# Projectiles

**Apparatus:** projectile launchers, plastic projectile spheres, rulers, projectile accessories, scissors, pulley cord, meter sticks, paper, plumb bobs, large clamps

**Purpose:** In this lab we will use kinematic theory to find the muzzle velocity of a spring gun and then predict the range of a projectile launched from an angle.

**Intro:** In today's lab you will use the freefall equations of motion in two dimensions to analyze the flight of a projectile. One version of the kinematics equations you can use is given below.

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$$
$$x = v_{ix}t$$
$$v_{ix} = v_i \sin\theta$$
$$v_{iy} = v_i \sin\theta$$

**Think:** Does $a_y = g$ or $a_y = -g$?
**Think:** Is $\Delta y$ positive or negative? Does it depend on the situation?
**Think:** What does each variable mean? You are expected to explain this in any write-up…
**Think:** why is it reasonable to use $x$ instead of $\Delta x$ for horizontal displacement?

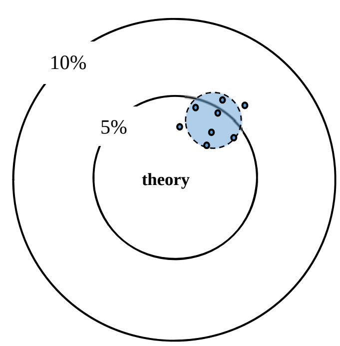
**Comments on error analysis**
Standard deviation ($\sigma$) is often used as an estimate of the uncertainty in a set of repeated measurements.

$$\sigma = standard\ deviation = \sqrt{\frac{\sum\left(x_i - x_{avg}\right)^2}{N-1}}$$

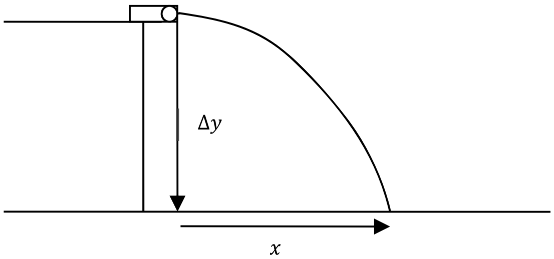
Note: the $N-1$ n the denominator indicates that you can't get a standard deviation of a single measurement.
It turns out that the standard deviation of a set of numbers gives you the error (the ±#) for that set of numbers.
When random errors dominate the precision of equipment we use

$$\%precision\ of\ an\ avg\ value = \frac{standard\ deviation\ of\ the\ measurements}{avg\ of\ the\ measurements} = \frac{\sigma}{avg} \times 100\%$$

**Target Diagram**
The figure at right shows a typical target diagram for this experiment.
Several measurements were taken and each measurement had at least three sig figs.
Because the individual measurements have several sig figs, each appears as a tiny dot.
Unfortunately, each measurement differs significantly from the others.
Since we took several measurements the target looks like it was hit with a shotgun blast.
Distance from the bull's-eye to the center of the shotgun blast represents % difference.
The spread in the dots (blue circle) represents % precision $\left(\text{in this case given by } \frac{\sigma}{average} \times 100\%\right)$. Notice % precision appears to be about 4% in this case.



**Part 1:** Finding the spring gun muzzle velocity.




Measure $\Delta y$ from the floor to the ball's launch position.
If you look on the projectile launcher, you should see a launch position indicator.

Think: should you measure from the top, bottom, or center of the launch position to get the most accurate $\Delta y$ measurement?

Verify you are able to shoot the ball without hitting anyone or any delicate lab equipment. The projectile range can be 3-4 meters in some cases!

Load a ball into to the cannon; I typically go all the way to the "long range" mode. When doing this you should hear three clicks.

**Before launching,** verify the range is clear then launch the ball *horizontally* (launch angle indicator of 0°).
A horizontal launch ensures that the ball's initial velocity (the muzzle velocity) has no vertical component.
HINT: Put paper on the ground where the ball lands. It will leave an imprint clearly indicating where to measure!
Now derive a result for the initial speed of the ball (the muzzle velocity) $v_i$ in terms of $\Delta y$, $g$, and $x$.

**WATCH OUT!** When loaded horizontally the ball can roll out of the launcher cradle inside the launcher. This messes up your data significantly. You *may* have to angle the launcher ever so slightly ($< 0.5°$) to keep the ball from rolling out of the launch cradle.

Measure $x$ from the launch position to the point of impact.
Think: should you measure from the left, right, or center of the launch position to get the most accurate $x$ measurement?

Since there will be variations, shoot the ball and measure $x$ five times.

