Rotation

WARNING: Today's lab requires the massing of objects that could damage our electronic balances. Do not mass these large objects on the electronic balances; use the old fashioned one in the SE corner of M205.

Apparatus: Rotation set disks and rings, rotation set aluminum rods and point masses, hanging mass sets, pulley cord, stopwatches, scissors, digital calipers, meter sticks, pulleys for rotation sets, rotation set turntable, large triple beam balance (from SE corner of M205)

Today you will be comparing theoretical moments of inertia to experimentally determined ones. This will be done by observing an object undergoing rotational motion and comparing observations to predictions using kinematics, forces, and torques.

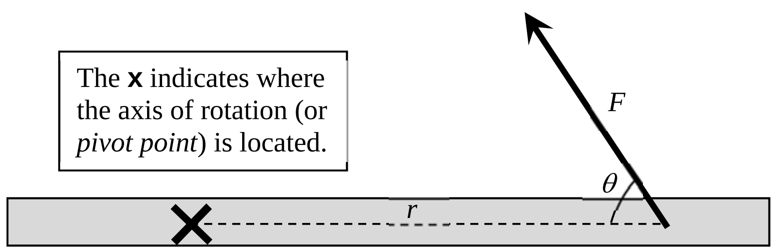
Linear motion equations and angular motion equations look very similar as seen in the table below:

$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v_x^2 = v_{0x}^2 + 2a_x(\Delta x)$	$\omega^2 - \omega_0^2 = 2\alpha(\Delta\theta)$
$v_x = v_{0x} + a_x t$	$\omega = \omega_0 + \alpha t$

The variables are defined in your class notes or in the text somewhere. In circular motion the following relationships exist between the above sets of variables:

$s = r\theta$
$v = \omega r$
$a_t = \alpha r$

A torque can be exerted on an object and cause it to rotate. A torque is essentially (in a sloppy, non-technical kind of way) a force that causes rotation. To determine the size of a torque consider the diagram and equation below:



The torque τ caused by the force F is given by the equation

$$\tau = rF \sin\theta$$

where r is the distance from the pivot point to the point where the force is applied and θ is the angle between the r-vector and the F vector as shown in the figure.

The analogy between rotational equations and translational equations holds for Newton's 2nd Law as well. The rotational equivalent to mass is the ***<u>moment of inertia</u>*** given by the letter *I*. The N2L eqt'n and its rotational analog are listed below.

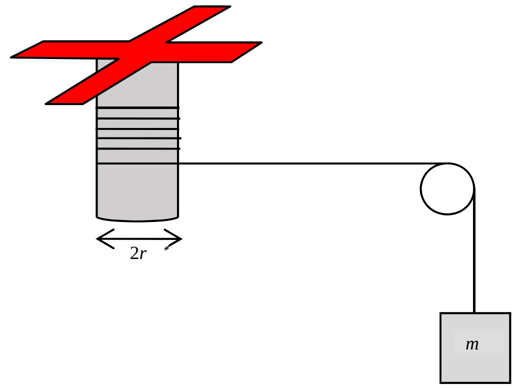
$\sum F = ma$	$\sum \tau = I\alpha$

It can be shown that a falling object (dropped from rest) of mass m attached to a spindle of radius r causes the object to rotate in accordance with the following equations:
The first equation is the 1D kinematics equation which describes the linear fall of the mass towards the floor.

1) $y = \frac{1}{2}at^2$	2) $a = r\alpha$	3) $mg - T = ma$	4) $rT = I\alpha$

The second equation relates the linear acceleration of the falling mass to the angular acceleration of the spindle. These two are related because they are connected by a string.

The third equation comes from doing the sum of forces on the falling mass. Show an FBD in your notebook and derive this equation. The fourth equation comes from the sum of torques on the spindle. Show a torque picture and derive this result in your notebook as well. Note that only one force causes the spindle to rotate and that force is the tension in the string. Because the string comes off the spindle at a right angle, the angle θ from the torque equations is 90° (which in turn makes $\sin\theta$=1).



By fooling around a bit with the third and fourth equations you should be able to derive an expression for I in terms of m, g, Δy, t, and r. Derive this equation in your notebook. Your final result should end up in the form $I_{exp}=mr^2(\ldots-1)$.

In today's lab you will measure Δy with a meter stick and t with a stopwatch. To get r you can measure the radius of the spindle with calipers and to get m you can use the balances in the lab. You can then use this information to determine the moment of inertia of the turntable and spindle.

Use five measurements to get an average and standard deviation for I_{exp}.
I suggest using a small hanging mass m for the turntable and a larger m when you have heavy objects on the turntable. Also, try spinning the pulley; if it is sticking try asking for some lubricant.

To further study rotational motion, additional experiments can be performed using the ring, the disk and the point mass and rod attachments. **WARNING: do not mass these large objects on the electronic balances, use the old fashioned one.**

Let half the groups do *both* Version A & B while the other half do Version C.
VERSION A: Determine I_{exp} for a disk and ring. Don't forget to subtract of the I_{exp} of the turntable.
VERSION B: Determine I_{exp} for the rod with the point masses close to the center then again with the point masses further from the center. Don't forget to subtract of the I_{exp} of the turntable (although it may be negligible).
VERSION C: Determine determine I for *all* possible positions of the point masses. Subtract off the turntable's moment of inertia. Plot I as a function of point mass position x. Show both theory and exp curves (type II graph).

It can be shown that the ***theoretical* moments of inertias** for these objects are as follows:

$I_{Disk} = \frac{1}{2}M_{Disk}R^2_{Disk}$	$I_{Ring} = \frac{1}{2}M_{ThickRing}\left(R^2_{Outer} + R^2_{Inner}\right)$	$I_{Rod} = \frac{1}{12}M_{Rod}L^2_{Rod}$	$I_{pnt} = M_{pnt}R^2_{pnt}$

Repeat the experiment using the disk & the cylinder. You can obtain **theoretical values** for the moments of inertia by massing the objects (with the old-fashioned balance) and measuring their dimensions with a meter stick.

You will obtain an experimental value for the moment of inertia using the method described above. Subtract from this value the moment of inertia of the turntable found previously. Compare this result to the theoretical moment of inertia of the disk, ring, or bar with point masses using a percent difference.

Conclusions:

1. Estimate the precision and compare it to the %difference for the moment of inertias of your two objects. Are the theoretical formulas for moments of inertia in good agreement with your experimental observations?
2. Which of your objects should have a larger angular acceleration? Explain why. Was this shown to be true?
3. Technically, we ignored friction in both the pulley and in the turntable. By neglecting these would that tend to make the percent differences more positive or more negative? Does this make sense based on the sign of your %differences?
4. Derive an equation for the speed of the hanging mass using $E_i = E_f$. Verify your result makes qualitative sense by describing what should happen to v in the following cases:
    a. By increasing the height the equation shows the velocity __________(increases/decreases/stays the same)
    b. By increasing the moment of inertia the equation shows the velocity __________(increases/decreases/stays the same)
    c. By increasing the radius of the spindle the equation shows the velocity __________(increases/decreases/stays the same)

By increasing the hanging mass the equation shows the velocity __________(increases/decreases/stays the same)