

**Ch1****1.1**

a	3	d	1	g	3
b	3	e	3	h	3
c	3	f	2	i	1

**1.2**

a	$2.03 \times 10^2$	b	$2 \times 10^2$	c	$3.030 \times 10^{-2}$
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**1.3 Tip: try typing in the number in your calculator on “float” mode then change the mode to “scientific”.**

Number	# of sig figs	Scientific Notation
0.000354	3	$3.54 \times 10^{-4}$
80.5	3	$8.05 \times 10^1$
80.0	3	$8.00 \times 10^1$
1234	4	$1.234 \times 10^3$
12	2	$1.2 \times 10^1$
0.000000030	2	$3.0 \times 10^{-8}$
0.45600	5	$4.5600 \times 10^{-1}$
6700	2	$6.7 \times 10^3$
860	2	$8.6 \times 10^2$
860.0	4	$8.600 \times 10^2$

**1.4**

Scientific Notation	# of sig figs	Number
$1.23 \times 10^{-4}$	3	0.000123
$3.00 \times 10^0$	3	3.00
$1.000 \times 10^{-1}$	4	0.1000
$2.0 \times 10^5$	2	200000

**1.5 Tip: try typing in the number in your calculator on “float” mode then change the mode to “engineering”.**

Number	Engineering Notation
0.000354	$354 \times 10^{-6}$
80500	$80.5 \times 10^3$
0.00000003	$30 \times 10^{-9}$
1234.0	$1.2340 \times 10^3$
0.45600	$456.00 \times 10^{-3}$
860.0	$860.0 \times 10^0$
12	$12 \times 10^0$
$9.09 \times 10^7$	$90.9 \times 10^6$
$6.20 \times 10^{-11}$	$62.0 \times 10^{-12}$
$1.23 \times 10^{-3}$	$1.23 \times 10^{-3}$ (no change!)
$2.46 \times 10^4$	$24.6 \times 10^3$

**1.6 Use the mode trick on your calculator then use the equation sheet to convert the exponent to a prefix.**

Number	Engineering Notation	Eng. Not. With Prefix	Comment
0.0434 m	$43.4 \times 10^{-3} \text{ m}$	43.4 mm	
501000 V	$501 \times 10^3 \text{ V}$	501 kV	
0.00000000020 C	$200 \times 10^{-12} \text{ C}$	200 pC	To reduce ambiguity might say 0.20 nC
90000000 F	$90 \times 10^6 \text{ F}$	90 MF	
0.77 g	$770 \times 10^{-3} \text{ g}$	770 mg	To reduce ambiguity might say 0.77 g
120000000000 Bq	$120 \times 10^9 \text{ Bq}$	120 GBq	To reduce ambiguity might say 0.12 TBq
12 ft	12 ft	12 ft	
$8.08 \times 10^8 \text{ J}$	$808 \times 10^6 \text{ J}$	808 MJ	
$1.50 \times 10^{-13} \text{ N}$	$150 \times 10^{-15} \text{ N}$	150 fN	To reduce ambiguity might say 0.150 pN
$1.23 \times 10^{-8} \text{ bel}$	$12.3 \times 10^{-9} \text{ bel}$	12.3 nbel	
$2.46 \times 10^3 \text{ cd}$	$2.46 \times 10^3 \text{ cd}$	2.46 kcd	

**DO NOT USE THE PREFIX AND THE “10 to the”.**

As an example, consider the first row. **DO NOT** write  $43.4 \times 10^{-3} \text{ mm}$ .

If you include both the red and green parts, you are effectively double counting the  $10^{-3}$ .

If units are included in the problem statement, include the units in you answer.

A prefix makes no sense without a unit attached to it!

In case you were curious:

- V stands for Volts (SI unit of electric potential)
- C stands for Coulombs (SI unit of charge)
- F stands for Farads (SI unit of capacitance)
- Bq stands for Becquerel (SI unit of radioactivity)
- J stands for Joules (SI unit of energy)
- N stands for Newton (SI unit of force)
- bel is a unit equivalent to  $1/10^{\text{th}}$  of a decibel
- cd stands for candela (SI unit for luminous intensity)

## 1.7

	Statement	Final Answers in <b>RED BOLD</b>
a	$L = 0.40$ m in <i>scientific</i>	$4.0 \times 10^{-1}$ m
b	$L = 0.40$ m in <i>engineering with appropriate prefix</i>	$400 \times 10^{-3}$ m = <b>400 mm</b>
c	$w = 0.0808$ m in <i>scientific</i>	$8.08 \times 10^{-2}$ m
d	$w = 0.0808$ m in <i>engineering with appropriate prefix</i>	$80.8 \times 10^{-3}$ m = <b>80.8 mm</b>
e	$t = 750 \times 10^{-7}$ m in <i>scientific</i>	$7.5 \times 10^{-5}$ m
f	$t = 750 \times 10^{-7}$ m in <i>engineering with appropriate prefix</i>	$75 \times 10^{-6}$ m = <b>75 <math>\mu</math>m</b>

**WATCH OUT!** Please ensure your answers are *exactly* identical to the ones shown above. Ask about any differences, no matter how subtle. Some common sticking points are listed below.

- Leading zeros are never significant.
- Trailing zeros are significant when at least one lies to the right of a decimal.
  - The trailing zero in 0.40 *is* significant.
  - The trailing zero in 750 *is not* significant.
- The underbar can be used to indicate a trailing zero is significant.
  - The **first trailing zero** in **400** is significant; the **second trailing zero** is not.
- Include units if units are in problem statement. A prefix means nothing without a unit attached to it.
- Do not write the “**ten to the**” and the prefix! We use the prefix to avoid writing the “**ten to the**”.
- Be sure mu ( $\mu$ ) looks different from m. To write mu, draw a “u” then add a line on the left side.

## 1.8

a)  $\frac{120.0}{3} + 70 = \underline{40.0} + \underline{70} = \underline{110} = 1.1 \times 10^2$

$$\begin{array}{r} \underline{40.0} \\ + \underline{70} \\ \hline 110.0 \end{array}$$

b)  $13 + 0.741 = \underline{13} + \underline{0.741} = \underline{13.741} = 1.4 \times 10^1$

$$\begin{array}{r} \underline{13} \\ + \underline{0.741} \\ \hline 13.741 \end{array}$$

c)  $65.02 - 64.99 = 0.03 = 3 \times 10^{-2}$

$$\begin{array}{r} \underline{65.02} \\ - \underline{64.99} \\ \hline 0.03 \end{array}$$

d)  $(\underline{12.0} - \underline{9.99}) \times \underline{8.00} \times 10^6 = (\underline{2.01}) \times \underline{8.00} \times 10^6 = \underline{16.08} \times 10^6 = 1.6 \times 10^7$

$\begin{array}{r} \underline{12.0} \\ - \underline{9.99} \\ \hline 2.01 \end{array}$	
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e)  $\frac{(\underline{13.10} - \underline{13.00})^2}{(\underline{800} + \underline{300})} = \frac{(\underline{0.10})^2}{\underline{1100}} = \frac{0.010}{\underline{1100}} = \underline{9.09} \times 10^{-6} = 9.1 \times 10^{-6}$

$\begin{array}{r} \underline{13.10} \\ - \underline{13.00} \\ \hline 0.10 \end{array}$	$\begin{array}{r} \underline{800} \\ + \underline{300} \\ \hline 1100 \end{array}$
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1.9  $10^{\text{E}3} = 10 \times 10^3 = 10^4 = 10,000$  whereas  $10^3 = 1000$

1.10 The answer is  $1.8 \times 10^2$  km. If you goofed on this one, you're not the only one...see work below.

$$\begin{array}{r} 10120 \text{ km} \\ -9943.0 \text{ km} \\ \hline 177.0 \text{ km} \end{array}$$

When adding or subtracting you keep the leftmost *column*...not the number of sig figs.

Consider aligning the decimal points (as shown in bold above) so the leftmost column is obvious.

Units are required if they are included on numbers in physics problems!

### 1.11

- The sum is  $17540.221 \text{ N} = 17.54022 \times 10^3 \text{ N} = 17.54022 \text{ kN}$ .
  - The difference is  $0.427 \text{ N} = 427 \times 10^{-3} \text{ N} = 430 \text{ mN}$ .
  - The sum is  $0.00168121 \text{ m} = 1.68121 \times 10^{-3} \text{ m} = 1.6812 \text{ mm}$ .
  - The difference is  $0.00000327 \text{ m} = 3.27 \times 10^{-6} \text{ m} = 3.3 \text{ }\mu\text{m}$ .
- Include units if units are included in problem statement...seems obvious yet many forget.
  - Do not write the “ten to the” and the prefix! We use the prefix to avoid writing the “ten to the”.
  - Be sure mu ( $\mu$ ) looks different from m. To write mu, draw a “u” then add a line on the left side.

1.12 We are asked to compute

$$\begin{aligned} R &= 229.5(F) - E \\ R &= 346.5(20 \text{ mockingbird}) - 4.97 \times 10^3 \text{ mockingbird} \\ R &= 6930 \text{ mockingbird} - 4970 \text{ mockingbird} \\ &\quad 6930 \text{ mockingbird} \\ &\quad -4970 \text{ mockingbird} \\ &\quad \hline &\quad 1960 \text{ mockingbird} \\ &\quad 1.960 \times 10^3 \text{ mockingbird} \\ &\quad \textit{To Kill a Mockingbird} \end{aligned}$$

Talk about a lot of work for a lousy joke...

To be clear, there is no such unit...if you don't like it, talk into the millionth of a phone over there.