Remember: when the launch is horizontal the launch angle is $\theta = 0°$. Also watch out for + or – signs.
The variable $g$ is the MAGNITUDE of acceleration ($g = +9.8\,\frac{\text{m}}{\text{s}^2}$).
**Think:** if the ball goes downward will $\Delta y$ be + or -?

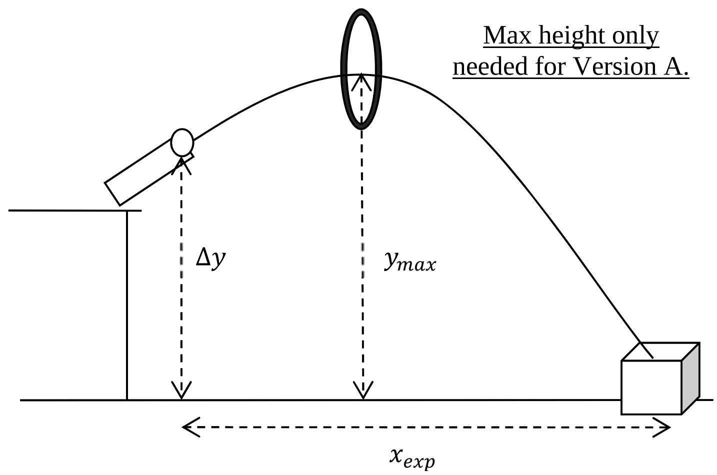
Calculate the muzzle velocity $v_i$ for each value of $x$.
Determine the average value of $v_i$.
Determine the standard deviation of your five values of $v_i$ using the formula shown on the previous page.
**Show your work (by hand) in your lab book for the standard deviation.**
Check your result with the computer using the AVERAGE command and STDEV command in Excel.

**Part 2:** Use the muzzle velocity $v_i$ from part 1. Predict how far (horizontally) from the launcher the ball travels.

**First determine an algebraic expression for the range in terms of the launch angle, the launch height, and the muzzle velocity. This derivation should be in your lab notes for credit.**
To do this, first determine an algebraic expression for $t$ from the $\Delta y$ equation.
equation. This can be done with the quadratic formula.
Then plug in this value of time into $x = v_{ix}t$.
Lastly, don't forget that $v_{ix}$ and $v_{iy}$ depend on the angle so you'll need to put in $\sin\theta$ and $\cos\theta$ in some places.
Your final result for the **theoretical** range should look like (you need to figure out the terms with ?'s):

$$x_{th} = v_i \cos\theta \frac{v_i \sin\theta \pm \sqrt{? - 2?}}{g}$$

**VERSION A:** Set the launcher on the table and angle the launch at 40 degrees. Predict the position it will land on the ground. Predict the location of the max height (both *x* and *y* position of max height). Place a bucket at the spot the ball should land and a hoop at the max height. Once you have predicted the locations, obtain a hoop and bucket from your instructor and show them that the projectile goes through the hoop at the max height and lands in the bucket. Is it repeatable?




**VERSION B:** Set the launcher on a table and angle the spring gun to launch at 40, 45, & 50 degrees. Create an Excel worksheet that computes the theoretical range for each angle. Compare experiment to theory.

**VERSION C:** Set the launcher on a table and angle the spring gun to launch at 39, 45, & 49 degrees. Create an Excel worksheet that computes the theoretical range for each angle. Compare experiment to theory.

**VERSION D (Challenging but really fun):** Assume muzzle velocity and launch height are known. Your instructor will give you $(x, y)$ coordinates of a target to hit. It is your job to create an Excel worksheet which computes all launch angles which will cause the ball to hit the target. Once your worksheet is done, consider checking it against a simulation (Projectile Phet). Use $h = 10.0\text{m}$, $v_0 = 30.0\frac{\text{m}}{\text{s}}$, and target location $(x, y) = (94.1\text{m}, 0)$.

**To get experimental values of range:**
Now actually launch the ball at each angle. For each launch angle, verify that the height $\Delta y$ has not changed! Remember that the launch height is where the ball leaves the cannon, not at the height of the table. Place a piece of paper on the floor where the ball is expected to land. Get 5 experimental values of the range for each angle. The average of these values will be your experimental range ($x_{exp}$). The standard deviation will give you the error in the experimental range ($\sigma_x$).

**For all versions your notes should include the following in your calculations section:**

- Derive the eqt'n for muzzle velocity $v_i$ algebraically
- Work out ONE sample calculation of the muzzle velocity
- Work out ONE calculation of $\sigma$ by hand from part 1.
- Derive the eqt'n for predicted range (for non-zero angles) $x_{th}$.
- Work out ONE sample calculation of the predicted range (for a single non-zero angle).
- Estimate your % precision and explain your reasoning.
- Show a target diagram for the %difference and %precision for the smallest angle.

