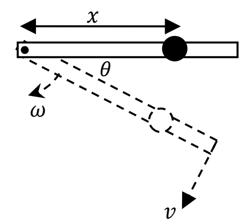
# Rotation Madness

**Apparatus:** Enough materials for two copies each of options 3 & 5, one copy each of options 1, 2, & 4




**OPTION 1:** Create a system wherein a rod (meter stick) is free to pivot about one end of the rod. You should be able to hold the rod parallel to the ground and release it from rest. The rod should be able to swing down until the rod is vertical. Furthermore, you should be able to securely affix a point mass to the rod at an arbitrary location. The variables used in this option are rod length ($L$), rod mass ($m$), point mass ($M$), point mass distance from pivot ($x$), and the angle of the rod from *horizontal* ($\theta$). Notice this is not the usual angle we use. I think it will be easier to understand your experimental data if we use this angle. Note: I chose to call this angle positive ($\theta > 0$) in my derivation. I also set the *initial* height to zero. Together these imply you should get negative heights for all $\theta < 180°$. Finally, if you make the point mass an integer multiple of the rod mass, say $M = 2m$, the math will be a lot cleaner.

You should be able to drop the rod with the point mass at various locations. For each location, use a rotary motion sensor to record $t$, $\omega$, and $\theta$ data. I recommend using $x = 10.0, 20.0, \ldots 100.0$ cm. If time permits, get a few extra data sets at $x = 15.0$ & $25.0$ cm as the graph is usually most interesting there.

Note: if the point mass is too close to the axis of rotation, it is no longer sensible to consider it as a point mass! Doing a little math one can show that treating a sphere as a point mass introduces less than 1% error when the sphere is positioned more than 3.2 diameters from the axis of rotation. How can you use this information? First determine the size of your point mass with a ruler. Ensure your point mass is at least four times this size from axis for all *x* values you use in the experiment.

**GOALS:**

1) Is angular acceleration constant for a swinging system?
2) Do energy methods accurately predict the rotational motion
    a. dependence of the rotation rate upon angle
    b. rotation rate at bottom of swing
    c. position of point mass which produces the fastest rotation

**Understand the problem theoretically for arbitrary angle**

1. Determine an expression for the center of mass position ($r_{\text{CM}}$) of the system at an arbitrary angle $\theta$. Answer in terms of $L$, $m$, $M$, $x$, and $\theta$.
2. Determine the moment of inertia of the rod in terms of $L$, $m$, $M$, and $x$.
3. Derive the theoretical angular velocity when the rod is some arbitrary angle $\theta$ from the vertical. Answer in terms of $L$, $m$, $M$, $g$, $x$, and $\theta$. Use energy methods.
4. Derive the theoretical angular acceleration in two ways
    a. Take the derivative of your theoretical expression for $\omega$. You'll need to use the chain rule on any terms involving $\theta$.
       For example: $\frac{d}{dt}\sin\theta = \cos\theta\left(\frac{d\theta}{dt}\right) = \cos\theta\,(\omega)$.
    b. Do it again using a torque problem. Hint: don't forget that the gravitational force is $(M + m)g$ and it is applied at the center of mass!
    c. Is the use of the instantaneous pivot is applicable in this case? Explain.
5. Determine an expression for the rotation rate at the bottom of the swing (assuming the rod was released from rest while parallel to the ground). Write your expression in terms of $L$, $m$, $M$, $x$, and $g$. Take an appropriate derivative to determine the value of $x$ which gives max speed at the bottom of the swing.
6. **For <u>each</u> value of $x$**, use a rotary motion sensor to obtain $\theta, \omega, \alpha,$ & $t$ data.

**GOALS:** (consider showing goal slide multiple times to help people stay on track in your talk)

1) Is angular acceleration constant for a swinging system?
2) Do energy methods accurately predict swinging motion
    a. dependence of the rotation rate upon angle
    b. rotation rate at bottom of swing
    c. position of point mass which produces the fastest rotation

Maybe discuss the procedure briefly then immediately revisit your goals so everyone is clear on what you are trying to do. Maybe this is a good time to ask the class to make a guess…when people guess they care more about finding out the answer and will listen better. Note: in any procedure show a picture of your equipment with something in picture for scale. Fortunately, your apparatus itself is a perfect way to show the scale of the device…

Then do the FBD/torque problem for arbitrary angle. Use this result to derive the angular acceleration as a function of angle.

To discuss point 1, show plots of angular displacement versus time and angular velocity versus time.

- Think: the slope of one of these plots gives you your answer. The concavity of the other plot gives you a double check.
- Think about the bottom of the swing…what is tangential acceleration there? What does this imply about angular acceleration at the bottom of the swing?
- Think: when should the slope of the angular velocity plot be steepest?
- According to your coordinates, when should the signs on each plot be positive/negative?
- Think about the signs and slopes of the angular displacement plot compared to the angular velocity plot.

Now do your energy problem derivation.

To discuss point 2a, create a plot of $\omega$ vs $\theta$.

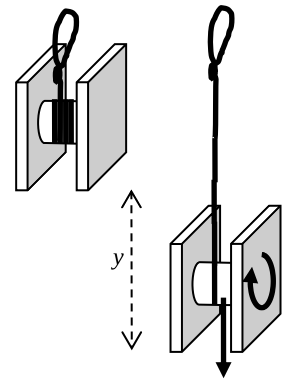
- You should be able to create a table of theoretical values using step 3 from the previous page.
- You should have experimental data from tracker.
- Show the theory as a smooth line and the experimental points as dots. Include theory equation on plot.
- Ask for ideas about getting a numerical value to describe the agreement of experiment to theory.

To discuss 2b and 2c, create a plot of $\omega$ vs $x$.

- You should be able to create a table of theoretical values using step 3 from the previous page.
- You should have experimental data from photogate measurements.
- Show the theory as a smooth line and the experimental points as dots. Include theory equation on plot.
- Ask for ideas about getting a numerical value to describe the agreement of experiment to theory.
- Point out to the class which value of $x$ is predicted to cause the fastest rotation at the bottom of the swing.