1.13 **Style 1:**  $Perimeter = (2 \times 4.0) + (2 \times 6.0) = 8.0 + 12.0 = 20.0 = 2.0 \times 10^1 \text{ m}$

**Style 2:**  $Perimeter = 4.0 + 4.0 + 6.0 + 6.0 = 20.0 = 2.00 \times 10^1 \text{ m}$

$Area = 4.0 \times 6.0 = 24.0 = 2.4 \times 10^1 \text{ m}^2$

Notice the style of calculating perimeter affects the number of sig figs in the result!

This is an indication that sig fig rules are not the be all end all of error analysis techniques. A better way is to use error propagation techniques discussed in a lab portion of the course. In this case the second style seems more reasonable.

1.14 We are told

$a \left( \frac{\text{m}}{\text{s}^2} \right)$	$v_{0x} \left( \frac{\text{m}}{\text{s}} \right)$	$t \text{ (s)}$
$-1.23 \times 10^{-1}$	$5.85 \times 10^{-3}$	$0.0987$

$$\begin{aligned}\Delta x &= \frac{1}{2} a_x t^2 + v_{0x} t \\ \Delta x &= 0.5000 \left( -1.23 \times 10^{-1} \frac{\text{m}}{\text{s}^2} \right) (0.0987 \text{ s})^2 + \left( 5.85 \times 10^{-3} \frac{\text{m}}{\text{s}} \right) (0.0987 \text{ s}) \\ \Delta x &= -0.0005991 \frac{\text{m}}{\text{s}^2} \cdot \text{s}^2 + 0.0005774 \frac{\text{m}}{\text{s}} \cdot \text{s} \\ \Delta x &= -0.0005991 \text{ m} + 0.0005774 \text{ m} \\ &\quad -0.0005991 \text{ m} \\ &\quad +0.0005774 \text{ m} \\ \Delta x &= -0.0000217 \text{ m} \\ \Delta x &= -2.17 \times 10^{-5} \text{ m} \\ \Delta x &= -21.7 \times 10^{-6} \text{ m} \\ \Delta x &= -21.7 \mu\text{m} \\ \Delta x &= -22 \mu\text{m}\end{aligned}$$

1.15

a) Rearranging the formula gives

$$\begin{aligned}v_{iy}^2 &= v_{fy}^2 - 2a_y \Delta y \\ v_{iy} &= \pm \sqrt{v_{fy}^2 - 2a_y \Delta y}\end{aligned}$$

$a_y \left( \frac{\text{m}}{\text{s}^2} \right)$	$v_{fy} \left( \frac{\text{m}}{\text{s}} \right)$	$\Delta y \text{ (m)}$
$-9.8$	$9.2 \times 10^1$	$8.7 \times 10^1$

It is often crucial to remember the  $\pm$  when taking the square root!

b) Plugging in gives

$$v_{iy} = \pm \sqrt{\left( 9.2 \times 10^1 \frac{\text{m}}{\text{s}} \right)^2 - 2 \left( -9.8 \frac{\text{m}}{\text{s}^2} \right) (8.7 \times 10^1 \text{ m})}$$

Notice the second term becomes positive!

$$\begin{aligned}v_{iy} &= \pm \sqrt{8464 \frac{\text{m}^2}{\text{s}^2} + 1705 \frac{\text{m}}{\text{s}^2} \cdot \text{m}} \\ v_{iy} &= \pm \sqrt{8464 \frac{\text{m}^2}{\text{s}^2} + 1705 \frac{\text{m}^2}{\text{s}^2}} \\ &\quad \frac{8464}{+1705} \\ &\quad 10169 \\ v_{iy} &= \pm \sqrt{10169 \frac{\text{m}^2}{\text{s}^2}} \\ v_{iy} &= \pm \sqrt{10169} \sqrt{\frac{\text{m}^2}{\text{s}^2}} \\ v_{iy} &= \pm 100.8 \frac{\text{m}}{\text{s}} \\ v_{iy} &= \pm 101 \frac{\text{m}}{\text{s}}\end{aligned}$$

**Tip:** many engineers write an extra sig fig when the first digit is 1. Said another way, many engineers stop at the line in bold above because the first digit in the final answer is 1.

$$1.16 V_{sphere} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \left(\frac{D}{2}\right)^3 = \frac{\pi}{6}D^3$$

$$1.17 V_{cyl} = \pi R^2 H = \pi \left(\frac{D}{2}\right)^2 H = \frac{\pi}{4}D^2 H$$

## 1.18

- a) We find  $a_1$  has 2 sig figs while  $a_2$  has 3 sig figs. *Leading* zeros (at the *left* side of the number) are never significant. *Trailing* zeros (at the *right* end of the number) are significant **IF** at least one of the trailing zeroes is to the right of the decimal.

- b) The conversion is shown below.

Note: because 100 cm = 1 m and 1000 mm = 1 m we know 1000 mm = 100 cm or 10 mm = 1 cm.

$$0.060 \frac{\text{mm}}{\text{s}^2} \times \frac{1 \text{ cm}}{10 \text{ mm}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{(3600 \text{ s})^2}{(1 \text{ hr})^2} = 2551 \frac{\text{ft}}{\text{hr}^2} = 2.6 \times 10^3 \frac{\text{ft}}{\text{hr}^2}$$

**TIP:** if unit is *squared* in the conversion remember to square the units **AND THE NUMBERS** in that conversion factor. Yes...I'm trying to shout at you so you don't miss this on test day.

**TIP:** unless the conversion factor specifies the number of sig figs with an underbar, conversion factors are typically assumed to have infinite sig figs. Most conversions will not change the number of sig figs.

- c) The conversion is shown below.

$$49500 \frac{\text{ft}}{\text{hr}^2} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{(1 \text{ hr})^2}{(3600 \text{ s})^2} = 0.001164 \frac{\text{m}}{\text{s}^2} = 1.164 \times 10^{-3} \frac{\text{m}}{\text{s}^2} = 1.164 \frac{\text{mm}}{\text{s}^2}$$

Note: because 10 mm = 1 cm, an alternative solution method is shown below.

$$49500 \frac{\text{ft}}{\text{hr}^2} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{10 \text{ mm}}{1 \text{ cm}} \times \frac{(1 \text{ hr})^2}{(3600 \text{ s})^2} = 1.164 \frac{\text{mm}}{\text{s}^2}$$

**TIP:** even though sig figs indicate you should round to 3 places, engineers often keep an extra sig fig if the first digit of a number is a 1. Also, you could write this as  $1.164 \times 10^0 \frac{\text{mm}}{\text{s}^2}$ ...but why?

$$1.19 v = 60 \frac{\text{mi}}{\text{hr}} \times \frac{1609 \text{ m}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 26.8 \frac{\text{m}}{\text{s}} \approx 27 \frac{\text{m}}{\text{s}} \text{ (a convenient approximation is } \div 2 \text{ then } -10\%)$$

$$1.20 \rho = 1.010 \times 10^2 \frac{\text{g}}{\text{in}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)^3 \times \left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = 6163 \frac{\text{kg}}{\text{m}^3} = 6.163 \times 10^3 \frac{\text{kg}}{\text{m}^3}$$

Note: while it appears the conversion from in to cm should reduce the number of sig figs it does not. This conversion is defined as 1 in = 2.54 cm. Typically, unit conversions do not change the number of sig figs.

**Challenge:** I know the density of aluminum is 2700 kg/m<sup>3</sup> while the density of iron is about 8000 kg/m<sup>3</sup>. We are told we have part aluminum and part iron. One can write the following systems of equations:

$$\begin{aligned} m_{Tot} &= m_1 + m_2 \\ V_{Tot} &= V_1 + V_2 \\ \rho_{Tot} &= \frac{m_{Tot}}{V_{Tot}} = \frac{m_1 + m_2}{V_{Tot}} = \frac{V_1 \rho_1 + V_2 \rho_2}{V_{Tot}} = \frac{V_1 \rho_1 + (V_{Tot} - V_1) \rho_2}{V_{Tot}} = \frac{V_1}{V_{Tot}} (\rho_1 - \rho_2) + \rho_2 \end{aligned}$$

$$\text{This gives } \frac{V_1}{V_{Tot}} = \frac{\rho - \rho_2}{\rho_1 - \rho_2} = \frac{6163 - 8000}{2700 - 8000} \approx 35\%$$

A much simpler solution is to solve for  $x$  in the expression  $\rho_{Tot} = x\rho_1 + (1 - x)\rho_2$ . Same result.

1.21

$$13.5 \frac{\text{lbs}}{\text{day}^2} \times \frac{453.6 \text{ g}}{1 \text{ lbs}} \times \frac{1 \text{ kg}}{10^3 \text{ g}} \times \frac{(1 \text{ day})^2}{(24 \text{ hrs})^2} \times \frac{(1 \text{ hr})^2}{(3600 \text{ s})^2} = 8.203 \times 10^{-10} \frac{\text{kg}}{\text{s}^2} = 8.20 \times 10^{-10} \frac{\text{kg}}{\text{s}^2}$$

**Remember to square the number and the units for terms in red!!!**

Don't forget: unless otherwise specified, most of your conversions have infinite sig figs.

When I say a number has infinite sig figs, I mean the sig figs of that number will have no effect on the final result.

Notice the conversion from pounds to grams does have sig figs indicated. It is an exception to the rule.

Either of the answers shown in bold (with underline or without) are fine for Jorstad's exams.

