force of zero). Describe which regions of motion should exhibit *positive* net force (and thus acceleration). Relate this to the concavity on your xt-plots and the slopes on your vt-plots. Repeat all of this analysis for regions with *negative* net force. **Tip:** if using a spring, should the spring force is dominate the force equation at small or large stretch?
- At max or min position the *velocity* is zero but force (and thus acceleration) *magnitude* is maximized. Relate this to slope of xt-plot and value of v on vt-plot.
- At equilibrium position *force* (and acceleration) are zero but *speed* is a max (*velocity* is max or min). In addition, you were told to set the equilibrium position at the origin. Show the class the max speed is a points on the xt-plot by emphasizing the steepness of the slope each time the object passes equilibrium.
- Show the xt-plot. Ask the class to determine when the object is *moving forwards* and *slowing down*. Explain one can get the sign of v from the sign of the slope and the sign of a from the concavity. Now ask, "When is the object *moving backwards* and *speeding up*?"
- Show the vt-plot. Ask the class to determine when the object is moving forwards and speeding up. Show them the answer and explain how one can tell using the sign of v and the slope of vt-plot. Repeat by asking them when the object is moving backwards and slowing down. If they aren't getting it, do it again.
- Show all three plots first without theory data then with theory data superimposed.

Think carefully about your summary slide. What are the key takeaways you want to emphasize to the class? Make your summary slide then rework your goal slide so it matches up well with your summary. Because this particular experiment is such a good teaching example, **remind your instructor to give you an extra 2 minutes to talk and point out this sentence to her or him. Remind them just before your talk as well.**

Oscillations: SET THE ORIGIN OF YOUR TRACKER COORDINATE SYSTEM AT EQUILIBIRIUM.
Assuming you set the origin of your Tracker vid at the equilibrium position the theoretical equations of motion are

$$x_{th}(t) = A\cos(\omega t + \phi)$$
$$v_{th}(t) = -\omega A\sin(\omega t + \phi)$$
$$a_{th}(t) = -\omega^2 A\cos(\omega t + \phi)$$
$$\text{where } \omega = \frac{2\pi}{T_{th}}$$

In these equations A is the amplitude, ω is angular frequency (units are rad/sec but I think of them as RPMs), ϕ is a phase angle (shifts the starting point of the oscillation) and T_{th} is the theoretical period from the previous page.

The example shown below is for Option 4b. Row 1 indicates the data required to determine T_{th} (using equations from the table on the previous page). Row 4 allows you to input model parameters for amplitude and phase angle. .
If you used option 4a, you would not need the angle θ in the first row.
If you used option 4c, your first row would instead include L, g, T_{th} and ω.

For all three cases the rest of the analysis is nearly identical. The amplitude (A) is the amount you displaced the object from equilibrium prior to release. For now assume the phase angle (ϕ) is zero. Starting in row 8 you can see my fake Tracker data in columns A through D. I then used the theoretical position equations (above on this page) to generate columns E through G.

Finally, if needs be, you may need to adjust the phase angle. If you are wondering what the phase angle does, consider what happens to x_{th} the when $t = 0$.

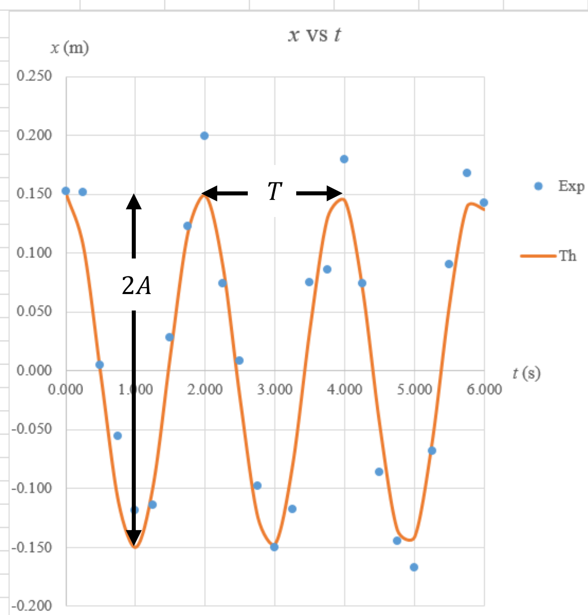
$$x_{th}(0) = A\cos(\phi)$$

The phase angle determines the initial position of the theoretical oscillation!!! It is appropriate to adjust the phase angle to ensure your theoretical equations line up as well as possible with the experimental data. This is an artifact of missing a few frames of video or modifying your coordinate system in Tracker. It is not appropriate to adjust the amplitude or period as those parameters are predicted by physics.