Before giving your talk, remind yourself of all assumptions you made in your theoretical calculations.
Quantify your % error and compare it to a % difference. Qualitative agreement? Quantitative agreement?

**OPTION 2:** Predict $v$ vs $y$ and $a$ for various yo-yo's as they fall in terms of its mass ($m$), center-of-mass moment of inertia ($I_{\text{CM}}$), $g$, and the spindle radius ($r$). Start with a yo-yo from rest. Record a video and use Tracker to collect $t$, $y$, and $v$ data so various plots can be made. Repeat the experiment with different yo-yo's. **I'm thinking three yo-yo's plus two solid cylinders (with different diameters) and/or two thin walled pipes (with different diameters).**




**GOALS:**

1) Do energy methods accurately predict speed versus displacement for falling yo-yo?
2) Is an unraveling yo-yo accurately modeled as rolling without slipping?
3) For two yo-yo's with equal moment of inertia, how does spindle size affect acceleration?

**Understand the problem theoretically for fall distance $y$**

1. Record the dimensions and masses of all different yo-yo's, cylinders, and pipes. You might also take a photo of all of them side by side for your talk. You can number them in this photo to make it easier to discuss things at the end of your talk. Include a ruler for scale in any photo. Note: from this info you can generate values of the moment of inertia for each object using the tables in your workbook. These moments of inertia can then be used to predict the theoretical acceleration (or velocity) of each yo-yo.
2. Derive the theoretical velocity after falling a distance $y$ using an energy problem. Use $I_{\text{CM}}$ for the moment of inertia (instead of your result for part 1) to keep your work simpler.
3. Derive the theoretical *translational* acceleration using a torque problem. Be sure to distinguish between $I_{\text{CM}}$ and $I_{\parallel}$.
   a. Do this once using the center of mass as the pivot point.
   b. Do this again using the instantaneous pivot point.
   c. Do it a third time using your result from part 1 and kinematics. Note: you will <u>not</u> need to explain 2c in your talk but it is definitely worth doing for exam practice and general understanding of your situation.
4. Take a video of each object falling. Record a video and use Tracker to collect $t$, $y$, and $v$ data so various plots can be made. Note: the cylinders and pipes will probably be quite sensitive to the way the string is wound. Try to get the string wrapped up near the center of mass. Try a few times to get a decent video for the pipes and cylinders then move on.
5. On the big wooden yo-yo, check the mass of the string with a balance…it may not be negligible. We will still assume it is but this may help explain discrepancies later.

In your talk you always want your goal slide first.
Then explain the procedure. Include a picture of your various yo-yo's including a ruler for scale.
Mention how you got your theoretical moments of inertia.
Don't go into gory detail on the math…do list final numerical results? Then revisit and beef up the goals…
**GOALS REVISITED:**

1) Do energy methods accurately predict speed versus displacement for falling yo-yo?
2) Is an unraveling yo-yo accurately modeled as rolling without slipping?
    a. Should the acceleration magnitude be more than, less than, or equal to $g$ as it falls? Why?
    b. Should the acceleration of a falling yo-yo be constant as it falls? Why or why not?
3) For two yo-yo's with equal moment of inertia, how does spindle size affect acceleration? Why?

After revisiting goals, show energy derivation.
To discuss point 1, show plots velocity versus fall distance.

- Your work for part 2 on the previous page should help you tabulate theoretical data.
- Your Tracker videos should give you experimental data.
- Show the experimental data as dots and the theory as smooth lines. Include theory equation on plot.
- Might start with one case and show clearly. Then show all cases on same plot if easy to color code. May need to be creative to get through these quickly…we can discuss options once plots are made.

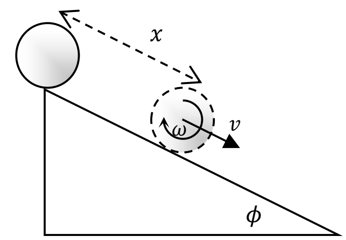
Now show torque derivation using both styles (see parts 3a and 3b on previous page).
Consider showing goal slide again after derivation. The torque derivation for the acceleration should tell you the answers to goals 2a, 2b, and 2c. Consider pointing this out the class at this time.

Use your results to create a table of predicted accelerations for each yo-yo/cylinder/pipe. Show this on a slide.
Then show a $vt$-plot for a single yo-yo. Use a trendline to get the acceleration. Then animate in a line showing the theoretical acceleration on the same plot (ask if you need help).
Then show all yo-yo's on the same plot with experimental data as dots and theory data (<u>not</u> trendlines) as smooth lines of matching color.
Show a slide with a table comparing your experimental accelerations to theoretical accelerations. Include a percent difference.
Note: This comparison directly address Goal 2a.
Note: if the slopes are constant for each $vt$-plot that addresses Goal 2b.

Finally, address Goal 3 by comparing two yo-yo's with nearly identical mass but different spindle radius. Your derivation of *translational* acceleration from part 3 on previous page should help answer which should accelerate faster. Then make a $vt$-plot with data from those two yo-yos. Hopefully the steeper slope matches what theory suggests!

**OPTIONAL:** Use your technique to predict the moment of inertia of yo-yo's of more complicated design. In particular, use your system to find the moment of inertia of a fourth yo-yo (black with rubber tubing on it). Use your previous work to estimate % uncertainty on your measurement of this oddball yo-yo.

Tie it together by revisiting the goal slide one last time and summarize how you answered each goal question.
Before giving your talk, remind yourself of all assumptions you made in your theoretical calculations.
Quantify your % error and compare it to a % difference. Qualitative agreement? Quantitative agreement?




**OPTION 3 (two versions means two groups):** Start the objects from rest. You will try at least 6 different objects. **One group uses 3 solid spheres (bowling ball plus 2 other sizes) and 3 spherical shells (basketball plus 2 other sizes). The other group uses 3 thin rings with differing radii and 3 sold disks with differing radii.**

Predict $v$ vs $x$ and for an arbitrary round shape (disk, ring, ball, etc) as it rolls down an incline. Compare to data from Tracker.
Also, use $v$ vs $t$ data from Tracker to experimentally determine $a_{exp}$ for each case. Use $a_{exp}$ to determine and experimental value of the moment of inertia $I_{exp}$ for each rolling object. Compare $I_{exp}$ to theoretical values predicted by formulas from a textbook.