1.22

$$12.3 \frac{\text{in}}{\text{hr}^2} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} \times \frac{(1 \text{ hr})^2}{(3600 \text{ s})^2} = 2.411 \times 10^{-8} \frac{\text{m}}{\text{s}^2} = 24.11 \times 10^{-9} \frac{\text{m}}{\text{s}^2} = 24.11 \frac{\text{nm}}{\text{s}^2} = 24.1 \frac{\text{nm}}{\text{s}^2}$$

**Remember to square the number and the units for terms in red!!!**

Either of the answers shown in bold (with underline or without out) are fine for Jorstad's exams.

**1.23** Rearrange  $\rho = \frac{m}{V}$  to solve for mass. One finds  $m = \rho V$ .

By looking at the units we know  $V = 4.0 \text{ cm}^3$  &  $\rho = 2.7 \frac{\text{g}}{\text{cm}^3}$ .

$$m = \rho V = \left(2.7 \frac{\text{g}}{\text{cm}^3}\right) \times (4.0 \text{ cm}^3) = 10.8 \text{ g} = 11 \text{ g} = 1.1 \times 10^1 \text{ g}$$

**1.24** Rearrange  $\rho = \frac{m}{V}$  to solve for volume. One finds  $V = \frac{m}{\rho}$ .

If I have  $m = 4.2 \times 10^2 \text{ kg}$  of aluminum with  $\rho = 2700 \frac{\text{kg}}{\text{m}^3}$  I find

$$V = \frac{m}{\rho} = \frac{4.2 \times 10^2 \text{ kg}}{2700 \frac{\text{kg}}{\text{m}^3}} = \frac{420 \text{ kg}}{2700} \times \frac{\text{m}^3}{\text{kg}} = 0.156 \text{ m}^3 = 0.16 \text{ m}^3 = 1.6 \times 10^{-1} \text{ m}^3$$

1.25

$$10^5 \frac{\text{ft}^3}{\text{s}} \times \frac{12^3 \text{ in}^3}{1^3 \text{ ft}^3} \times \frac{2.54^3 \text{ cm}^3}{1^3 \text{ in}^3} \times \frac{1^3 \text{ m}^3}{100^3 \text{ cm}^3} \times \frac{1000 \text{ kg}}{1 \text{ m}^3} \times \frac{3600 \text{ s}}{1 \text{ hr}} \times \frac{24 \text{ hr}}{1 \text{ day}} = 2.45 \times 10^{11} \frac{\text{kg}}{\text{day}}$$

$$\approx 3 \times 10^{11} \frac{\text{kg}}{\text{day}}$$

**1.26** For part a), we know that the volume of the nut is given by

$$\begin{aligned}
 V_{nut} &= V_{box} - V_{cyl} \\
 V_{nut} &= LWH - \pi R^2 H \\
 V_{nut} &= s^2 t - \pi \left(\frac{D}{2}\right)^2 t \\
 V_{nut} &= s^2 t - \frac{\pi}{4} D^2 t \\
 V_{nut} &= \left(s^2 - \frac{\pi}{4} D^2\right) t
 \end{aligned}$$

For part b) we want the mass of the nut. Since we have an equation for volume, use density to convert!

$$\begin{aligned}
 \rho &= \frac{m}{V} \rightarrow m = \rho V \\
 m_{nut} &= \rho V_{nut} \\
 m_{nut} &= \rho \left(s^2 - \frac{\pi}{4} D^2\right) t
 \end{aligned}$$

Since this is messy, let's first convert everything to cm! Why? The density has units of  $\frac{\text{g}}{\text{cm}^3}$ .

$t = 4.50 \times 10^{-3} \text{ m} = 0.450 \text{ cm}$	$D = 50.3 \text{ mm} = 5.03 \text{ cm}$	$s = 2.00 \text{ in} = 5.08 \text{ cm}$
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Now look at the term in parentheses and try to get the sig figs correct.

$$s^2 - \frac{\pi}{4} D^2 = (5.08 \text{ cm})^2 - \frac{\pi}{4} (5.03 \text{ cm})^2 = 25.81 \text{ cm}^2 - 19.87 \text{ cm}^2 = 5.935 \text{ cm}^2$$

**WATCH OUT!** When you subtract two numbers the number of sig figs can change! Remember when subtracting (or adding) you keep the *column* of sig figs...not the *number* of sig figs. Ask your instructor if you don't get it.

$$\begin{aligned}
 m_{nut} &= \rho \left(s^2 - \frac{\pi}{4} D^2\right) t \\
 m_{nut} &= 19.3 \frac{\text{g}}{\text{cm}^3} (5.935 \text{ cm}^2) 0.450 \text{ cm} = 51.55 \text{ g} = 0.052 \text{ kg} = 5.2 \times 10^{-2} \text{ kg}
 \end{aligned}$$



1.27

$$\begin{aligned}
 12 \mu\text{\AA} &= 12 \times 10^{-6} \text{\AA} \\
 \frac{12 \times 10^{-6} \text{\AA}}{1 \text{\AA}} \times \frac{10^{-10} \text{ m}}{1 \text{\AA}} \times \frac{1 \text{ fm}}{10^{-15} \text{ m}} &= 1.2 \text{ fm} \\
 A_{\text{sphere}} &= 4\pi R^2 = 4\pi \left(\frac{D}{2}\right)^2 = \pi D^2 = \pi (1.2 \text{ fm})^2 = 4.52 \text{ fm}^2
 \end{aligned}$$

Another style might look like this:

$$\begin{aligned}
 12 \mu\text{\AA} &= 12 \times 10^{-6} \text{\AA} \\
 A_{\text{sphere}} &= \pi D^2 = \pi (12 \times 10^{-6} \text{\AA})^2 = 452 \times 10^{-12} \text{\AA}^2 \\
 \frac{452 \times 10^{-12} \text{\AA}^2}{1^2 \text{\AA}^2} \times \frac{(10^{-10})^2 \text{ m}^2}{(10^{-15})^2 \text{ m}^2} &\times \frac{1^2 \text{ fm}^2}{(10^{-15})^2 \text{ m}^2} = 452 \times 10^{-2} \text{ fm}^2 = 4.52 \text{ fm}^2
 \end{aligned}$$

**1.28** For problems talking about mass, density and/or volume, get the density and volume equations involved. Furthermore, for an object with a hole cut out, I typically start with trying to get a formula for the volume.

$$\begin{aligned}
 V_{\text{plate}} &= V_{\text{cyl}} - V_{\text{box}} \\
 V_{\text{plate}} &= \pi r^2 t - LWH \\
 V_{\text{plate}} &= \pi \left(\frac{D}{2}\right)^2 t - \left(\frac{D}{3}\right) \left(\frac{D}{3}\right) t \\
 V_{\text{plate}} &= \frac{\pi}{4} D^2 t - \frac{1}{9} D^2 t \\
 V_{\text{plate}} &= \left(\frac{\pi}{4} - \frac{1}{9}\right) D^2 t
 \end{aligned}$$

You could be a purist and keep all those exact numbers out front, but, in this case, a decimal number is often easier for humans to quickly understand.

$$V_{\text{plate}} = 0.6743 D^2 t$$

Now that we know the volume of the plate, we can relate this to mass and density using

$$\begin{aligned}
 \rho_{\text{plate}} &= \frac{M_{\text{plate}}}{V_{\text{plate}}} \\
 \rho_{\text{plate}} &= \frac{M}{0.6743 D^2 t}
 \end{aligned}$$

It is considered bad form to leave that decimal number in the denominator. To move it to the top do the following:

$$\begin{aligned}
 \rho_{\text{plate}} &= \frac{1}{0.6743} \cdot \frac{M}{D^2 t} \\
 \rho_{\text{plate}} &= 1.483 \frac{M}{D^2 t}
 \end{aligned}$$

**NOTICE NO UNITS ARE INCLUDED ON THIS ANSWER.**

This is the correct way to answer an *algebraic* problem.

Also notice the number is out front, multiplied by the algebraic information (not in the denominator).

This is standard form for algebraic problems as it allows the user of this equation to plug in their own numbers using their desired units.

Think about it: what if you plugged in numbers like  $M = 1.25 \text{ kg}$ ,  $D = 0.254 \text{ m}$ , &  $t = 0.056 \text{ m}$ ? The numbers that people will plug in produce the correct units for the output...not the number 1.483.

## 1.29

- a) In physics, it is often useful to split up a tricky geometry into several smaller shapes with easy to remember formulas. Rather than look up the area of a hexagon, why not split it into six equilateral triangles?

Rather than memorize the height of an equilateral triangle, one could derive it. Since the triangles are equilateral, we know the angle shown in the figure is  $60^\circ$ . Using SOH CAH TOA gives

$$\tan 60^\circ = \frac{\text{opp}}{\text{adj}} = \frac{h}{\frac{s}{2}}$$

$$h = \frac{s}{2} \tan 60^\circ$$

I don't have tangent for the trig wheel memorized but I do know sine and cosine. I know

$$\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

This gives

$$h = \frac{\sqrt{3}}{2} s$$

The area of *one* triangle is

$$A_{tri} = \frac{1}{2} \text{base} \times \text{height} = \frac{1}{2} (s) \left( \frac{\sqrt{3}}{2} s \right) = \frac{\sqrt{3}}{4} s^2$$

The area of the entire hexagon is

$$A_{hex} = 6A_{tri} = 6 \left( \frac{\sqrt{3}}{4} s^2 \right) = \frac{3\sqrt{3}}{2} s^2$$

The *volume* of the solid hexagon is  $V_{hex} = A_{hex}t = \frac{3\sqrt{3}}{2} s^2 t$ .

The volume of the hole is  $V_{cyl} = \pi r^2 t = \pi \left( \frac{D}{2} \right)^2 t = \frac{\pi}{4} D^2 t$ .

