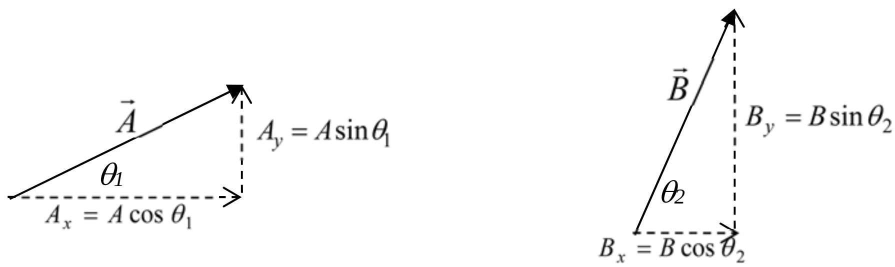
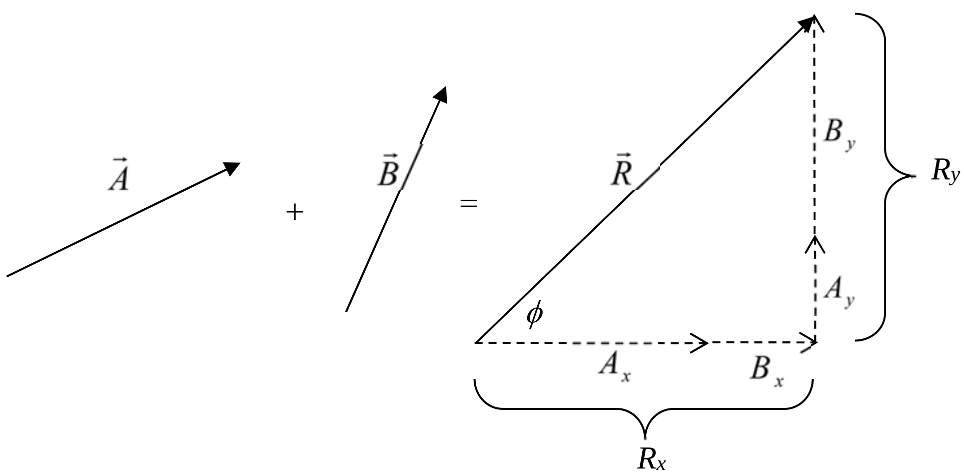
Vectors

Apparatus: Force table, force table pulleys (plastic & metal), ring, slotted masses, mass hangers (5 gram), scissors, strings, rulers, and protractors, pulley cord.

Purpose: The purpose of this lab is to practice vector addition using force vectors. Mathematically vector addition can be performed as follows.





where $R_x = A_x + B_x$**,** $R_y = A_y + B_y$**,** $R = \sqrt{R_x^2 + R_y^2}$ **,and** $\phi = \tan^{-1}\left(\dfrac{R_y}{R_x}\right)$**.**

Note: there is often confusion on the following point: the **scalar** components of the vector $\vec{A}$ are A_x and A_y (with no vector symbol or i-hat/j-hat). The **vector** components of the vector $\vec{A}$ are $\vec{A}_x = A_x\hat{i}$ and $\vec{A}_y = A_y\hat{j}$ **.**

If we have 2 vectors, **A** and **B**, which can be written as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} \qquad \text{and} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j}$$

then the sum of **A** and **B** is a resultant vector **R**,

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

where

$$R_x = A_x + B_x \qquad \text{and} \qquad R_y = A_y + B_y\,.$$

Finally the magnitude of R and the direction of R (given by the angle ϕ can be determined using

$$R = \sqrt{R_x^2 + R_y^2} \qquad \text{and} \qquad \phi = \tan^{-1}\left(\frac{R_y}{R_x}\right).$$

Experiment 1: We will use components (method 1) and graphical addition (theory method 2) to add vectors.

Since force relates linearly to mass we will cheat this week and say that the amount of grams applied to a string is the force. Technically force is measured in Newton's which we will learn about in subsequent lectures.

> **WATCH OUT!** The little gray mass hangers have a mass of 5 grams.
> **VERSION A:** use 30 grams at 0º and 40 grams at 30º.
> **VERSION B:** use 30 grams at 0º and 40 grams at 45º.
> **VERSION C:** use 80 grams at 0º and 60 grams at 30º.
> **VERSION D:** use 80 grams at 0º and 60 grams at 45º.

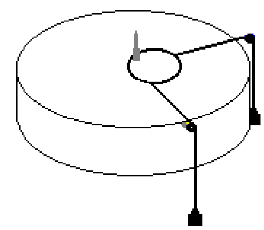
Sketch a picture and figure out A_x, B_x, A_y, and B_y.
Don't worry about drawing perfectly to scale for this picture.
Figure out R_x, R_y, R, and ϕ.
The resultant, in this case, means that your two masses at add up to an equivalent single mass of R grams pulling at the angle ϕ.
Add or subtract 180 degrees to your angle.
This is now your *first* **theoretical result** for the mass (m_{th1}) and angle (θ_{th1}) which should balance the two masses.