**My criteria for submitting a sample calculation**:

1) Write down the eqt'n
2) Then plug in the numbers without doing any math
3) Show the unrounded final answer (with units)
4) Show the rounded final answer with units and box around it
5) Work down the page…not in columns.
6) You needn't show work for *every* data set, just show work for *one* trial.

**Conclusions**

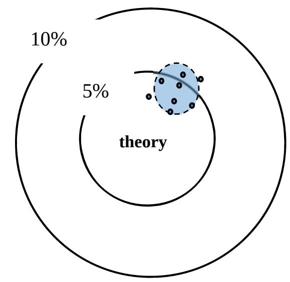
1. Were the experimental results found to be in good agreement with theoretical predictions (how did %precision compare to %difference)? Some helpful comments appear on the next page.
2. If air resistance is considered negligible, can today's lab be considered as an example of freefall? Defend your answer.
3. VERSION A use part a, VERSION B & C use part b
    a. Use your %error and a calculated value of the max height to estimate the smallest hoop size you could use and reasonably expect to pass through the hoop at max height 70% of the time.
    b. Use your %error and a calculated value of the max range to estimate the smallest target you could place at the max range and reasonably expect to hit the target 70% of the time.
4. In part I of the experiment air resistance was neglected in the theoretical calculation of the muzzle velocity. However, air resistance was included in the experimental value obtained for $x$. Is the actual experimental muzzle velocity to be slightly higher or slightly less than our computed value?
5. Assume the following conditions: $\Delta y = -1.0$ m, $v_i = 6.3\,\frac{\mathrm{m}}{\mathrm{s}}$, & $g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$. Use your range formula to show that a 39.3° launch angle flies 1.4% farther than a 45.0° launch angle. Notice that 45° will give maximum range only when the launch height equals the impact height!

**Going Further**

Use MATLAB to make a contour plot showing the range of the projectile for initial heights ranging from 0.0 to 18.0 meters for all angles between 0.0° and 90.0°. I suspect you might try incrementing the initial height in 0.5 meter increments and the angle in 5.0° increments. If you are feeling frisky, I suppose you could even consider angles between 0.0° and 180.0°. Think: for launch angles greater than 90.0° why does it make sense to get negative values for range?

**Comments on % precision versus % difference**

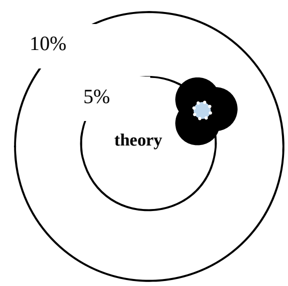
- Think of % precision as the number of sig figs on your final result. If you have a lot of sig figs on your final result it is very precise (lots of sig figs). This can be true even if your answer is "way off".
- Think of % percent difference as quantifying how off you are from theory.
- If % difference is greater than % precision, the experiment is NOT in good agreement with theory. You have small bullet holes, closely grouped hitting far from the bull's eye.
- If % difference is less than (or *approximately* equal to) % difference, the experiment is in good agreement with theory. Most of the bullets are hitting the bull's eye. Note: in experiments with large % precision, (huge bullets) and you are very likely to agree with theory (to hit the target), but that doesn't mean your results are very useful.
- We strive for experiments with small bullets (good precision) and hit the target (good agreement)




The *upper* figure at right shows one possible target diagram for this experiment.
Several measurements were taken and each measurement had at least three sig figs.

Unfortunately, each measurement differs significantly from the others.
Since we took several measurements the target looks like it was hit with a shotgun blast.
Because the individual measurement have several sig figs, we see each measurement appear as a tiny dot.

Distance from the bull's-eye to the center of the shotgun blast represents % difference.
The spread in the dots (blue circle) represents % precision $\left(\frac{\sigma}{average} \times 100\%\right)$.
Notice the % precision looks like it is about 4% based on the scale of the 5% ring.




The *lower* figure at right shows another possible target diagram for this experiment.
In this case, the measurements each have nearly identical values.

Unfortunately, each individual measurement might have only 1 or 2 sig figs.
Each bullet radius is larger than the blue circle defined by $\frac{\sigma}{average} \times 100\%$.

Distance from the bull's-eye to the center of the shotgun blast represents % difference.
In this case we should use sig fig rules or propagation of error techniques to compute % precision. In this case the % precision looks like it is about 5% based on the scale of the 5% ring.