The volume of metal in the nut is thus

$$V_{nut} = V_{hex} - V_{cyl} = \left( \frac{3\sqrt{3}}{2} s^2 - \frac{\pi}{4} D^2 \right) t$$

- b) Rather than convert all dimensions to cm, I will convert the density to  $\frac{\text{g}}{\text{mm}^3}$ . I'm doing this because I thought it might be good practice for physics exams. Most people probably convert all mm to cm since you need only shift the decimal point one place to the left. Either way, the result should be the same...

$$2.7 \frac{\text{g}}{\text{cm}^3} \times \frac{(1 \text{ cm})^3}{(10 \text{ mm})^3} = 0.0027 \frac{\text{g}}{\text{mm}^3} = 2.7 \times 10^{-3} \frac{\text{g}}{\text{mm}^3}$$

Then use  $\rho = \frac{m}{V} \rightarrow m = \rho V$

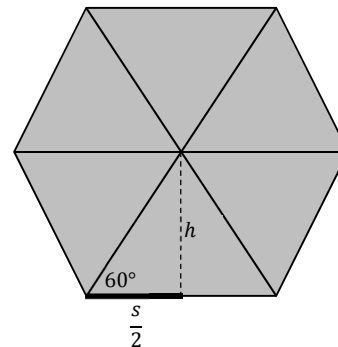
$$m_{nut} = \rho V_{nut}$$

$$m_{nut} = \rho \left( \frac{3\sqrt{3}}{2} s^2 - \frac{\pi}{4} D^2 \right) t$$

$$m_{nut} = \left( 2.7 \times 10^{-3} \frac{\text{g}}{\text{mm}^3} \right) \left( \frac{3\sqrt{3}}{2} (6.0 \text{ mm})^2 - \frac{\pi}{4} (10.7 \text{ mm})^2 \right) (2.0 \text{ mm})$$

$$m_{nut} = \left( 2.7 \times 10^{-3} \frac{\text{g}}{\text{mm}^3} \right) (93.53 \text{ mm}^2 - 89.92 \text{ mm}^2) (2.0 \text{ mm})$$

$$m_{nut} = \left( 2.7 \times 10^{-3} \frac{\text{g}}{\text{mm}^3} \right) (3.61 \text{ mm}^2) (2.0 \text{ mm}) = 0.01949 \text{ g} = 19 \text{ mg} \approx 20 \text{ mg}$$



## 1.30

- a) To determine the diameter of the initial drop using back-up values

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\pi}{6}d^3$$

$$d = \sqrt[3]{\frac{6V}{\pi}}$$

$$d = \sqrt[3]{\frac{6(5 \times 10^{-10}\text{m}^3)}{\pi}} \approx 1 \text{ mm}$$

For reference, this is about the size of the little ball used in the writing tip of ball point pens.

- b)  $V = \pi\left(\frac{D}{2}\right)^2 t = \pi\frac{D^2}{4}t$  which can be solved for  $t$  to give  $t = \frac{4V}{\pi D^2}$ . Check the units...
- c) Using back-up values:

$$V = 0.5 \times 10^{-6}\text{L} \times \frac{1000\text{cm}^3}{1\text{L}} \times \frac{(1\text{m})^3}{(100\text{cm})^3} = 5 \times 10^{-10}\text{m}^3$$

Don't forget  $D = 60 \text{ cm} = 0.6 \text{ m}$ . Plugging in gives

$$t = \frac{4V}{\pi D^2}$$

$$t = \frac{4(5 \times 10^{-10}\text{m}^3)}{\pi(0.6\text{m})^2}$$

$$t = 1.76 \times 10^{-9} \text{ m}$$

$$t \approx 2 \text{ nm} = 20\text{\AA}$$

Notice the diameter of the slick is about 600 times the diameter of the drop!!!

Side note: in theory you could use this information and some chemistry knowledge to estimate the size of the atoms in the olive oil molecule.

**1.31** For problems talking about mass, density and/or volume, get the density and volume equations involved. Furthermore, for an object with a hole cut out, I typically start with trying to get a formula for the volume.

$$\begin{aligned}
 V_{plate} &= V_{tri-plate} - V_{cyl} \\
 V_{plate} &= \frac{1}{2} \text{base} \cdot \text{height} - \pi r^2 \cdot \text{height} \\
 V_{plate} &= \frac{1}{2} s^2 x - \pi \left( \frac{D}{2} \right)^2 x \\
 V_{plate} &= \frac{1}{2} s^2 x - \pi \left( \frac{\frac{s}{4}}{2} \right)^2 x \\
 V_{plate} &= \frac{1}{2} s^2 x - \pi \left( \frac{s}{8} \right)^2 x \\
 V_{plate} &= \frac{1}{2} s^2 x - \frac{\pi}{64} s^2 x \\
 V_{plate} &= \left( \frac{1}{2} - \frac{\pi}{64} \right) s^2 x
 \end{aligned}$$

You could be a purist and keep all those exact numbers out front, but, in this case, a decimal number is often easier for humans to quickly understand.

$$V_{plate} = 0.4509 s^2 x$$

Now that we know the volume of the plate, we can relate this to mass and density using

$$\begin{aligned}
 \rho_{plate} &= \frac{M_{plate}}{V_{plate}} \\
 \rho_{plate} &= \frac{M}{0.4509 s^2 x} \\
 \rho_{plate} &= 2.218 \frac{M}{s^2 x}
 \end{aligned}$$

**NOTICE NO UNITS ARE GIVEN FOR THIS ANSWER.**

This is the correct way to answer an *algebraic* problem.

Also notice the number is out front, multiplied by the algebraic information (not in the denominator).

This is standard form for algebraic problems as it allows the user of this equation to plug in their own numbers using their desired units.

Think about it: what if you plugged in numbers like  $M = 1.25 \text{ kg}$ ,  $s = 0.254 \text{ m}$ , &  $x = 0.056 \text{ m}$ ? The numbers that people will plug in produce the correct units for the output...not the number 2.218.

**1.32** This problem talks about volume, mass and density. I would first derive the equation for the volume of the object, then shove that into the density equation, re-read the problem, and finally solve for something.

$$V_{total} = V_{cyl} - V_{hemi}$$

$$V_{total} = \pi R^2 h - \frac{1}{2} \left( \frac{4}{3} \pi r^3 \right)$$

Notice the radii are not the same; I used big  $R$  for the cylinder and little  $r$  for the hemisphere.

Because the problem statement gave diameter of cylinder use  $R = \frac{D}{2}$

$$V_{total} = \frac{\pi}{4} D^2 h - \frac{2}{3} \pi r^3$$

We also know  $\rho = \frac{m}{V}$  which can be rearranged to give  $V = m/\rho$ . This gives

$$\frac{\pi}{4} D^2 h - \frac{2}{3} \pi r^3 = \frac{m}{\rho}$$

Now solve this for little  $r$ . Convert all numbers to similar units. Then plug in those numbers to get a result.

$$\frac{2}{3} \pi r^3 = \frac{\pi}{4} D^2 h - \frac{m}{\rho}$$

$$r^3 = \frac{3}{2\pi} \left( \frac{\pi}{4} D^2 h - \frac{m}{\rho} \right)$$

$$r = \sqrt[3]{\frac{3}{2\pi} \left( \frac{\pi}{4} D^2 h - \frac{m}{\rho} \right)}$$

Convert mass $m$ to g	$5.55 \text{ lbs} \times \frac{0.4536 \text{ kg}}{1 \text{ lbs}} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 2517.5 \text{ g}$ <p>The kg to g conversion has infinite sig figs.</p>
Convert height $h$ to cm	$2.40 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 6.096 \text{ cm}$ <p>The in to cm conversion has infinite sig figs (this is how inch is defined)</p>
Convert diameter $D$ to cm	$4.00 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}} = 10.16 \text{ cm}$
Leave density $\rho$ as is in $\frac{\text{g}}{\text{cm}^3}$	$\rho = 8.73 \frac{\text{g}}{\text{cm}^3}$

$$r = \sqrt[3]{\frac{3}{2\pi} \left( \frac{\pi}{4} (10.16 \text{ cm})^2 (6.096 \text{ cm}) - \frac{2517.5 \text{ g}}{8.73 \frac{\text{g}}{\text{cm}^3}} \right)}$$

$$r = \sqrt[3]{\frac{3}{2\pi} (494.22 \text{ cm}^3 - 288.37 \text{ cm}^3)}$$

$$r = \sqrt[3]{\frac{3}{2\pi} 205.85 \text{ cm}^3}$$

$$r = 4.615 \text{ cm} \approx 4.62 \text{ cm} = 4.62 \times 10^{-2} \text{ m} = 46.2 \text{ mm}$$

Problem asked for sci notation or engr with appropriate prefix (last two bold answers). Some things to check:

- The units worked out (that's a good sign).
- The radius of the hemisphere is less than the height and the diameter of the hemisphere is less than the diameter of the cylinder. This means such a hole could actually be removed from the cylinder.
- If problem statement gives decimal numbers with units, answers have decimal numbers with units.
- If using stored numbers on calculator you get 46.149 mm=46.1 mm. Both answers differ from unrounded final answer by 50 microns, about 1/2 of a human hair. Arguing over this is literally like splitting hairs!