m (kg)	g (m/s^2)	θ (deg)	θ (rad)	Δx_{eq} (m)	k (N/m)	T_{th} (s)	ω (rad/s)
0.500	9.80	30.0	0.524	0.475	5.16	1.96	3.21

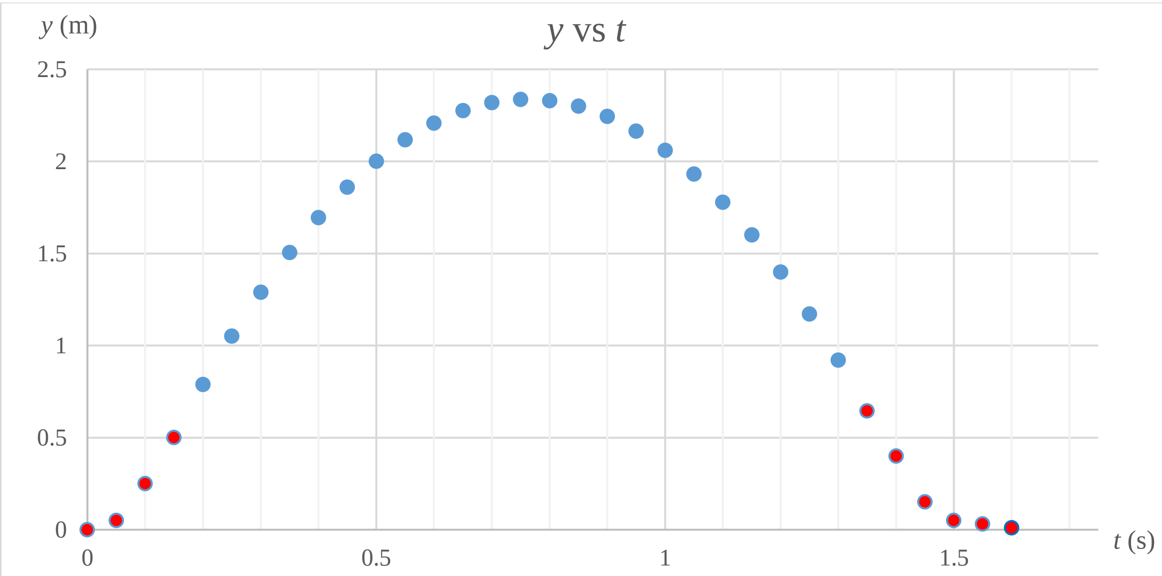
A (m)	ϕ (rad)
0.150	0

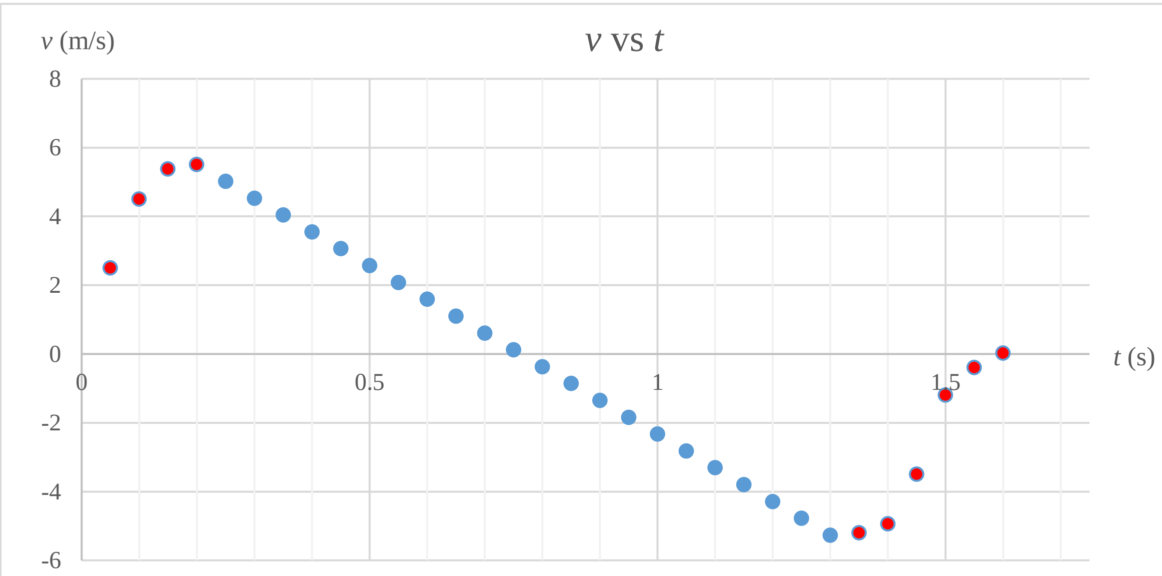
t (s)	x (m)	v (m/s)	a (m/s^2)	x_{th} (m)	v_{th} (m/s)	a_{th} (m/s^2)
0.000	0.152			0.150	0.000	0.232
0.250	0.151	-0.294		0.104	-0.347	0.161
0.500	0.005	-0.415	0.093	-0.005	-0.481	-0.008
0.750	-0.056	-0.248	0.597	-0.112	-0.322	-0.173
1.000	-0.119	-0.116	1.083	-0.150	0.034	-0.232
1.250	-0.114	0.294	1.180	-0.096	0.369	-0.149
1.500	0.028	0.474	0.098	0.016	0.479	0.024
1.750	0.123	0.343	-1.146	0.118	0.296	0.183
2.000	0.199	-0.099	-1.447	0.149	-0.067	0.230
2.250	0.074	-0.381	-0.489	0.088	-0.390	0.136
2.500	0.009	-0.343	0.127	-0.026	-0.474	-0.041
2.750	-0.098	-0.318	0.609	-0.124	-0.269	-0.193
3.000	-0.150	-0.039	1.536	-0.147	0.101	-0.227
3.250	-0.118	0.450	0.890	-0.079	0.409	-0.123
3.500	0.075	0.406	-0.483	0.036	0.467	0.056
3.750	0.086	0.209	-0.858	0.130	0.240	0.201
4.000	0.179	-0.023	-1.479	0.144	-0.134	0.223
4.250	0.074	-0.531	-0.829	0.070	-0.426	0.109
4.500	-0.086	-0.438	0.735	-0.047	-0.458	-0.072
4.750	-0.145	-0.163	1.180	-0.135	-0.210	-0.209
5.000	-0.168	0.152	1.359	-0.141	0.166	-0.218
5.250	-0.069	0.516	0.642	-0.061	0.441	-0.094
5.500	0.090	0.473	-0.824	0.057	0.446	0.087
5.750	0.168	0.104		0.139	0.179	0.215
6.000	0.142			0.137	-0.197	0.212



Option 5 Notes:
Throw the ball as close to perfectly vertical as possible.
Somehow try to release and catch the ball at the same height. Perhaps you could have your forearm bump into a stationary 2 by 4 as you are about to release the ball. The idea is to leave your forearm touching the board until the ball comes back down. Hopefully your hand is then in nearly the same spot as you release and catch the ball. I have no clue if this will work…feel free to try other ideas.
Try making several videos and pick the best one after several attempts.

Copy and paste your data for time, position, velocity, and acceleration from Tracker into Excel.
Make yt, vt & at plots. I expect they might look a bit like the ones shown below. Note: I totally faked this data just to give you an idea of the shape of the plot. Do not expect your numbers to be similar to mine. Notice I chose to color the dots differently for times the ball is in freefall versus touching the hand. Now read the next page.





There are several major things you should be able to do with your plots:

1) Copy your data and split your plots into two stages
    a. The first stage should include all times the hand is throwing the ball.