**GOALS:**

1) Do energy methods accurately predict speed versus displacement for rolling objects?
2) In general, what factors determine which object gets downhill fastest? Mass, radius, shape, frictional coefficients, etc.

**Understand the problem theoretically:**

1. Get the mass and radius of each object. Take a photo of all together and number them. Use textbook formulas and this data to generate theoretical moments of inertia for each object. We won't need the gory details in the talk but we do need a number for each $I_{th}$ at some point. Note: for rolling objects we expect all moments of inertia take the form $I_{\text{CM}} = kMR^2$. Know the value of $k$ for each object…
2. Derive the theoretical velocity after rolling distance *x* using an energy problem. Use *I* for the moment of inertia to keep your work simpler. Note: for rolling objects we expect all moments of inertia take the form $I_{\text{CM}} = kMR^2$. *At the end of your derivation*, use this fact to clean things up.
3. Do a torque problem and solve for $I_{\text{CM}}$ in terms of the translational acceleration ($a$, not $\alpha$). Be sure to distinguish between $I_{\text{CM}}$ and $I_{\parallel}$. OPTIONAL: use $I_{\text{CM}} = kMR^2$ to clean up the formula. If you do this you get a prediction for $k$ in terms of $a$ and it should look a lot simpler.
   a. Do this once using the center of mass as the pivot point.
   b. Do this again using the instantaneous pivot point.
   c. Do it a third time using your result from part 1 and kinematics. Note: you will <u>not</u> need to explain 2c in your talk but it is definitely worth doing for exam practice and general understanding of your situation.
4. Take a video of each rolling object experiment. Use Tracker to get $t$, $x$, and $v$ data for making plots.
   **Think:** how could you use this information to also make $\theta$ vs $t$ and $\omega$ vs $t$ plots. I'm not sure if you *need* to do that, but understanding how you *might* do this is worth considering.
   **Think:** look back at the picture and you will notice I said the angle was $\phi$. Be ably to clearly explain what $\theta$ and $\phi$ mean in this experiment. If not sure…take a guess then check with me.
5. At some point you will need a precise value of the angle. Rather than use a protractor or angle indicator…use height and length of the ramp along with some trig to get the angle. That said, double check this number with a protractor or phone.
6. Make video of two different races to show class (ask who wins each race before starting vids). Could use this as a final slide to see who was listening?
   a. One of two objects with same shape but differing radii.
   b. One race with two objects of differing shape but same radii.
   c. Ask bonus question, in which case, if either, do objects have same rotation rate at finish line?

Before giving your talk, remind yourself of all assumptions you made in your theoretical calculations.
You'll want to emphasize this case is <u>not</u> on verge of slipping; that is why you had to do sum of forces in step 3a.
Emphasize objects with identical shapes have same translational speed/accel but not same rotational speed/accel.
Quantify your % error and compare it to a % difference. Qualitative agreement? Quantitative agreement?

**GOALS REVISTED:**

1) Do energy methods accurately predict speed versus displacement for rolling objects?
2) In general, what factors determine which object gets downhill fastest? Mass, radius, shape, frictional coefficients, etc.

Explain procedure.
Include picture showing each object with number. Include a ruler for scale.
Consider animating in $I_{\mathrm{CM}}$ and/or $k$ for each object?
Consider revisiting goal slide after explaining procedure to segue into energy problem? Remind us of goal 1?

Then do energy derivation slide.
To discuss Goal 1, show plots of $v$ vs $x$.

- Your work for part 2 on the previous page should help you tabulate theoretical data.
- Your Tracker videos should give you experimental data.
- Show the experimental data as dots and the theory as smooth lines. Include theory equation on plot.
- Might start with one case and show clearly. Then show all cases on same plot if easy to color code. May need to be creative to get through these quickly…we can discuss options once plots are made.
- Emphasize objects with identical shapes have same translational speed but not same rotational speed.

Consider a revisit to Goal Slide to remind us what we've learned and what is left to answer?

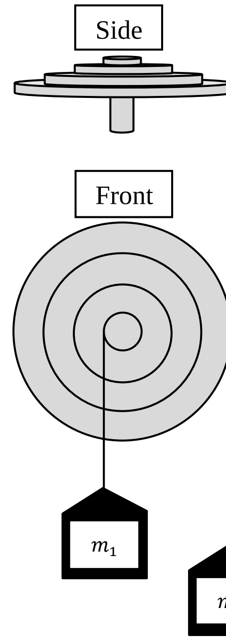
Do torque derivations (once with CM pivot and once with instantaneous pivot). Distinguish between $I_{\mathrm{CM}}$ and $I_{\parallel}$.
To discuss goal 3, first show plot of $v$ vs $t$ for single object.

- Use Tracker data to get plot.
- Add trendline to get slope (slope is the experimental translational acceleration, $a_{exp}$).
- Animate in algebraic equation from torque derivation onto this plot. It will either say $I_{\mathrm{CM}} = \cdots$ (or $k = \cdots$).
- Finally animate in a *numerical* value for $I_{\mathrm{CM}}$ (or $k$).

Summarize the entire process you just did to find $I_{\mathrm{CM}}$ (or $k$) for this one object.
Perhaps show the data for *all six* objects on one $vt$-plot?

Perhaps a final table summarizing theoretical values of $I_{\mathrm{CM}}$ (or $k$). Include pics of each object if space allows?
Include theoretical value, experimental value, and % difference.

Revisit the goal slide one last time and make sure you hit everything.




**OPTION 4:** Obtain one of the unusual pulleys that look the figure shown at right.
For now, assume the pulley is a uniform disk with center of mass moment of inertia $I_{\text{CM}}$. Two hanging masses $m_1$ and $m_2$ are attached at radii $r_1$ and $r_2$ respectively. Assume we may ignore *axle* friction for the pulley. Assume there is sufficient friction between the strings and *the edge of the pulley* such that the strings do not slip relative to the pulley as it turns. Finally, assume $m_2$ travels downwards.