## 1.33

**Be careful. When you write  $v$  it doesn't mean the same thing as  $[v]$ .**

- Here  $v$  is the variable speed which includes both a number and units when you substitute in a value.
- Here  $[v]$  implies you should write only *the units of speed*.
- After you write the units of speed, the square brackets disappear.
- Notice the following tricky points:
  - $m$  (italicized) implies the variable for mass which has *units* of kg
  - the unit  $m$  (not italicized) stands for meters, the SI unit of length
  - $[m] = \text{kg}$  while  $[x] = m$

$$\text{a) } [K] = \left[\frac{1}{2}\right] [m][v^2] = [m][v]^2 = \text{kg} \cdot \left(\frac{\text{m}}{\text{s}}\right)^2 = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

$$\text{b) } [\rho] = \frac{[m]}{[v]} = \frac{\text{kg}}{\text{m}^3}$$

$$\text{c) } [P_D] = \left[\frac{1}{2}\right] [\rho][v^2] = [\rho][v]^2 = \frac{\text{kg}}{\text{m}^3} \cdot \left(\frac{\text{m}}{\text{s}}\right)^2 = \frac{\text{kg}}{\text{m} \cdot \text{s}^2}$$

$$\text{d) } [\omega] = \sqrt{\left[\frac{g}{L}\right]} [\tan \theta] = \sqrt{\left[\frac{g}{L}\right]} = \sqrt{\frac{\frac{\text{m}}{\text{s}^2}}{L}} = \sqrt{\frac{\text{m}}{\text{s}^2} \times \frac{1}{\text{m}}} = \sqrt{\frac{1}{\text{s}^2}} = \frac{1}{\text{s}} = \text{rad}$$

**Note: the radians unit is inserted by convention.** To be clear, no units is equivalent to radians.

**If you missed this it is unimportant until chapter 6 or so...**

## 1.34

- a) We see from the equations that  $[m][a] = [-1][b][v^2]$ . Rearranging and ignoring the dimensionless  $[-1]$  gives

$$[b] = \frac{[m][a]}{[v^2]} = \frac{[m][a]}{[v]^2} = \frac{\text{kg} \cdot \frac{\text{m}}{\text{s}^2}}{\left(\frac{\text{m}}{\text{s}}\right)^2} = \frac{\text{kg} \cdot \frac{\text{m}}{\text{s}^2}}{\frac{\text{m}^2}{\text{s}^2}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \times \frac{\text{s}^2}{\text{m}^2} = \frac{\text{kg}}{\text{m}}$$

- b) Solving first for  $G$  gives

$$[G] = \frac{[F][r]^2}{[m][M]} = \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} = \frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m}^2}{\text{kg}^2} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m}^2 \times \frac{1}{\text{kg}^2} = \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$$

- c) The left side of the equation is the position variable  $x$ .

The position variable is measured in units of meters (**m**).

Therefore, we expect all terms on the right side of the equation should have units of meters (**m**).

The 1<sup>st</sup> term on the right hand side has units of **m** but the 2<sup>nd</sup> term has units of  $\frac{\text{s}^2}{\text{m}^{1/2}}$ .

The equation is not dimensionally correct.

- d) We find

$$[x] = [A][\cos \omega t] = [A] \times \text{no units} = [A] = m$$

Notice the function is dimensionless and thus drops out.

To determine the units of  $\omega$ , use the fact that the argument of the function is dimensionless (or radians).

$$[\omega t] = \text{rad}$$

Note: if this was something other than a trig function you would say no units instead of radians.

Continuing we find

$$[\omega t] = [\omega][t] = \text{rad}$$

$$[\omega] = \frac{\text{rad}}{[t]} = \frac{\text{rad}}{\text{s}}$$

- e) Since  $-at$  is the argument of the exponential function we find  $[\alpha] = \frac{1}{[t]} = \frac{1}{\text{s}}$ .

**1.35** The variable  $\mu_s$  has *no units*. We will learn later  $\mu_s$  is called the *coefficient of static friction*.

$$\begin{aligned}
 v &= \sqrt{\frac{rg}{\mu_s}} \\
 v^2 &= \frac{rg}{\mu_s} \\
 \mu_s &= \frac{rg}{v^2} \\
 [\mu_s] &= \frac{[r][g]}{[v^2]} \\
 [\mu_s] &= \frac{\text{m} \cdot \frac{\text{m}}{\text{s}^2}}{\frac{\text{m}^2}{\text{s}^2}} \\
 [\mu_s] &= \text{m} \cdot \frac{\text{m}}{\text{s}^2} \cdot \frac{\text{s}^2}{\text{m}^2} \\
 [\mu_s] &= \text{no units!!!}
 \end{aligned}$$

**1.36** When given an equation, we assume the units of *each term* in the equation must be equivalent.

In this case we thus assume *the units of F* must match *the units of* the term  $ax$ .

To be clear, force does *not* equal  $ax$  ( $F \neq ax$ )...the units of force *should* equal the units of  $ax$  ( $N = [ax]$ ).

It is fastest to write the above statement with [ ] notation:  $[F] = [ax]$ .

Similarly,  $[F] = [bx^3]$ .

One finds

$$\begin{aligned}
 [a] &= \frac{N}{m} = \frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{m} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{1}{m} = \frac{\text{kg}}{\text{s}^2} \\
 [b] &= \frac{N}{m^3} = \frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{m^3} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{1}{m^3} = \frac{\text{kg}}{m^2 \cdot \text{s}^2}
 \end{aligned}$$

**1.37** When given an equation, we assume the units of *each term* in the equation must be equivalent.

In this case we thus assume *the units of F* must match *the units of* the term  $cv$ .

To be clear, force does *not* equal  $cv$  ( $F \neq cv$ )...the units of force *should* equal the units of  $cv$  ( $N = [cv]$ ).

It is fastest to write the above statement with [ ] notation:  $[F] = [cv]$ .

Similarly,  $[F] = [dv^3]$ .

One finds

$$\begin{aligned}
 [c] &= \frac{N}{\frac{m}{s}} = \frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{\frac{m}{s}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{s}{m} = \frac{\text{kg}}{s} \\
 [d] &= \frac{N}{\frac{m^2}{s^2}} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{s^2}{m^2} = \frac{\text{kg}}{m}
 \end{aligned}$$

## 1.38

- a) If the equation is simple (one term on each side), first isolate the variable of interest. This is not required but it is just the style I like. Then I get the units of each variable. Sometimes this avoids some mistakes associated with fractions within fractions.

$$\begin{aligned}
 PV &= nRT \\
 R &= \frac{PV}{nT} \\
 [R] &= \frac{[P][V]}{[n][T]} \\
 [R] &= \frac{\text{Pa} \cdot \text{m}^3}{\text{mol} \cdot \text{K}} \\
 [R] &= \frac{\frac{\text{N}}{\text{m}^2} \cdot \text{m}^3}{\text{mol} \cdot \text{K}} \\
 [R] &= \frac{\text{N} \cdot \text{m}}{\text{mol} \cdot \text{K}} \\
 [R] &= \frac{\frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{m}}{\text{mol} \cdot \text{K}} \\
 [R] &= \frac{\text{kg} \cdot \text{m}^2}{\text{mol} \cdot \text{K} \cdot \text{s}^2}
 \end{aligned}$$

If you already know  $1 \text{ J} = 1 \text{ N} \cdot \text{m}$  you might recognize the more commonly used units  $[R] = \frac{\text{J}}{\text{mol} \cdot \text{K}}$ .

- b) With this more complicated looking equation is

$$\left[ P + a \left( \frac{n}{V} \right)^2 \right] \left( \left( \frac{V}{n} \right) - b \right) = RT$$

it is sometimes wise to look at just a part of it to figure out the units. I like to say you can only add apples to apples and oranges to oranges. This means the units of the two *red* parts must be equal. Likewise, the units of the two *blue* parts must be equal. Note: the units of the red part do not necessarily equal the units of the blue part. Terms in an equation must have the same units if they are added (or subtracted) together.

Since the two red parts are equal we can write

$$[P] = [a] \left[ \left( \frac{n}{V} \right)^2 \right]$$

Notice the units of the two terms are equal...but the two terms themselves are not equal!!!

$$\begin{aligned}
 [a] &= \frac{[P]}{\left[ \left( \frac{n}{V} \right)^2 \right]} \\
 [a] &= \frac{[P]}{\left[ \frac{n^2}{V^2} \right]} \\
 [a] &= \frac{\frac{\text{N}}{\text{m}^2}}{\frac{\text{mol}^2}{\text{m}^6}} = \frac{\text{N}}{\text{m}^2} \cdot \frac{\text{m}^6}{\text{mol}^2} \\
 [a] &= \frac{\text{N} \cdot \text{m}^4}{\text{mol}^2}
 \end{aligned}$$

You could go further but this is good enough for now.

$$[b] = \left[ \frac{V}{n} \right] = \frac{\text{m}^3}{\text{mol}}$$

Notice what happens if you set  $b = 0$ ...



**1.39** Looking at the units on each term gives

$$[F] = \frac{[m]^k [v]^n}{[r]^q}$$

$$\frac{\text{kg}^1 \cdot \text{m}^1}{\text{s}^2} = \frac{(\text{kg})^k \left(\frac{\text{m}}{\text{s}}\right)^n}{(\text{m})^q}$$

We know the powers of kg, m, and s must all match on both sides of the equation. This sets up three equations for the exponents. One finds

$$\begin{aligned} k &= 1 && \text{from kg} \\ n - q &= 1 && \text{from meters} \\ n &= 2 && \text{from sec} \end{aligned}$$

We find that force is given by

$$F = \frac{mv^2}{r}$$

This formula happens to be correct. It is important to note that this procedure can get the exponents correct but cannot determine the size of any multiplicative constants like  $\pi$  or  $\frac{1}{2}$ . If one has no clue about how to start modelling a problem, perhaps this procedure could give you a push in the right direction.