    b. The second stage should include all times the ball is in freefall.
    c. The third stage should include all times the hand is catching the ball.
    d. If you are confused about the exact transition times, assume the hand is still touching the ball.
2) Compare acceleration of stage two to freefall theory
    a. Use a *linear* trendline on the stage two vt-plot. Think about what each number in the trendline means. Do the numbers seem reasonable? When you put your plots up in front of the class, that is what I will be thinking…
    b. Use a quadratic trendline on the stage two xt-plot. Again consider the numbers in the trendline: are they reasonable?
    c. Compare a_{th} (what you expect for freefall) to the slope of your vt-plots (a_{exp})…watch the signs!
    d. Use a percent difference to compare a_{th} to a_{exp} for this stage
3) Determine *average* force exerted by hand for stages 1 & 3
    a. While the hand is touching the ball Newton's 2nd Law tells us $F - mg = ma$. In this equation I am assuming F is the magnitude of the *average* force exerted by the hand.
    b. Solving this equation for F gives $F = m(g + a)$
    c. Notice the force exerted by the hand is NOT simply given by ma! Newton's second law says *net* force equals ma.
    d. Use trendline on stage 1 & 3 vt-plots to get values for a. Use a balance to get a value for m. Estimate the force of the hand during the throw and during the catch. Are they approximately the same? Did anything in the video indicate one should be larger than the other for any reason?
4) Use the entire experiment yt-plot (all three stages in single plot) to describe the motion
    a. Ask the class "Over what time intervals is the ball slowing down?" Wait thirty seconds for them to answer. Then explain the answer. The ball is slowing down whenever acceleration and velocity have opposite signs. The sign of acceleration is determined by the concavity of the yt-plot. The sign of the velocity is determined by the slope of the yt-plot.
    b. Now ask them when the ball is at rest. Explain the answer.
5) Use the entire experiment vt-plot (all three stages in single plot) to describe the motion
    a. Ask the class "Over what time intervals is the ball speeding up?" Wait thirty seconds for them to answer. Then explain the answer. The sign of acceleration is determined by the slope of the vt-plot. The sign of the velocity is determined by the value on the vt-plot.
    b. Now ask them when the ball is at rest. Explain the answer.

