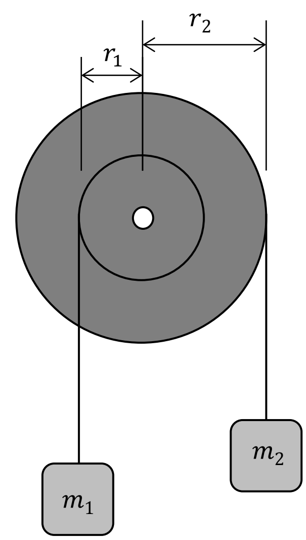
Torque & Rotational Energy: Mixed Lecture/Lab Day Activities

Goals:

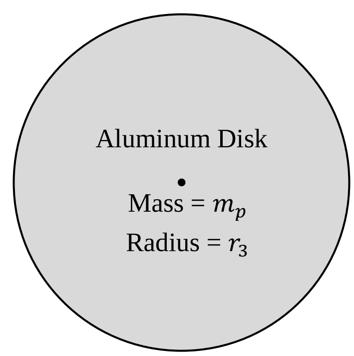
1) Work Atwood's Machine with massive pulley using torque together as a class (or watch [this vid]).
2) Discuss how things differ if strings attach at two different radii.
3) Collect data for experiment 1 (Atwood's with massive pulley using different radii).
4) Work swinging pendulum problem (uniform rod) using energy together as a class.
5) Discuss how things differ when using compound object.
6) Collect data for experiment 2 (rod pivoted at end with two point masses).

Experiment 1 – Theory

Consider the variation on the Atwood's machine shown at right.
The axis of rotation is indicated by the white dot at the center of the pulley.
Notice m_1 & m_2 hang on each side of the pulley at different radii!



After the masses are mounted, we will mount the aluminum disk to the pulley.
Because the pulley is made of lightweight plastic, we may effectively treat the pulley as having the same moment of inertia as the aluminum disk.



Derive theoretical angular acceleration of the pulley:

1. Draw FBDs and write force equations for each hanging mass.
   a. **WATCH OUT!** Because we used different radii $a_1 \neq a_2$!
   b. **WATCH OUT!** Because we used a massive pulley $T_1 \neq T_2$!
   c. **WATCH OUT!** The system accelerates; $T_1 \neq m_1 g$ and $T_2 \neq m_2 g$!
   d. Rewrite the accelerations a_1 & a_2 in terms of α and r_1 & r_2.
   e. Multiply the m_1 equation by r_1 and the m_2 equation by r_2.
   This seems odd now, but it will simplify the algebra later…
2. Draw an FBD and write the torque equation for the pulley.
   a. Assume $I_{pulley} = I_{disk} = \frac{1}{2} m_p r_3^2$.
   b. WATCH OUT! The tensions
3. Determine α_{th}
   a. Stack the three equations and add them.
   b. On the right side you should be able to factor out α.
   c. If you remembered to do step 1e, the internal forces should drop out.
   d. Solve for α and rename it α_{th} (*theoretical* angular acceleration).

Experiment 1 – Data Collection

Record the masses and radii. Tip: to get the radii it is easier to measure diameter and divide by 2.
Plug these numbers into your equation for α_{th} to get a numerical result.
Instructions for the process to get a value for *experimental* angular acceleration (α_{exp}) are written on the next page.
After you complete this process, determine the percent difference between α_{th} & α_{exp}.

Results summary for Experiment 1 - Show the following:

1. Theoretical formula for α_{th}.
2. Numerical value of α_{th} (with units, rounded to three sig figs).
3. Plot of ω vs t showing trendline and R^2 value.
4. Numerical value of α_{exp}.
5. Percent difference, rounded to 1 sig fig (use 2 sig figs if the % difference is larger than 10%).

Short version of instructions for getting $\boldsymbol{\alpha_{exp}}$:

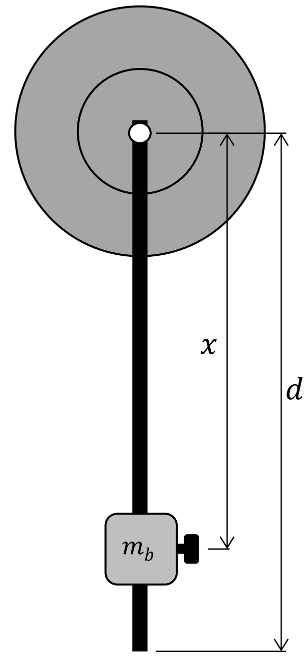
1. Start Data Studio and create an experiment.
2. Connect the rotary motion sensor to the Interface Box using yellow in digital channel 1, black in channel 2.
3. On screen, look for a yellow circle around digital channel 1. Click on it and add the rotary motion sensor.
4. Ensure the computer will display angular velocity in radians per second (check top left of screen).
5. Drag that angular velocity icon down to the "Table" icon in the bottom left of the screen.
6. Record a plot of ω vs t. The slope is α_{exp}!