**1.40** This problem is similar to the previous problem. Considering the units of each side of the equation gives

$$[T] = [k][L]^a [m]^b [g]^c$$

$$s = (\text{no units}) m^a \text{kg}^b \left(\frac{\text{m}}{\text{s}^2}\right)^c$$

$$s = m^a \text{kg}^b \left(\frac{\text{m}}{\text{s}^2}\right)^c$$

I notice that kg and m do not appear on the left side. I could choose however to write the equations as

$$s \cdot m^0 \cdot \text{kg}^0 = m^a \text{kg}^b \left(\frac{\text{m}}{\text{s}^2}\right)^c$$

Lastly, I choose to write the last term on the right side without any fraction.

$$s^1 \cdot m^0 \cdot \text{kg}^0 = m^{a+c} \text{kg}^b s^{-2c}$$

Now it is easier to equate the exponents on each side of the equation.

$$\begin{aligned} 1 &= -2c \\ 0 &= a + c + d \\ 0 &= b \end{aligned}$$

From these we find the exponents are  $a = 1/2$ ,  $b = 0$ , and  $c = -1/2$ . We find

$$T = k L^{1/2} m^0 g^{-1/2}$$

$$T = k \sqrt{\frac{L}{g}}$$

Notice the mass doesn't affect the period! Later on in physics you can show  $k = 2\pi$ . If you don't believe it, try it at home. Get a long string (say 2 m  $\approx$  6 ft) and try swinging various objects for small angles of oscillations. For example, you could try a hair brush, a shoe, a hammer, etc. Wildly different masses give the same period! This problem breaks down if air resistance becomes important; you might not get the same period if you use a Styrofoam ball.

**1.41** Similar to the previous problem

$$\begin{aligned}
 [Re] &= [\rho]^1 [D]^a [\mu]^b [v]^c \\
 \text{no units} &= \left(\frac{\text{kg}}{\text{m}^3}\right)^1 \text{m}^a \left(\frac{\text{kg}}{\text{m} \cdot \text{s}}\right)^b \left(\frac{\text{m}}{\text{s}}\right)^c \\
 \text{no units} &= \text{kg}^1 \text{m}^{-3} \text{m}^a \text{kg}^b \text{m}^{-b} \text{s}^{-b} \text{m}^c \text{s}^{-c} \\
 \text{no units} &= \text{kg}^{1+b} \text{m}^{-3+a-b+c} \text{s}^{-b-c}
 \end{aligned}$$

From this we find  $a = 1$ ,  $b = -1$ , and  $c = 1$ . Thus we may write the Reynolds number as

$$Re = \frac{\rho D v}{\mu}$$

Why care? Suppose you want to design and test an airplane wing (in this case air is the fluid). Instead of trying to fly using the wing with no testing, one could create a model wing and test it in a wind tunnel with comparable Reynolds number.

More abstractly, the process of finding dimensionless parameters amongst a set of experimental parameters can help experimental scientists identify methods to explain complicated phenomena with the minimum effort. For more on this you might do a web search for “dimensional analysis MIT” and, within the first result, search for “utility”.

**1.42** The variable  $\psi$  has units of  $\frac{\text{m}}{\text{s}}$ .

**1.43** The given equation is

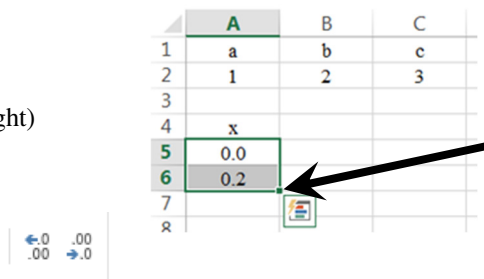
$$\begin{aligned}
 \frac{x}{v^3} &= k \sqrt{\frac{a}{r t^2}} \\
 \frac{[x]}{[v^3]} &= [k] \sqrt{\frac{[a]}{[r][t^2]}} \\
 \frac{\text{m}}{\frac{\text{m}^3}{\text{s}^3}} &= [k] \sqrt{\frac{\frac{\text{m}}{\text{s}^2}}{\text{m} \cdot \text{s}^2}} \\
 \text{m} \cdot \frac{\text{s}^3}{\text{m}^3} &= [k] \sqrt{\frac{\text{m}}{\text{s}^2} \cdot \frac{1}{\text{m} \cdot \text{s}^2}} \\
 \frac{\text{s}^3}{\text{m}^2} &= [k] \frac{1}{\text{s}^2} \\
 [k] &= \frac{\text{s}^5}{\text{m}^2}
 \end{aligned}$$

**1.44** The terms  $m \frac{v^2}{r}$  and  $ma$  both have units of  $\frac{\text{kg} \cdot \text{m}}{\text{s}^2}$ . The terms  $\frac{1}{2} m v^2$  and  $mgh$  both have units of  $\frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$ .

**1.45** While using your calculator to solve equations is nice, solving them with Excel or Matlab is even better. These tools are more practical when you consider you must eventually show your work in a report or presentation. The following instructions were designed for Excel 2013. Most of these instructions will work for other versions as well. Don't forget you can also use the help file if, for instance, you can't figure out how to insert a scatter chart.

To quickly generate a sequence of numbers do the following:

- Type the first 2 in the sequence
- Highlight those two cells
- Mouse over the bottom right corner (shown by arrow in figure at right)
- The cursor should change to a solid black cross *without arrows*.
- Click and drag down until the desired number is reached.
- Adjust the sig figs by playing with the controls that look like this near the top middle of the screen.



Suppose we want Excel to compute  $f(x) = ax^2 + bx + c$  for  $x = 0$  to 2.

Do the following:

- Create the column of  $x$  values using the trick described above. I chose to use increments of 0.1 as you can see in the data table below.
- Make a column for  $f(x)$
- In cell B5 I typed “ $=\$A\$2*A5^2+\$B\$2*A5+\$C\$2$ ”. Note: upper or lower case letters both work just fine.
- Grab the bottom right corner of B5 and drag it down. Be sure to check you have a black cross without arrows before clicking!
- Adjust sig figs as needed.

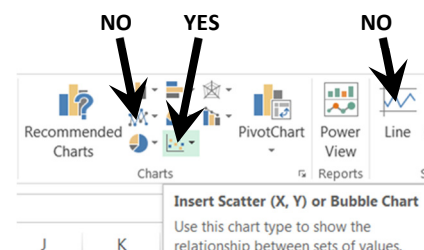
	A	B	C	D	E
1	a	b	c		
2	1	2	3		
3					
4	x	f(x)			
5	0.0	3			
6	0.1				
7	0.2				
8	0.3				
9	0.4				

Once the data looks right it is worth trying two things to prevent future problems.

- Make a new column for  $f(x)$  using the same procedure with the following exception. This time, forget to type the equals sign before the start of all that jibberish in B5. Notice it won't work without the equals sign.
- Make a new column for  $f(x)$  using the same procedure with the following exception. This time, type everything the same in B5 except change “ $\$C\$2$ ” to “ $C2$ ”. Fill down. Notice the equation works but gives the wrong answers. This is obviously the more dangerous error because you might not notice and think you've done things correctly! **To avoid this error, check a random row or two for correctness with your calculator.**

To make a plot do the following

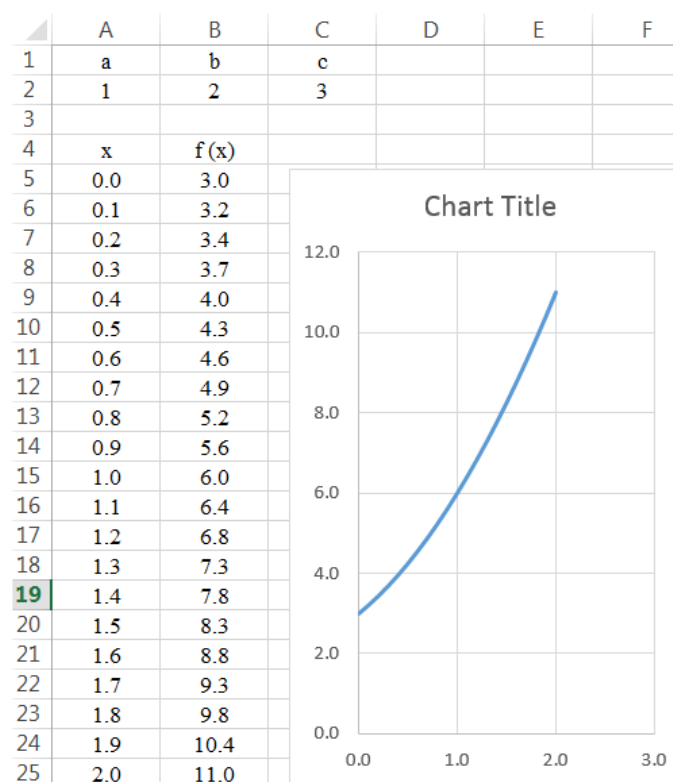
- Use the mouse to highlight the first column of data, numbers only. Press and hold control.
  - To highlight data in a column, place the cursor in the middle of the top cell in the column. The cursor should look like a white cross (not the black cross used to fill down).
  - Then click and drag down.
- Highlight the second column of data, numbers only.
- Ensure both columns have the same number of rows highlighted.
- Go to the top left of the screen and find the INSERT tab. Click on it.
- Near the middle of the top of the screen you should see something like the figure at right. Insert a SCATTER plot. Do NOT insert a LINE plot. TO see the type of chart you are inserting, mouse over the various charts until you find scatter. Check the next page to compare your chart to mine.