NOTE: the strings are not wound up the same way!
NOTE: rather than tie knots, it may help to tape the strings to the pulley to make it easy to change radii.
NOTE: it may be useful to let $m_2 \approx 15$ g and $m_1 \approx 10$ or 12 g. You may need to tape together custom hanging masses. If too light, try $m_2 \approx 50$ g and $m_1 \approx 20$ g or $m_2 \approx 200$ g and $m_1 \approx 100$ g?

Start with $m_2$ on the outer radius and $m_1$ on the inner radius. Release the system from rest. Record a video of either the pulley spinning or one of the masses translating. Using this video in tracker you can get $t$, $x$, and $v$ data (for a translating mass) or $t$, $\theta$, and $\omega$ data (for rotating pulley).

**Update:** Try putting a photogate pulley in line with the string as pictured at right.
Then use data studio to record $t$, $\theta$, & $\omega$ data.
Think: you should use units of rad & $\frac{\text{rad}}{\text{s}}$ if you plan to use $v = r\omega$ & $a = r\alpha$.

Repeat with the same $m_1$ & $m_2$ with $m_2$ still on the outer radius but move $m_1$ the next radius.
Keep repeating with $m_2$ always on the outer radius but keep moving $m_1$ the next radius.
Notice for the last trial both masses will be on the same radius.
If time permits retake all four data sets <u>using half the radius for $m_2$</u>.
In the end you should ideally have 8 sets of $t$, $\theta$, & $\omega$ data (but check with instructor after 4).

**GOALS:**
1) Do energy methods accurately predict the rotation rate of a pulley versus angular displacement?
2) Do torque methods accurately predict the rotate rate of a pulley versus time?
3) Is our pulley approximately a disk (as far as moment of inertia is concerned)?

**Understand the problem theoretically:**
1. Derive the theoretical rotation rate $\omega$ after pulley rotates through angle $\theta$. Answer algebraically in terms of $I$, $m_1$, $m_2$, $r_1$, $r_2$, $g$, and $\theta$.
2. Derive the theoretical angular acceleration using a torque problem. Answer algebraically in terms of $I$, $m_1$, $m_2$, $r_1$, $r_2$, $g$, and $\theta$.
    a. Do this using the center of mass as the pivot point.
    b. Do it again using your result from part 1 and kinematics.
    c. Is the instantaneous pivot is applicable in this case? Be prepared to explain why or why not.
3. Measure all the radii with calipers if possible. If the largest radius is too big for calipers wrap a string around it a few times and measure the length to determine the radius. Note: don't forget to divide by two if you measure the diameters! We may have jumbo calipers in the back…
4. Don't forget to record the masses of all items (including the pulley itself) using a balance!

**GOALS:**

1) Do energy methods accurately predict the rotation rate of a pulley versus angular displacement?
2) Do torque methods accurately predict the rotate rate of a pulley versus time?
3) Is our pulley approximately a disk (as far as moment of inertia is concerned)?

Things to emphasize/think about before giving presentation:

- String should not slip on edge of pulley. This allows us to relate translational variables to rotational variables.
- Axle friction is assumed negligible (in practice you'll observe the friction is probably non-negligible).
- The strings are at different radii. As the pulley spins through angle $\theta$ each string will experience the same angular displacement $\theta$ but different arclengths of string are wound (or unwound). This means the masses will travel different amounts for the same angular displacement!
- Similarly, the masses will travel at different speeds as they rise and fall. They will also have different accelerations!
- The string connects the super pulley to the big pulley at radius $r_2$. The edges of these two pulleys have the same *translational* speed. The two pulleys do not have the same *angular* speed. The *angular* speed of the *big* pulley should equal the *translational* speed of the super pulley divided by $r_2$. Thoroughly understand this tricky spot.

After this rather long complicated procedure, revisit goal slide and remind us what goals are.
Now do energy derivation to determine $\omega$ in terms of $I$, $m_1$, $m_2$, $r_1$, $r_2$, $g$, and $\theta$.
Show plot of $\omega$ versus $\theta$:

- Get experimental data from super pulley (or Tracker vids).
- Get theoretical prediction from step 1 on previous page.
- Show theory as smooth line and experiment as points. Include theory equation on slide.

Now do torque derivation to determine $\alpha_{th}$ in terms of $I$, $m_1$, $m_2$, $r_1$, $r_2$, $g$, and $\theta$.
For one trial (one of your 10 data sets), show plot of $\omega$ versus $t$:

- Get experimental data from super pulley (or Tracker vids).
- Do a trendline to get the slope of $\omega$ versus $t$ (this is $\alpha_{exp}$)
- Consider animating in a smooth theoretical line using theoretical prediction from step 2 on previous page. Think: you should know the value of $\alpha_{th}$ and $\omega_{th}(t) = \omega_i + \alpha_{th}t$.
- Summarize what was predicted by theory and what was found in experiment. Compare with a percent difference.

Now show $\omega$ versus $t$ data for the *first* half of the data sets *on a single plot*. Include only the experimental data and theoretical data (no trendlines). Coordinate the colors to make it easy to interpret the data sets. Comment on any unusual discrepancies.
Now show $\omega$ versus $t$ data for the first half of the data sets on a single plot. Include only the experimental data and theoretical data (no trendlines). Coordinate the colors to make it easy to interpret the data sets. Comment on any unusual discrepancies. Ask the class why one (or more) of the slopes is negative)? Make sure you know why…

Note: you may have one data set that has no acceleration. Explain why ignoring axle friction causes major problems when analyzing this data set.

Consider making a summary table showing the predicted/theoretical values of angular acceleration, the experimental angular accelerations, and the % difference. Consider taking the absolute value of each % difference then averaging them.

