Work and Energy Conservation

Apparatus: small bases, right angle clamps, medium rods, small rods, scissors, calipers, pulley cord, aluminum cylinders, spheres with hooks, springs, photogate heads, photogate interface cables, meter sticks, PASCO Science Workshop 750 Interface & Power Supplies, hanging mass sets, pulleys, protractors, tape

For today's lab we are making big diagrams and comparing computations to real-life results as quickly as possible.

- When doing derivations for each project, work *down* the page (do not do work in columns).
- If you run out of room, continue work on a new page (regardless of blank space produced).
- Start each new project on a new page (regardless of blank space produced).
- Use the top 25% of the first page for each project to draw before and after pictures of the scenario.
    - It may even help to include front and side views to clarify the pictures.
    - Include important variables in your diagrams.
    - Significant pieces of equipment (photogates, protractor, meterstick, etc.) should also be labeled.
    - Your figures should be enhanced versions of the figures I started for you.
    - Think: what would someone else require to recreate the experiment from your figure alone?

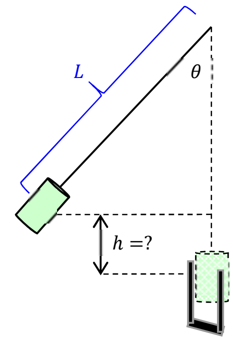
Percent *difference* versus percent *precision* (percent *error*):
Percent *difference* is a comparison of your experimental result to a theoretical (or accepted) result. In most of today's experiments you will derive theoretical results from conservation of energy equations. To be clear, we must sometimes calculated the expected theoretical results using some measured values (i.e. string lengths, masses, etc).

$$\% \, difference = \frac{exp - th}{th} \times 100\%$$

Percent *precision* is an estimate of the quality of your experimental apparatus. At my undergrad university I always called this percent *error* instead of percent precision. In general, the techniques used to determine experimental precision can vary wildly from experiment to experiment. As such, I'll try to explain the relevant techniques as you go through this handout.

Why care about these two numbers?
The % precision estimates how much you expect experimental value *could* differ from the theory value.
The % difference estimates how much the experimental value *actually* differs from the theory value.

1) If your % difference is *less than* (or *approximately equal to*) your % precision, the experimental result is about as close as one could expect it to be. The results agree quantitatively with theory.
2) If your % difference is *significantly more than* your % precision, the errors associated with measuring are not enough to explain the discrepancy between theory and experiment. The experimental result is NOT is good quantitative agreement with theory.



Project 1 (40 minutes): Our goal is to determine speed of a cylinder at the bottom of its swing. **WATCH OUT:** L should be measured from the center of mass of the cylinder to the point where the string attaches to the rod.

Use a table clamp, two rods, a right angle clamp, and a metal cylinder of diameter D on a string to create a simple pendulum (see figure at right). Measure L (I recommend about half a meter). Ensure the *center* of the cylinder breaks the beam of the photogate. Ensure you can record the release angle θ from the *vertical*.

Using $\theta = 60.0°$, release the pendulum from rest and record time in the photogate. Acceleration is negligible at the bottom of the swing, especially for the amount of time the cylinder requires to pass through the photogate. Determine cylinder velocity v_{exp} using time in the gate and D.

Derivations for Project 1:

1. Make a figure and use geometry/trig to derive an *algebraic* expression for h in terms of L & θ.
2. Use $E_i + W_{\substack{non-con \\ external}} = E_f$ to determine an expression for v_{th} at the bottom of the swing.
   a. Draw before & after figures separate from the figure used to determine h.
   b. Provide a statement explaining why $W_{\substack{non-con \\ external}} = 0$ for this problem.
   In particular, explain why string tension does zero work in this problem.
   c. Clearly label your reference level for gravitational potential energy.
   d. Start your work with this expression: $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$
   e. Show about 3 additional lines of work getting to your result for v_{th} (note: in this case $v_{th} = v_f$).
   Your final result must be algebraic in terms of $L, g, \& \theta$.
   f. Finally, plug in numbers to your algebraic result to determine a numerical value for v_{th}.
3. Show work estimating % precision for v_{exp}.
   a. You should have measurements of $D = (some\ \#) \pm \delta D$. You are expected to accurately estimate δD with units.
      i. Measure the cylinder at a few different spots to get an idea of the range of values you get. Use this range of values to estimate the value δD.
      ii. If you are always getting identical results, use the sig figs of the measuring device (1 in right most column) as an estimate of the value for δD.
   b. You should know t or v from Data Studio. We will assume this contributes negligible error.
   c. You should know $\theta = 60.0° \pm \delta\theta$. You are expected to estimate $\delta\theta$.
   d. You should know $L = 0.500\ \text{m} \pm \delta L$. You are expected to estimate δL.
   e. An overly cautious estimate of percent precision associated with measuring v_{exp} is given by

$$\%\ precision\ v_{exp} = \frac{\delta v_{exp}}{v_{exp}} \times 100\% = \left(\sqrt{\left(\frac{\delta D}{D}\right)^2 + \left(\frac{\delta\theta}{\theta}\right)^2 + \left(\frac{\delta L}{L}\right)^2}\right) \times 100\%$$