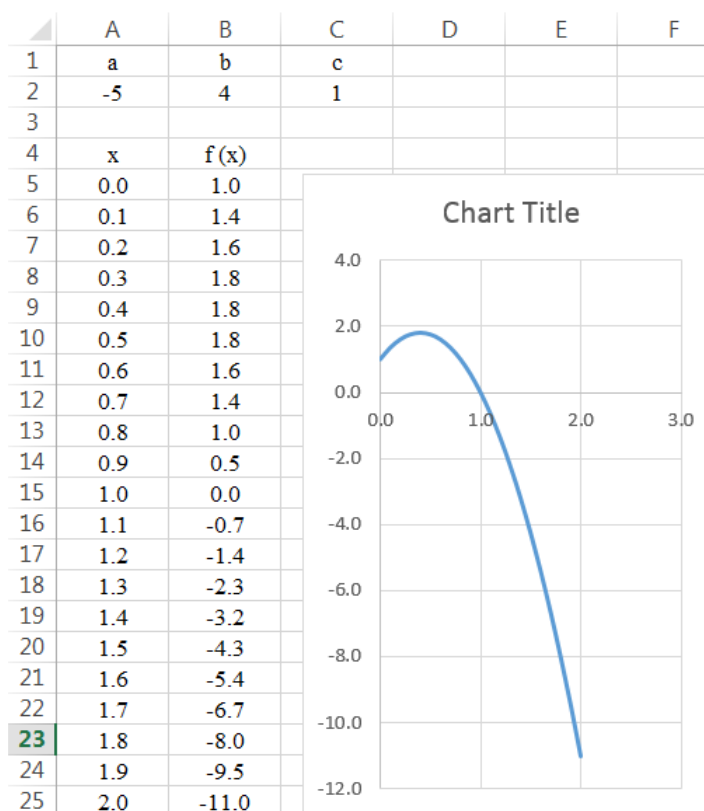
**Do NOT insert a LINE plot. Use a SCATTER plot!!!!!!!**

A line plot does not allow you to set your own increment size on the  $x$ -data. Forgetting this is confusing because your graph may seem like the correct shape but all the numbers, including the slope, will be incorrect!

Hopefully your work looks like the upper figure at right. I resized and moved the chart a bit. Other than that, you should have the same stuff.



Now comes the cool part. Cells A2, B2, and C2 form a row of *constants*. You can now change the constants and the graph should auto-update. Try changing the values to  $A = -5$ ,  $B = 4$ , and  $C = 1$ . Assuming you have done all your work correctly, you should now see a table and graph that look like the lower one at right.



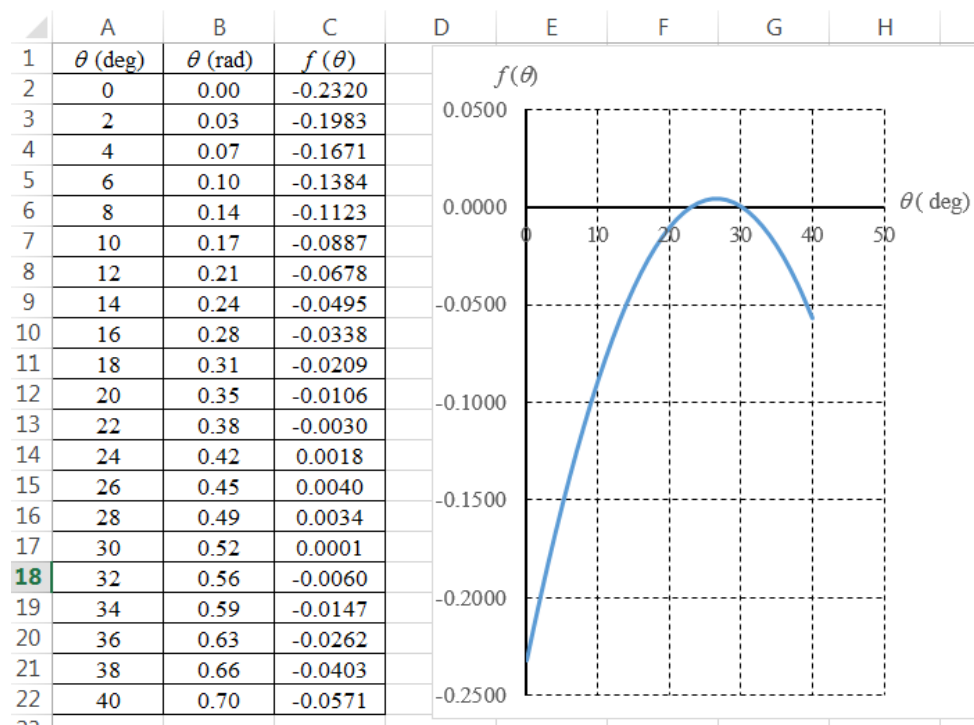
1.46

$$\begin{aligned}
 2 \cos \theta - 2.232 &= -\sin \theta \\
 (2 \cos \theta - 2.232)^2 &= \sin^2 \theta \\
 4 \cos^2 \theta - 8.928 \cos \theta + 4.9818 &= \sin^2 \theta \\
 4 \cos^2 \theta - 8.928 \cos \theta + 4.9818 &= 1 - \cos^2 \theta \\
 5 \cos^2 \theta - 8.928 \cos \theta + 3.9818 &= 0 \\
 \cos \theta &= \frac{-(-8.928) \pm \sqrt{(-8.928)^2 - 4(5)(3.9818)}}{2(5)} \\
 \cos \theta &= \frac{8.928 \pm \sqrt{79.709 - 79.636}}{10} = \frac{8.928 \pm 0.270}{10} = \frac{0.9198}{0.8658} \\
 \cos^{-1} 0.9198 &= 23.10^\circ = 23.1^\circ \\
 \theta &= \text{OR} \\
 \cos^{-1} 0.8658 &= 30.02^\circ = 30.0^\circ
 \end{aligned}$$

Notice two sig figs are lost when doing the subtraction inside the square root. One of them is gained back doing the addition after the square root is taken.

Another style is to create the function  $f(\theta) = 2 \cos \theta - 2.232 + \sin \theta$  and plot it in Excel or Matlab. The solutions for  $\theta$  occur whenever the data table changes sign (when the plot crosses the horizontal axis). Notice on the graph we see  $f(\theta)$  cross the horizontal at  $\theta = 23.1^\circ$  and at  $\theta = 30.0^\circ$ . **WATCH OUT:** I had to convert degrees to radians because Excel computes sine and cosine using radians.

To make the graph, see the instructions on the next page...

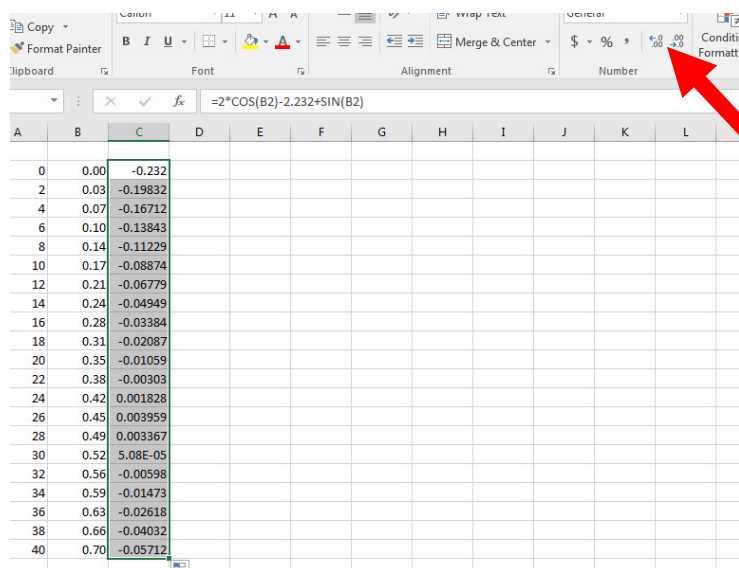


To get the numbers 0, 2, 4, ...40 I typed in the first two numbers (0 and 2). Then I highlighted those two cells. Next I put the cursor over bottom right corner of the cell (A3 in this case). The cursor changed from white to black (with no arrowheads). I clicked and held down the left mouse button. Then I dragged down until I got to cell A 22. At this point I finally release the mouse button.

In cell B2 I typed “=RADIANS(a2)” and filled down. To fill down, once again mouse over the bottom left corner of the cell (now you should be using cell B2). The cursor will once again change to a black plus with no arrows. Once this occurs, press and hold the left mouse button and drag downwards as many cells as required. Let go of the mouse button when done.

In cell C2 I typed “=2\*cos(b2)-2.232+sin(b2)” and filled down.

To change the sig figs, I hit there sig figs buttons until I had the desired numbers. See the figure immediately below. Try both and one of them will work.



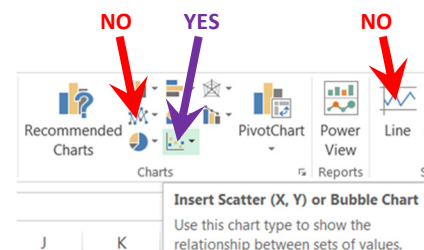
These are  
the sig fig  
buttons in  
Excel

To get the theta symbol, I used the letter q in the Symbol Font.