Now use the tail-to-tip graphical method to connect your vectors.
To do this, first give yourself some room on the graph paper.
Draw your vectors to scale. To do this:

1) Set a ruler along side your graph paper
2) For a large distance (10 cm or 5"), see if your graph paper lines up better with cm or inch markings.
3) Suppose you find there are 6 blocks per inch. You want each block to correspond to 5 grams. That means 6 blocks = 30 grams = 1 inch. Your conversion will be different than this.
4) To draw a line that corresponds to 48 grams I use my conversion:
$$48 \text{ grams} \frac{1 \text{ inch}}{30 \text{ grams}} = 1.6 \text{ inches}$$
5) Unfortunately, inches are not measured in metric so I can convert the decimal portion to sixteenths by doing the following:
$$\frac{0.6 \text{ inches}}{0.0625} = \frac{9.6}{16} \text{inches} = \frac{10}{16} \text{inches}$$

Draw the first vector.
Be sure to draw a little arrow tip on the appropriate end of the arrow.
At the tip of the first vector, draw a new coordinate system (make it tiny so it won't get in the way of your vectors).
Use your protractor first to mark the new angle of the second vector.
Convert your grams to inches (or cm) for the second vector.
Now draw the second vector with the appropriate length (given by your conversion) in the proper direction (given by your protractor.
Be sure to draw a little arrowhead at the end of your second vector to indicate the tip of the second vector.

Determine the resultant:
Draw a line FROM THE TAIL OF THE FIRST VECTOR TO THE TIP OF THE LAST VECTOR.
Measure the length of this vector with a ruler and use your scale to convert that length to grams.
Measure the angle with a protractor.
Add or subtract 180 degrees from your angle.
This is your *second* **theoretical result** for the mass (m_{th2}) and angle (θ_{th2}) which should balance the two masses.



Lastly, set up the force table with the appropriate mass at each angle.
Set up a third pulley at the predicted angle with the predicted mass.
Slightly adjust the angle and hanging mass until the ring is properly centered.
Determine the minimum mass that balances the ring.
Determine the maximum mass that balances the ring.
Adjust the angle slightly if necessary to ensure the ring is centered then record it in your data as θ_{exp}.
The **experimental value** of the mass (m_{exp}) is the average of your minimum and maximum (see the example below if this is confusing you).
The error in recording your mass, δm_{exp}, is determined by the range of values which balanced the ring.

For today's lab, let us assume that the %precision is given by the percent error in measuring m_{exp} ($\delta m_{exp}/m_{exp}$x100%).
Record this as the precision in your notes.
Find the percent difference between the experimental value and each of the two theoretical methods.

Example: The first theoretical method predicted m_{th1}=62g. The minimum and maximum balancing masses were 58 g and 64 g respectively. This gave m_{exp}=61g with δm_{exp}=3g. More succinctly this is stated as m_{exp}=61±3g. The percent difference is -2% with a percent precision of 5%. Since the percent precision was greater than the percent difference the experiment was in good agreement with theory.

Notice that in the example I used third person, past tense. The sentences are short and complete.
I never used words like:

"Measure the mass"
"I/We did this"

Lastly, to get the 2% and 5% numbers, I did the following:

$$\%diff = \frac{exp - th}{th} \times 100\% = \frac{61\text{g} - 62\text{g}}{62\text{g}} \times 100\% = 1.63\% = -2\%$$

and

$$\%precision = \frac{\delta m_{exp}}{m_{exp}} \times 100\% = \frac{3}{61} \times 100\% = 4.92\% = 5\%$$

Notice, in both cases, the units of grams drop out.
Notice, in both cases, the final answer is rounded to a single significant figure since they are both error estimates and any extra significant figures wouldn't be meaningful. Exception: if first digit is a 1, keep an extra sig fig.

Activity 2: Repeat the above procedure with different masses.

VERSION A: use 60 grams at 215° and 80 grams at 120°.
VERSION B: use 60 grams at 135° and 80 grams at 210°.
VERSION C: use 30 grams at 215° and 40 grams at 120°.
VERSION D: use 30 grams at 135° and 40 grams at 210°.

Summary:

- For each activity: show a small sketch (not to scale) of your math for the component-wise vector addition using the A_x, B_x, sin, cos, etc (theoretical method 1).
- For each activity: Show a sketch to scale and your work for the tail-to-tip Graphical Method (theoretical method 2).
- For each activity: record the actual mass and angle needed to balance the ring (experimental method).
- For activity 1, theory 1: calculate the %difference, the %precision, and draw a target diagram.
- For activity 1, theory 2: calculate the %difference, the %precision, and draw a target diagram.
- For activity 2, theory 1: calculate the %difference, the %precision, and draw a target diagram.
- For activity 2, theory 2: calculate the %difference, the %precision, and draw a target diagram.
- Answer conclusion questions (see below) in a numbered list. For a formal write-up this would be in paragraph form.

CONCLUSIONS:

1. State if the experimental results of vector addition were in good agreement with the theory. You should have four statements since you used two different experimental methods for two different experiments. Use verbiage similar to that in the example. Remember, if any part of the bullet hits the bull's-eye, your experiment is in good agreement with the theory!
2. How should friction in the pulleys affect your percent differences? Should it make the percent differences more positive or more negative? Explain why it does or does not affect the experiment?