It is typically wise to avoid data tables if you can present the information graphically. Perhaps a better thing to do would be to use each data set to calculate the moment of inertia of the pulley. Then get the average of those numbers (since that should be the same for each trial). You can determine an average and standard deviation to figure out an estimate on the error. Then compare this experimentally obtained value of the pulley's moment of inertia to a theoretical value assuming the pulley is a disk ($I_{th} = \frac{1}{2}MR^2$). You should then be able to find a percent difference to compare to a percent error.

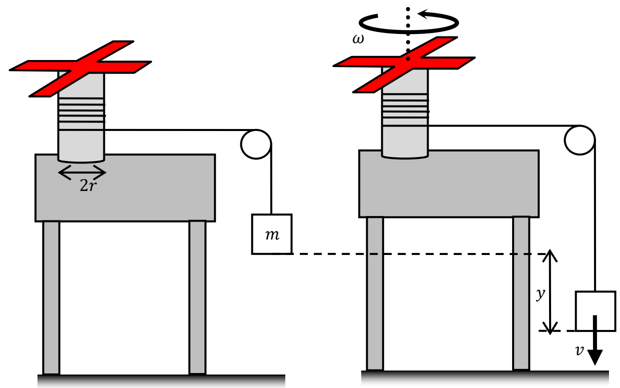
Discuss both qualitative agreement & quantitative agreement?

Revisit the goal slide one more time telling us how you answered those questions.
You might end by mentioning if you believe it was reasonable to treat the pulley as a solid disk.
Try a web search for images with the keywords "change drill press speed". You should see a practical application of this type of system. That might be a fun way to end your talk as well.

**OPTION 5 (two versions means two groups):**
**GOALS:**

1) Do energy methods accurately model motion in system with translational and rotational motion?
2) Test a system for determining moment of inertia of arbitrary objects.

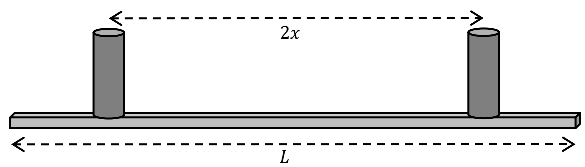
Connect a *small* hanging mass (50 grams?) to the spindle of the turntable. Release the hanging mass from rest. Acquire data for position and velocity versus time for the small hanging mass. Tip: rather than use tracker for this step, consider using a smart pulley and data studio. This will hopefully give you $t$, $y$, and $v$ data directly with no need for video capture.
Experimentally determine the magnitude of the acceleration of the hanging mass. Use that acceleration to determine an experimental value for the moment of inertia of the turntable.




TIP: adjust system such that mass $m$ barely impacts ground when string is fully unwound from spindle. This avoids the turntable spinning out tons of string to get caught in the axle.

Repeat the experiment with various objects on top of the turn table. Note: for the rest of the day you will probably want to use the same *large* hanging mass…probably 500 g or 1kg. For each different object on the turntable, acquire data for position and velocity versus time.
**Version A:** This version uses the disk, ring, and rod with point masses close to axis and rod with point masses far from axis. Also cut a rectangular wooden board to fix on the apparatus to use as a rectangular plate. If possible, also flip the board on its edge. Could you safely flip the ring on its edge as well? **Discuss with instructor before proceeding.**
**Version B**: Use the rod and point masses with all different positions of the point masses.

Rod with point masses, one object you will place on turntable




If you are doing things correctly, you should have $t$, $y$, and $v$ data for the hanging mass for the turntable with nothing on it as well as the turntable with each object on it.

The acceleration in each $vt$-plot should gives $a_{exp}$. Each $a_{exp}$ can be used to determine $I_{exp}$ for each object. **Don't forget to subtract off the moment of inertia of the turntable from the other objects!!!**

**Understand the problem theoretically:**

1. Determine the equation for $I_{th}$ of each object using the tables from text. You will need the masses, length, and radii of your various shapes. For the point masses you will have several different theoretical values since you will have several different values of $x$. Use your formulas to also compute numerical values of $I_{th}$ for each object you will place on the turntable.
2. Derive a theoretical velocity $v$ of the hanging mass after falling a distance $y$ using an energy problem. Use $I$ for the moment of inertia (instead of your result for part 1) to keep your work simpler.
3. Do forces on the hanging mass and torques on the turntable. Solve for moment of inertia ($I_{exp}$).
   a. Do this once using the center of mass as the pivot point.
   b. Do it again using your result from step 2 and kinematics.
   c. Does it make sense to do this problem using the instantaneous pivot instead?

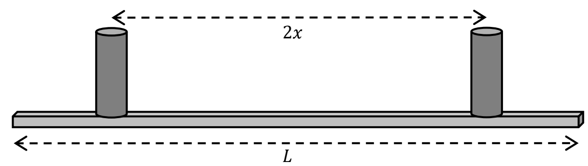
**GOALS:**

1) Do energy methods accurately model motion in system with translational and rotational motion?
2) Test a system for determining moment of inertia of arbitrary objects.

Briefly explain the procedure.

**VERSION A:** Include a picture showing all the various objects you placed on the turntable. Include a meter-stick for scale. Animate in numerical values of each objects moment of inertia (we are calling these values $I_{th}$).

**VERSION B:** Include a picture of turntable with rod and point masses on it. Also show a sketch similar to the one shown at right.




Explain why $I_{th} = \frac{1}{12}m_{rod}L^2 + 2\left(m_{p.m.}x^2\right)$.

Show a plot of $I_{th}$ vs $x$ as a smooth line with no points. Include numerical values for $m_{rod}$, $m_{p.m.}$, and $L$ somewhere on this slide.

Consider a revisit to the goal slide. Remind us of the purpose of your talk.

Now do energy problem to derive speed of hanging mass as function of distance fallen.
Mention if we plug in numerical values of $I_{th}$ into the equation for $v$ we can predict the shape of a plot of $v$ vs $y$.
Show plot of $v$ vs $y$:

- Theory comes from steps 1 & 2 on previous page.
- Experiment comes from Data Studio or Tracker data
- Experiment is dots, theory is smoothed line. Include theoretical equation on slide.

Revisit goal slide…was goal 1 met? Can you estimate an average % difference?