If you need more crap to fill time: Compare the area under the vt-plot to displacement on the xt-plot. Calculate the area under each vt-plot triangle with units. Estimate the displacement from these areas. Be sure to include the signs. Then, show how these displacement numbers match up to your xt-plot.

By the way: at some point you should give the definition of freefall. Remind students a ball thrown upwards is just as much in freefall as a dropped ball (as long as air resistance is negligible).

Since this talk has so many neat things it is crucial to finish your talk with a clear summary. Think carefully key takeaways you wish to emphasize in your summary slide. Afterwards, revise your initial goals to match well with your summary slide. If desired you can ask for an extra minute of time if you are doing a lot of cool stuff.

Option 6 Notes:

Ask for help if you want to include an FBD to fill time but it won't make any sense until chapter 10.

In theory a solid ball *rolling* on an incline has $a_{th} = \frac{5}{7} g \sin\theta$. We will derive this in chapter 10.

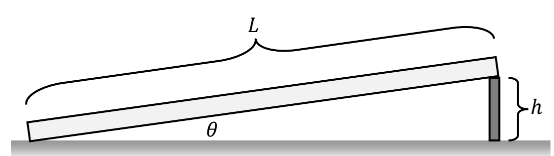
Please note this differs from a block *sliding* (with negligible friction) where $a = g\sin\theta$.

From experience you want to use a very small angle for this experiment. If the angle is too large, the ball moves quickly by the time it reaches the magnet. If the ball moves too quickly one cannot observe any effects caused by the magnet.

Try to use the smallest angle possible. If you have the option, try to use slow motion. Note: if you use slow motion video, release the ball much closer to the magnet to avoid excessive data and a painful tracking experience. Whichever style you choose (slo-mo vs regular speed), ensure you start the ball far enough from the magnet such that it travels for at least 20 frames of video before the magnet has any effect.

Copy and paste your data for time, position, velocity, and acceleration from Tracker into Excel.
Make xt, vt & at plots. There are several major things you should be able to do with your plots:

1) Estimate the range of the magnet
    a. At some time near the end of the motion the slope of the vt-plot should spike up. This should help you identify the time at which the magnet starts to affect the ball's motion.
    b. Use this time information with your xt-plot to estimate the distance between the ball and the magnet at this same instant in time. This is the max range of the magnet.
2) Copy your data and split your plots into two stages (graphs on option 5 are *similar* but not identical)
    a. The first stage should include all times the magnet appears to have no effect
    b. The second stage should include all times the magnet appear to have some effect
    c. If you are uncertain about a point near the transition time, include it in the second stage.
3) Compare acceleration of stage 1 to theoretical prediction
    a. Get the angle of your ramp by measuring the length of the ramp and the height of whatever block you used to lift it. Feel free to double check this with an angle indicator or an app on your phone but to get more than 1 sig fig you need to use the trig…



    b. Compare $a_{th} = \frac{5}{7} g\sin\theta$ to the slope of your vt-plots for times before the magnet has any effect. Use a percent difference to do the comparison. Usually this number is way off. I suspect it is easy to make small errors in the angle which cause huge % differences…
4) Estimate the *average* force exerted by magnet
    a. Once close the magnet the forces down the plane have magnitudes $mg\sin\theta$ and F. Here F is the magnitude of the *average* magnetic force.
    b. It can be shown (using Chapter 10 methods) a constant magnetic force acting at the center of the ball changes theoretical acceleration equation to $a = \frac{5}{7}\left(g\sin\theta + \frac{F}{m}\right)$.
    c. Rearrange the previous to show $F = m\left(\frac{7}{5}a - g\sin\theta\right)$.
    d. Get mass of ball from a balance and acceleration from slope of vt-plot. Be sure to use a stage where magnet is actually close enough to have an effect!
    e. Plug in numbers to get the magnitude of the average magnetic force.

Make a summary slide tying all this crap together. Think about what you most want students to learn. Once the summary slide is made, revise your goal slide to match.