Long version of instructions for getting $\boldsymbol{\alpha_{exp}}$:

1) Prepare Data Studio for data acquisition.
   a. Open the program and select "Create new Experiment".
   b. Ensure your Data Interface appears to the screen (check power and cable to computer connected).
   c. Click on Digital Channel 1 on the displayed Data Interface on screen.
   d. Select Rotary Motion Sensor.
   e. Connect your Rotary Motion Sensor to channels 1 (yellow plug) & 2 (black plug).
      i. Note: If you connect the cables backwards it flips the sign of the output. This is not a big deal. We can always multiply all the data by -1 later.
   f. Click on the sensor to adjust the quantities it will measure. Ensure you measure ω in $\frac{\text{rad}}{\text{s}}$.
   g. Find the ω (rad/s) icon in the upper left corner of the screen. Drag the ω icon down to the "Table" icon on the bottom left side of the screen.
   h. Hit start and give the system a spin to ensure you can record t & ω data in the table.
2) Hang two equal point masses $m_1 = m_2 = 200$ g at different radii as shown.
   a. Hang the first mass on the *outer* radius, taping the string to the pulley.
   b. Wind the pulley upwards until the hanging mass on the *outer* radius is close to the pulley.
   c. Now hang the mass on the *inner* radius, taping the string to the pulley.
3) Release the system from rest and hit the start button on data studio immediately after.
   a. It is ok if the system doesn't start perfectly from rest…we want the slope of the data.
   b. Stop data collection before any masses hit the ground or the pulley.
4) Copy and paste the data into Excel to determine an experimental value for α.
   a. Plot ω vs t.
      i. Label axes as "angular velocity (rad/s)" on the *vertical* & "time (s)" on the *horizontal*.
      ii. Get rid of the gridlines (right click on them and hit delete).
      iii. Add tick marks.
         1. Right click on an axis.
         2. Hit "format axis", then look around on the right side of the screen.
      iv. Reduce wasted space by formatting the axis values if necessary.
      v. Since we have a single data set, no title or legend is necessary.
   b. Right click one time on a data point to add a linear trendline.
   c. Ensure you display both the trendline equation and the R^2 value.
      i. You may need to right click the trendline and click "Format Trendline" to do this.
      ii. Then, on the right side of the screen, scroll way down and look for two checkboxes.
   d. THINK! The slope of your ω vs t plot is your experimental value of angular acceleration α_{exp}!

Experiment 2 - Theory

Video of worked example of a simpler scenario (rod *without* point mass).



Assumptions:

- The plastic disk with multiple radii has negligible mass.
- End of rod is connected to the pivot point.
- Distance x is to the center of the brass point mass.
- Axle fiction is negligible.

According to the manufacturer:

- The brass point mass is $m_b = 75$ g.
- Rod mass (black rectangle in figure) is $m_r = 27$ g.
- Rod length is $d = 38$ cm.

Derive theoretical angular speed of system at the bottom of the swing:

1. Determine an expression for the moment of inertia of the rod and point mass (I_{total}).
2. Determine an expression the radius from the pivot to the center of mass (r_{CM}).
3. Write an energy equation and solve for ω_{th} (skim these bullets before starting):
    a. Assume the rod & point mass are raised parallel to the ground before being released from rest.
    b. The final state will be the lowest point in the swing.
    c. With swinging motion the only forms of energy are *rotational* kinetic energy and gravitational potential energy.
    d. Gravitational energy is based on the center of mass location in each state.
    e. Work is zero.
        i. Frictionless pivot.
        ii. Rod exerts forces perpendicular to displacement.
    f. Solve the equation for ω using r_{cm} & I_{total} THEN plug in your expressions for r_{cm} & I_{total}.
    g. Rename this expression ω_{th}.
    h. Plug in numerical values so you also have a numerical result for ω_{th}.

Experiment 2 – Data Collection

Use Studio to plot ω vs t.

WATCH OUT! Data Studio will output angular *velocity*…we are looking for angular *speed*.

When you paste the data into Excel to plot it, you may need to multiply by -1.

Look for the maximum value of ω in the data and on the plot.

- To get a precise value, use the MAX function in Excel (I can show you how if you need help).
- Then look at the plot and ensure this MAX value seems to match the max value of ω on the plot.
- The maximum value is ω_{exp}!

Results summary for Experiment 2 - Show the following:

1. Theoretical formula for ω_{th}.
2. Numerical value of ω_{th} (with units, rounded to three sig figs).
3. Plot of ω vs t (no trendline…we do not know what function best models the data!).
4. Numerical value of ω_{exp}.
5. Percent difference, rounded to 1 sig fig (use 2 sig figs if the % difference is larger than 10%).