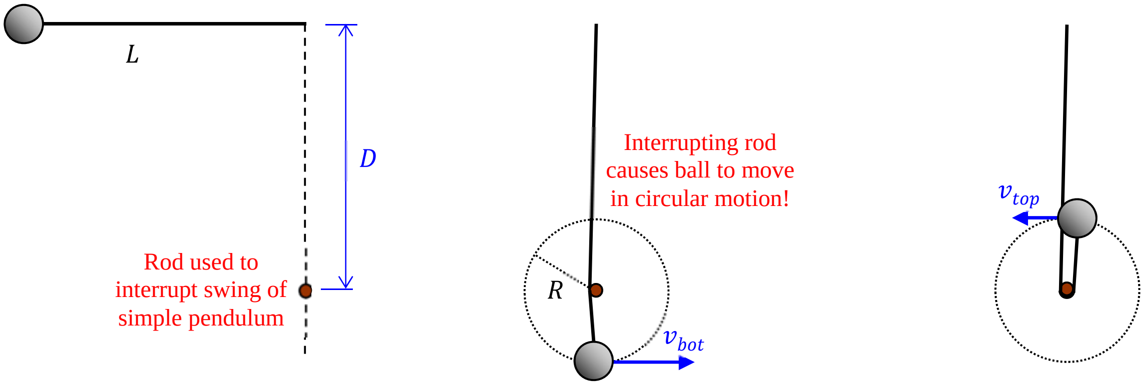
   f. You can now write your final experimental result as

$$v_{exp} = (some\ \#) \pm \%\ precision\ v_{exp}$$

Conclusions Project 1:

1) Was your experimental result in good agreement with the theoretical prediction?
2) Why is it useful to use a cylinder (instead of a sphere) as the pendulum in this experiment? Hint: consider how the calculation of v_{exp} would be affected for a misaligned sphere versus a misaligned cylinder.
3) Determine string tension (in N) at the bottom of the swing using an FBD. Show work for credit. Ignore error analysis for this question.

Project 2 (80 minutes): This apparatus (shown below) is called an interrupted pendulum. For this project use a sphere on the end of a string instead of a cylinder. Tip: don't use black string (it breaks); use braided string (white).



Our goal is to determine the largest possible value for D for which the sphere can still move in circular motion after the string impacts the rod. We can then compare this result to what is expected from conservation of energy!

Start with your pendulum held parallel to the ground as shown in the leftmost figure shown above.
Any sting length longer than $L = 0.500$ m should work (again, measure L to the *center* of the sphere).
Longer strings should give more precise results (but they are also more likely to hit you in the face).

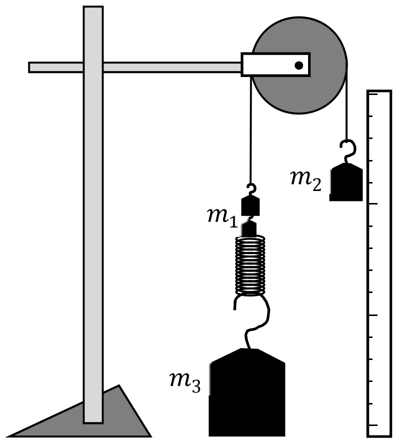
Determine D_{exp} by starting with a large value for D…only slightly shorter than L. Notice the string will impact the rod, causing the ball to rapidly spiral inwards. Try several times, each time with a slightly *shorter* D. At some point the string loses tension near the top of the circular motion. If the ball moves in projectiles over the rod, you need a slightly *longer* value for D. Without spending all day, determine the largest value of D for which the string loses tension *before* the ball reaches the top of circular motion. Record this as D_{exp} & estimate the associated error δD.

Derivations for Project 2:

1. Provide an equation explaining how the radius R of circular motion relates to D & L.
2. For the ball to remain in circular motion at the top of the loop, we know tension must be non-zero. Determine the *minimum* speed required (at the top of the loop) for the ball to remain in circular motion.
    a. Tip: use an FBD at the top of the circular motion.
    b. Think: what is string tension if the ball just barely makes it through the circular motion?
    c. Write your final answer for $v_{\substack{min \\ @top}}$ in terms of $L, D,$ & g.
3. Use $E_i + W_{\substack{non-con \\ external}} = E_f$ to determine an expression for D_{th}.
    a. Draw before & after figures (leftmost & rightmost shown above…ignore middle picture).
    b. Explain why $W_{\substack{non-con \\ external}} = 0$ for this problem (why does string tension do zero work)?
    c. Clearly label your reference level for gravitational potential energy.
    d. Start your work with this expression: $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$
    e. Show at least 6 additional lines of work getting to your result for D_{th}.
    Your final result must be a simplified fraction times L.
    f. Finally, plug in numbers to your algebraic result to determine a numerical value for D_{th}.