Now do your forces and torques derivation to derive a direct determination of $I_{exp}$.
Emphasize us that $r$ is the spindle radius…not the radius of the object on the turntable.
Emphasize you remembered to subtract off moment of turntable before comparing results.
Show a $vt$-plot for case only:

- Data points come from Data Studio or Tracker.
- Use a trendline to get the slope (this is $a_{exp}$).
- Animate in your equation that relates the acceleration to moment of inertia $I_{exp} = mr^2(\ldots)$.
- Animate in a numerical result for $I_{exp}$ based off of this one graph.

If time permits, show a single $vt$-plot with all data color coded and a legend. Color code the trendlines if possible but you can leave off the $I_{exp}$ equations and calculations for this plot.

**VERSION A:** create a table showing all shapes. Consider include tiny pics of each object if you can squeeze it in to make it painfully obvious which shape goes with which entry in the table. Include $I_{th}$, $I_{exp}$, and % difference for each shape. You can then estimate % errors.

Finally, revisit your goal slide and make sure you covered everything.
Before you talk, think about all the different assumptions you made. Is there axle friction? Is the friction between the edge of the pulley and the string? Why does friction between the string and the edge of the pulley do no work? Why did you include the rotational energy of the turntable but not the smart pulley? Is air resistance more of a factor on certain shapes? Is it completely negligible? To help you think about drag, first consider how fast things moved (translational speed) for your fastest trial in miles per hour?

**VERSION B** on next page…

**VERSION B:** Add in one final graph showing $I$ vs $x$

- The theory values (smooth line) should come from $I_{th} = \frac{1}{12} m_{rod} L^2 + 2\left(m_{p.m.} x^2\right)$
- Include theory equation on chart.
- Animate in experimental points (ask me how).
- Remind that class that each experimental point was determined by first getting $a_{exp}$ from Data Studio (or Tracker) and subsequently using $I_{exp} = mr^2(\ldots)$. Animate in this equation as well.
- Animate in
- Discuss why data shouldn't match perfectly for smallest value of $x$. State if the theory equation used above is an underestimate or overestimate for $I_{th}$ for small $x$. Explain why.

Before you talk, think about all the different assumptions you made. Is there axle friction? Is the friction between the edge of the pulley and the string? Why does friction between the string and the edge of the pulley do no work? Why did you include the rotational energy of the turntable but not the smart pulley? Is air resistance more of a factor on certain shapes? Is it completely negligible? To help you think about drag, first consider how fast things moved (translational speed) for your fastest trial in miles per hour?

**OPTION 6:** Program a computer simulation of any of the above experiments. Include sliders for typical parameters (e.g. in OPTION 1 make sliders for selecting the values for $m$, $M$, $x$, and $L$). Ensure that even an idiot like your instructor can pick some values and then hit play to watch the simulation.

**OPTION 7:** Propose something else requiring roughly equal effort AND possible with equipment available. We have a lot of weird stuff (and tons of tape) so we can probably work up something for most ideas.

**Prepare a PowerPoint Presentation.**

- State why is the audience supposed to care/listen, support claims with visual evidence, minimize unnecessary details/words or other distracting information
- Post your video to the web somehow (e.g. post it to YouTube) so it can be played during your presentation if there are any questions that come up. Edit out the unimportant stuff or know the appropriate times to use in the video so we don't need to watch a lot of worthless material.
- Each slide has one key point supported by pictures, an eqt'n, or a graph (not bulleted lists of words)
- Keep fonts big (20 points for axis labels, everything else bigger than that)
- Include a title slide that gives your names, a candid picture of you performing your experiment, a short title of your experiment.
- Include a goal slide.
- Include a derivation slide (or two). Try to minimize clutter. Show the important equations you started with and the also the final result but skip all the gritty details in between. You should have all those details written out on a piece of paper while you are presenting but not on the slide (just in case I ask a question). At the end of your derivation, consider re-emphasizing your goal since people have probably forgotten.
- Include a procedure slide. Briefly summarize the steps required to get your graphs or data. Don't say every little detail...if I really want to know something detailed I'll ask a question. If you have a big picture or sketch of your apparatus (or before and after picture) you can simply explain by pointing at the picture and telling us how it worked.
- If your video is really short, this might be a good time to show it to the class. You could escape out of PowerPoint, have your video ready to go in another window, play it, and then go back to your PowerPoint. Note: sometimes trying to embed the video in your PowerPoint can cause frustrations and delays. Please do not frustrate and delay everyone. Practice showing your video on at least two different computers before coming to class.
- Show any data or graphs. Mention how they relate to goal. Point out any unusual trials of your experiment. Discussion the experimental precision of your experiment relative to your % differences. Use 20 pt or larger font size so we can actually read your axis labels, etc.
- Have a final slide summarizing how you met (or didn't meet) your goal. Were the theoretical predictions in good agreement with the experimental results? Don't just say yes or no, discuss the % precision and % difference.
- Practice the presentation. Each group member should speak an equal amount. The total presentation time should be 8-10 minutes. Anyone exceeding the 10 minute time limit will be stopped and lose points.
- Save the file on each member's flash drive and, as a back-up, email it to yourself. Think, what if somebody gets sick? Make sure all team members have all the info.
- Consider reviewing http://writing.engr.psu.edu/speaking.html or google other sites to improve your talk.

Be prepared to give your presentation next week in lab. You will also be required to give feedback to your peers on their presentation.

**What is good science?**

I want to make your lab experience useful not only in this class but also after you transfer and eventually get a job. I asked myself, what is good science? What are the most important things I want you to take with you from lab?

- Form a good question (falsifiable claim)
- Design an experiment (tests claim, can be repeated by others, wisely restricts parameter space)
- Collect adequate data (data covers broad swath of restricted parameter space, multiple trials per case)
- Appropriate error analysis (honestly interpret data, even if errors are large or claim is shown to be false)
- Proper citations (be generous and honest about contributions from outside sources)

This is, of course, some variation on the classic scientific method.