Conclusions Project 2:

1) Estimate your % errors in L and D. Add these % errors to estimate the % precision for D_{exp}.
Compare this to your % difference and state if your experimental result is in good agreement with the theory of the conservation of energy.



Project 3 (40 minutes): Build the apparatus shown at right.
For the largest mass I used $m_3 = 1000$ g.
Mass m_3 must be so large that it never moves; it is merely an anchor for this project.
For the other masses use $m_1 = 150$ g & $m_2 = 200$ g.

Notice the following:

- The spring must be UNSTRETCHED when released from rest.
- When released from rest, the smaller mass must never impact the pulley.

Our goal is to release the system from rest, record maximum extension of the spring, then use that information to determine the spring constant. We can then compare to the manufacturer's rated value of $k_{mfr} = 3.4 \frac{\text{N}}{\text{m}}$.

Get data for use in the derivation:

1. Verify the spring is unstretched every time before doing anything.
2. Record the location of the *bottom* of m_2 before releasing the system from rest (spring unstretched).
3. Record a video of m_2 as it reaches the bottom of its motion. Use the video to determine the max extension! Again use the location of the *bottom* of m_2 to determine max spring extension (x_{max}).
4. Repeat the experiment 5 times and tabulate all 5 values for x_{max} in Excel.

Derivations for Project 3:

1. Use $E_i + W_{\substack{non-con \\ external}} = E_f$ to determine an expression for k_{th}. Workbook problems **8.10** & **8.19** may help
   a. Draw a before picture (system @ rest, spring unstretched) and an after picture (at max extension).
   b. Explain why $W_{\substack{non-con \\ external}} = 0$ for this problem (why does string tension do zero work)?
      - Hint: the reasoning is NOT the same as in the previous two projects…
   c. Label two *separate* reference levels for gravitational potential energy (one each for m_1 & m_2).
   d. Start your work with this expression:

$$\frac{1}{2}m_1v_{1i}^2 + m_1gh_{1i} + \frac{1}{2}m_2v_{2i}^2 + m_2gh_{2i} + \frac{1}{2}kx_i^2 = \frac{1}{2}m_1v_{1f}^2 + m_1gh_{1f} + \frac{1}{2}m_2v_{2f}^2 + m_2gh_{2f} + \frac{1}{2}kx_f^2$$

   e. Include a statement clarifying why the *initial* velocities are zero.
   f. Include a statement clarifying why the *final* velocities are zero.
   g. Show several lines of work getting to your result for k_{th}.
      Your final result must be in terms of m_1, m_2, x_{max}, and g.
   h. In Excel, use this algebraic formula to generate 5 values for k_{th}.
   i. Use the average and standard deviation functions to get an estimate for % precision.
      - Bullets 2b & 2c from the following link explain how
      - http://www.robjorstad.com/BriefErrorAnalysis.pdf

Conclusions Project 3:

1) Compare your experimental result to $k_{mfr} = 3.4 \frac{\text{N}}{\text{m}}$ using a % difference.
   a. State if you believe the manufacturer is providing springs at the stated specification.
   b. Mention your % difference & % precision in a single sentence defending your statement.

Turn-in checklist:
Use plain white or engineering paper for all work.
Use one side of the paper for all work.
In the upper right hand corner of the first page I expect the following:

- your full name listed (denoted as author)
- your partners full names
- your lab meeting time

Staple your final packet in the top left corner.

Notes for each project should start on a new page, regardless of how much blank space this causes.
Energy problems should include at least one before and one after picture.
These pictures should be large and well labeled (fill approximately the top quarter of the page).
Occasionally you will also draw FBD's; make these at least a ¼-page in size as well.

Algebra should be clear and easy to follow

- Work down the page, not left to right
- If you run out of room, start a new page (regardless of how much blank space this causes)

For this week's lab, each project includes one or more conclusion questions.
Answer these questions before moving on to the next Project.
If you have ample blank space below your algebraic work, you needn't start the conclusion questions on a new page.
Answer these questions using full sentences which make clear what question was asked.
Include the rationale behind your answers for credit.

For **Projects 1 & 2**, you are expected to do most calculations by hand.
Your calculations should show the following:

- The algebraic equation
- The numbers plugged in without any simplification or math done
- One or two intermediate steps as needed (showing rounding digits underlined with at least one extra digit)
- Unrounded final answer (with appropriate units)
- Rounded final answer (with appropriate units)

For **Project 3**, you can print out a data table from Excel and share it as a group (give each student a copy).

- Format the table well (units, sig figs, proper subscripts, italics, include borders around each data table)
    - *Italics* should be used for *variables* (but not for numerical subscripts)
    - Units are not italicized
- **Hit print preview before wasting tons of paper!** Improve your table layout before printing.

Everything else must be individually hand-written.