We now have several different well-accepted theories we can use to design falsifiable questions:
- Kinematics (constant acceleration, separation of variables, etc)
- Newton's laws ($\sum \vec{F} = m\vec{a}$ & $\sum \vec{\tau} = I\vec{\alpha}$)
- Work-Energy ($E_i + W_{ext\&n.c.} = E_f$)

Furthermore, in lab we typically measured the following parameters:
- Determine elapsed time with a stopwatch
- Determine velocity with a photogate or encoder (e.g. a smart pulley)
- Determine acceleration using one of the above tools plus distance
- Plot position and velocity versus time with Tracker
- Determine acceleration form the slope of a *vt*-plot made using Tracker

It makes sense, then to use our theories to predict elapsed time, final speed, acceleration, etc. These predictions can then be shown to be false (or not false) with our experimental equipment. By testing not just a single case but multiple heights/angles/masses we can see if the result is valid over a broad range of conditions. We can then do proper error analysis to determine percent error (% precision) of our experiment. By comparing the % error to the % difference we can state if our results are in good quantitative/qualitative agreement with the widely accepted theories. Eventually you will learn how to quantify your confidence in such statements. That is good science.

**Option1: WATCH OUT – This problem differs significantly from workbook problem 10.50.**
In the **10.50** problem, the angle is measured from the *vertical*. Heights involve a $1 - \cos\theta$ term.
In this problem, the angle is drawn to the *horizontal*. Heights involve $\sin\theta$.

I will assume $\theta > 0$ (angle downward is positive). If using $\theta < 0$, some signs flip (i.e. $y_{cm\,f} = +\frac{m\frac{L}{2}+Mx}{m+M}\sin\theta$ ).

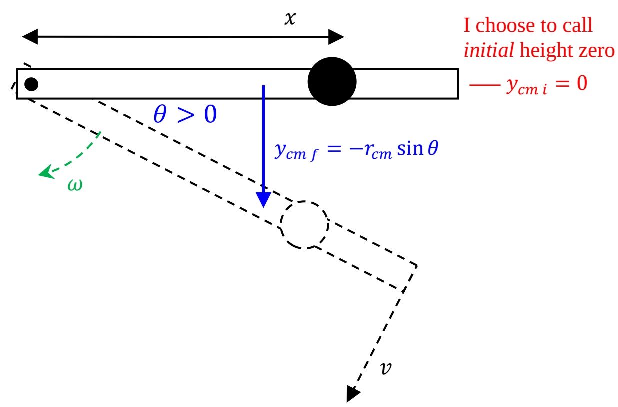
1) Radius from pivot to center of mass is $r_{cm} = \frac{m\frac{L}{2}+Mx}{m+M}$

If I choose to call initial height zero…

$$y_{cm\,f} = -\frac{m\frac{L}{2} + Mx}{m + M}\sin\theta$$

where $\theta$ is from the horizontal.

Notice this height is *negative* since it is *below* the initial position is zero and I assumed $\theta > 0$.




2) $I = \frac{1}{3}mL^2 + Mx^2$

3) Energy equation gives

$$U_i + K_i = U_f + K_f$$

$$0 + 0 = (m + M)g y_{cm\,f} + \frac{1}{2}I\omega^2$$

$$\omega = \sqrt{\left(\frac{mL + 2Mx}{I}\right) g\sin\theta}$$

where $I$ is moment of inertia of the rod & point mass combined

4) $\alpha = \left(\frac{m\frac{L}{2}+Mx}{I}\right) g\cos\theta$…you should be able to get this result using a torque problem or using $\alpha = \frac{d\omega}{dt}$.

5) At the bottom the angle is $\theta = 90°$.
The velocity is given by $v = r\omega$ where, for the end of the rod, $r = L$.
One finds $v = L\sqrt{\frac{mL+2Mx}{I}g} = L\sqrt{\frac{mL+2Mx}{\frac{1}{3}mL^2+Mx^2}g}$.
Notice that in theory you can predict a value of *v* for given values of *x*, *L*, *m*, and *M*.

12) You can use your $v$ vs $x$ plots to compare $v_{exp}$. Put error bars on the points so we can quickly see if the experimental results and in quantitative agreement with the theoretical line.

If not in good quantitative agreement, do the experimental points suggest good qualitative agreement? Is the shape of $v_{exp}$ vs $x$ similar to the shape of $v_{th}$ vs $x$? For $a_{th}$ and $a_{exp}$, just make a little table and include % difference & % error for each of the two cases.

Feeling über hard-charging? Try to make a theoretical plot of $\omega$ vs $t$. I think this will require a numerical technique such as Euler's Method. This is very interesting…and probably a bit challenging. Essentially, you start with a known initial value. Then use

$$\omega = \frac{\Delta\theta}{\Delta t} = \sqrt{\left(\frac{mL + 2Mx}{I}\right) g\sin\theta}$$

This gives

$$\Delta\theta = \Delta t\sqrt{\left(\frac{mL + 2Mx}{I}\right) g\sin\theta}$$

In theory, for a small enough step size, one can then predict the next value of theta $\theta_1 = \theta_0 + \Delta\theta$. This corresponds to the time $t_1 = 0 + \Delta t$. By repeating this process you can eventually get a column of data for both *t* and $\theta$ to compare to your experimental values.

**Option2:**

1) $I_{th} = I_{cyl} + 2I_{square} = \frac{1}{2}m_{cyl}R^2 + \frac{1}{3}m_{sq}s^2$ where $R$ is the radius of the cylinder and $s$ is the side length of the square.

2) $v_{th} = \sqrt{\frac{2gh}{1+I/mR^2}}$

3) $\alpha_{th} = \frac{g}{R}\left(\frac{1}{1+I/mR^2}\right)$

4) $t = \sqrt{\frac{2h}{g}(1 + I/mR^2)}$

9) From your plot of $\omega(t)$ you should be able to use a trendline to determine $\alpha_{exp}$. You can use the LINEST command in Excel to get an error associated with $\alpha_{exp}$. You can compare this to your theoretical value derived in part 3) with a % difference. Put error bars on the plot of $v$ vs $h$ so it is easy to see if the experimental points are in quantitative agreement with the smooth theoretical line.

If the experiment is not good agreement, one can still look for qualitative agreement. Also note if the experiment is in qualitative agreement by looking for the following two things: 1) is the slope of $\omega$ vs. $t$ roughly constant & 2) is the magnitude of the slope in the ballpark. Does the $v$ vs. $h$ plot take the shape of the square root function?

**Option3:**

1) $v_{th} = \sqrt{\frac{2gL\sin\phi}{1+I/mR^2}}$ where $\phi$ is the angle of incline as opposed to the usual $\theta$ for reasons that will become obvious later

2) $\alpha_{th} = \frac{g\sin\phi}{R}\left(\frac{1}{1+I/mR^2}\right)$

3) $t = \sqrt{\frac{2L}{g\sin\phi}(1 + I/mR^2)}$

9) From your plot of $\omega(t)$ you should be able to use a trendline to determine $\alpha_{exp}$. You can use the LINEST command in Excel to get an error associated with $\alpha_{exp}$. You can compare this to your theoretical value derived in part 3) with a % difference. Put error bars on the plot of $v$ vs $L$ so it is easy to see if the experimental points are in quantitative agreement with the smooth theoretical line.

If the experiment is not good agreement, one can still look for qualitative agreement. Also note if the experiment is in qualitative agreement by looking for the following two things: 1) is the slope of $\omega$ vs. $t$ roughly constant & 2) is the magnitude of the slope in the ballpark. Does the $v$ vs. $L$ plot take the shape of the square root function?

**Option 4:**

1) This is trickier than I first suspected. We know the two objects will rotate the same angle but the amount of height traveled by the blocks differs! We know that block 1 will travel distance $h_1 = r_1\theta$ while block 2 will travel distance $h = r_2\theta$ A ratio gives $h_1 = \frac{r_1}{r_2}h$. Similarly, if $m_1$ is travelling with speed $v$, $m_1$ is travelling with speed $v_1 = \frac{r_1}{r_2}v$. The energy equation becomes

$$m_2gh = m_1gh\frac{r_1}{r_2} + \frac{1}{2}m_2v^2 + \frac{1}{2}m_1\left(\frac{r_1}{r_2}v\right)^2 + \frac{1}{2}I\omega^2$$

The final result for speed is

$$v_{th} = \sqrt{2gh\frac{m_2 - m_1\frac{r_1}{r_2}}{m_2 + m_1\left(\frac{r_1}{r_2}\right)^2 + \frac{I}{r_2^2}}}$$

2) The two masses do not have the same acceleration! As before, if $m_2$ has acceleration $a$, then $m_1$ has acceleration $a_1 = \frac{r_1}{r_2}a$. Ultimately one finds

$$\alpha = g\frac{r_2m_2 - r_1m_1}{m_2r_2^2 + m_1r_1^2 + I}$$

3) $t = \sqrt{\frac{2h(m_2r_2^2 + m_1r_1^2 + I)}{r_2g(r_2m_2 - r_1m_1)}}$

4) $I_{exp} = \frac{g}{\alpha}(r_2m_2 - r_1m_1) - m_2r_2^2 - m_1r_1^2$

11) From your plot of $\omega(t)$ you should be able to use a trendline to determine $\alpha_{exp}$. You can use the LINEST command in Excel to get an error associated with $\alpha_{exp}$. For each video you can determine a value of $I_{exp}$. Assume the % error is the same as the % error associated with $\alpha_{exp}$. Average these $I_{exp}$'s values to give a result for the moment of inertia of the pulley.

While not strictly a disk, you may compare your experimental results to the theoretical value for a disk given by $I_{Disk} = \frac{1}{2}m_rR^2$ and give a % difference. Note that the error in the average will be the average % error divided by $\sqrt{N}$ where $N$ is the number of cases you tried. State if it is appropriate to treat the pulley as a disk by noting if the %difference < %error?

Put error bars on the plot of $v$ vs $h$ so it is easy to see if the experimental points are in quantitative agreement with the smooth theoretical line.

If the experiment is not good agreement, one can still look for qualitative agreement. Also note if the experiment is in qualitative agreement by looking for the following two things: 1) is the slope of $\omega$ vs. $t$ roughly constant & 2) is the magnitude of the slope in the ballpark. Does the $v$ vs. $h$ plot take the shape of the square root function?

Feeling frisky? You could track both masses and plot velocity versus time for both masses on the same plot. You could compare the slopes of the two lines and see if the relationship $a_1 = \frac{r_1}{r_2}a_2$ is in quantitative agreement with the values of your slopes!

**Option 5:**

1) For a rod with two point masses one finds $I_{total} = I_{rod} + 2I_{p.m.} = \frac{1}{12}m_{rod}L^2 + 2\left(m_{pm}x^2\right)$

A thick ring is one that has an inner radius noticeably different from the outer radius. For a thick ring the moment of inertia is given by $I_{ThickRing} = \frac{1}{2}m_r(R_{outer}^2 + R_{inner}^2)$. A solid disk can be found with the same formula noting that the inner radius is 0!

2) $v = \sqrt{\frac{2gh}{(1+I/mr^2)}}$ where *m* is the hanging mass and *r* is the spindle radius.

3) $\alpha = \frac{g}{r}\left(\frac{1}{1+I/mr^2}\right)$

4) $t = \sqrt{\frac{2h}{g}(1 + I/mr^2)}$

Note: re-arranging the equation 3) and substituting $\alpha = \frac{a}{r}$ gives

$$I_{exp} = mr^2\left(\frac{g}{a} - 1\right)$$

10) Use the slope of each *v* vs *t* plot to get a value for $a_{exp}$. The error for each $a_{exp}$ can be obtained using the LINEST command in Excel. You can determine $I_{th}$ using the equations from part 1). Watch out! Don't forget to subtract the moment of inertia of the turntable from the $I_{exp}$'s obtained for all the other graphs! Compare using a percent difference. **Version B:** plot *I* versus *x* showing the theory values as a smooth line and the experimental values as dots (with error bars).