To make the graph:

To make a plot do the following

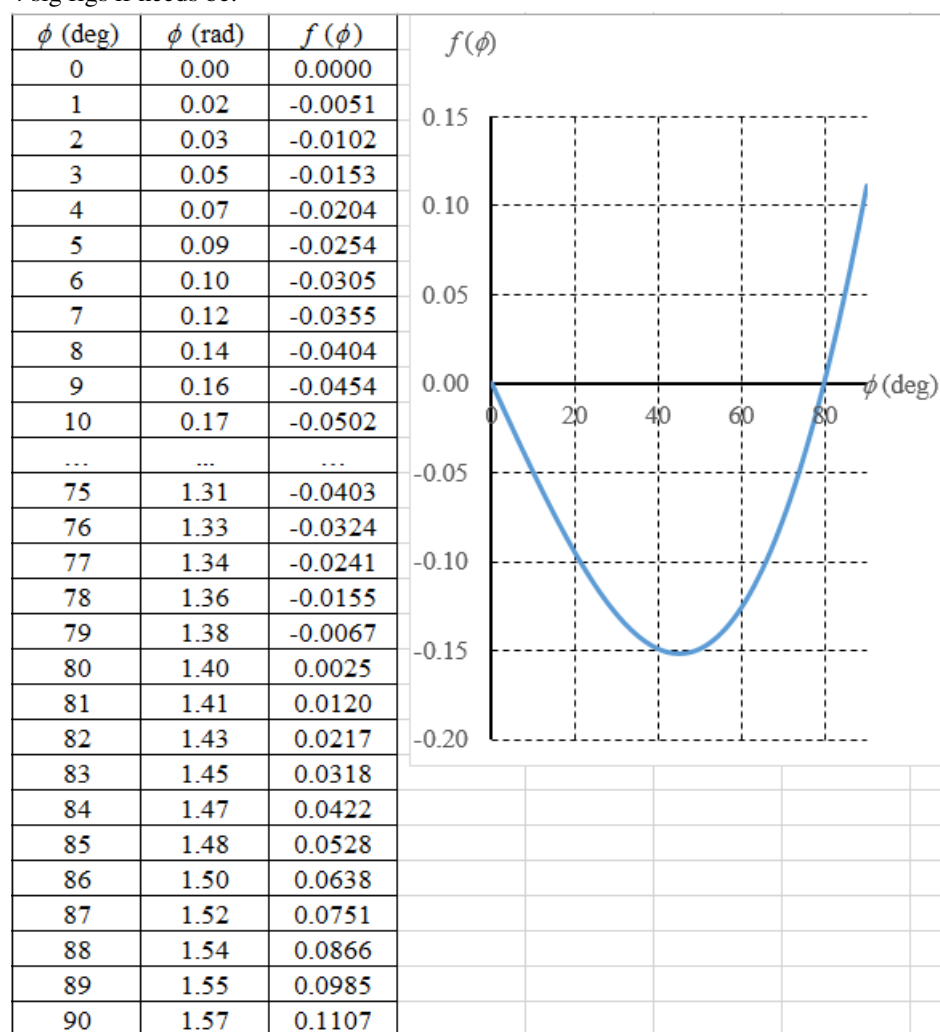
- Use the mouse to highlight the first column of data, numbers only. Press and hold control.
  - To highlight data in a column, place the cursor in the middle of the top cell in the column. The cursor should look like a *white* cross (not the *black* cross used to fill down).
  - Then click and drag down.
- Highlight the second column of data, numbers only.
- Ensure both columns have the same number of rows highlighted.
- Go to the top left of the screen and find the INSERT tab. Click on it.
- Near the middle of the top of the screen you should see something like the figure at right. Insert a SCATTER plot. Do NOT insert a LINE plot. TO see the type of chart you are inserting, mouse over the various charts until you find scatter. Check the next page to compare your chart to mine.



Do NOT insert a LINE plot. Use a SCATTER plot!!!!!!!!!!

**1.47** This is a transcendental equation. Numerical methods are *required* to find a solution. The plotting technique works just fine. I made the equation  $f(\phi) = \frac{\phi}{\sqrt{2}} - \sin \phi$  and went to town. Again, I converted degrees to radians using the Excel function RADIANS. Since most people tend to think in degrees more than they do in radians, I used units of degrees in the plot. The solution was found at approximately  $80^\circ$ .

Note: to get more precision (sig figs) on the result, I could redo the problem but zoom in on the area of interest around  $\phi = 80^\circ$ . For instance, we could go from  $79^\circ$  to  $80^\circ$  in  $0.01^\circ$  increments and figure out the crossing to about 4 sig figs if needs be.



## 1.48

- a)  $0.0020 \approx 0.2\%$
- b)  $0.033 \approx 3\%$
- c)  $0.3 = 30\%$
- d)  $0.166 \approx 1.7\% \text{ or } 2\%$

This answer might vary depending on instructor.

Many write 2 sig figs *for error calculations* when first digit 1.

- e)  $0.166 \approx 1.7\% \text{ or } 2\%$

Notice parts d) and e) have vastly different *absolute* errors yet equal *percent* errors.

- f)  $0.055 \approx 6\%$
- g)  $0.053 \approx 5\%$
- h)  $0.048 \approx 5\%$
- i)  $0.026 \approx 3\%$

**Compare parts *h* & *i*.** We see *i* has about half the percent error of *h* even though they have the same sig figs and absolute error!

**Percent errors are better (than sig figs) at describing the precision of measurements.**

When the first digit of a measurement is 1, sig fig rules overestimate error. This is why many engineers use 3 digits for most measurements but write 4 digits if the first number in a measurement is 1.

- j)  $0.1 \approx 10\%$

**In general, 2 sig figs numbers have up to 10% error while 3 sig fig numbers have up to 1% error.**



## 1.49

- a)  $3.\underline{35}1 \times 10^{-5} \text{ m}^3$ . The underbar indicates the correct column for rounding. I keep extra sig figs in case I need to use the result in another calculation. Keeping the extra sig figs avoids intermediate rounding error.
- b)  $5.0\underline{2}7 \times 10^{-3} \text{ m}^2$
- c) Since a table tennis ball uses very thin celluloid we can model the volume of the ball as

$$V = (\text{Surface Area}) \times \text{thickness} = 4\pi r^2 t = \pi D^2 t$$

The density of the ball is given by  $\rho = \frac{m}{V}$  which implies the volume is given by  $V = \frac{m}{\rho}$ . Putting it all together and solving for  $t$  gives

$$t = \frac{m}{\pi \rho D^2}$$

If you have trouble visualizing  $V = 4\pi r^2 t$ , imagine slicing a basketball and pressing the entire thing flat. It would look all funky and but have total area of  $4\pi r^2$  and  $t$  thick. Hope this helps.

- d) Notice the largest possible thickness is determined using the upper limit for  $m$  and the lower limit for  $D$ . The minimum thickness is determined using the lower limit for  $m$  and the upper limit for  $D$ . I found an upper limit on the thickness of 0.405 mm and lower limit of 0.377 mm or

$$t = 0.391 \pm 0.014 \text{ mm} \approx 0.391 \text{ mm} \pm 4\%$$

Note: a common convention is to use only one digit to express uncertainties. The exception to this rule occurs when the first error digit is 1. This explains why I kept two sig figs for the  $\pm$  number.

- e) For the mass of air use the spherical volume  $V = \frac{4}{3}\pi r^3 = \frac{\pi D^3}{6}$ . I found  $m_{\text{air}} \approx 0.0432 \text{ g}$  which is 1.6% of the entire mass of the ball. Enough to mess up the 3<sup>rd</sup> sig fig but not by much. This is essentially the same as the error tolerance of the ball so it is probably fine to ignore this.
- f) Using style 1 the volume equation is

$$V = \pi D^2 t$$

Using the more precise style gives

$$\begin{aligned} V &= \frac{\pi D^3}{6} - \frac{\pi(D-2t)^3}{6} \\ &= \frac{\pi}{6} [D^3 - (D^3 - 6D^2t + 12Dt^2 - 8t^3)] \\ &= \frac{\pi}{6} [6D^2t - 12Dt^2 + 8t^3] \\ &= \pi D^2t - 2\pi Dt^2 + \frac{4}{3}\pi t^3 \end{aligned}$$

Notice the inner *radius* is  $r - t$  but the inner *diameter* is  $D - 2t$ !

The difference in the volume calculations is thus given by

$$\Delta V = V_1 - V_2 = -2\pi Dt^2 + \frac{4}{3}\pi t^3$$

The result is negative implying  $V_1$  is an overestimate as we expect.

- g) The question suggests  $\frac{\Delta V}{V} = 1\%$ . Since  $\Delta V$  is tiny we may use either volume formula to calculate  $\frac{\Delta V}{V}$ . This gives

$$\frac{\Delta V}{V} = \frac{-2\pi Dt^2 + \frac{4}{3}\pi t^3}{\pi D^2 t} = -2\frac{t}{D} + \frac{4}{3}\frac{t^2}{D^2} = 0.01$$

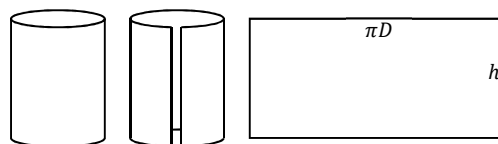
This sets up a quadratic equation

$$1.333x^2 - 2x - 0.01 = 0$$

where  $x = \frac{t}{D}$ . Using the quadratic formula gives  $x = 0.0050$  or  $t = 0.0050D = 0.2 \text{ mm}$ . This means we probably introduced about 2% error using  $V = \pi D^2 t$ . I was surprised this error was that significant. Notice however, this error is still half of the error associated with the uncertainties in  $m$  and  $D$  as indicated by part d.

1.50

- a) Imagine slicing the pipe vertically and rolling it flat as shown in the upper sequence of images shown at right. Notice the area of the sheet, and thus the sidewall is  $A_{side} = \pi Dh$



- b) It would be nice to use the thin-walled method shown in the lower sequence of images at right. Unfortunately, this equation is only valid when  $t \ll D$ . For typical 1-inch PVC pipes, the thickness is about 25% of the outer diameter. **We cannot treat this pipe as thin-walled.** It is best to use

$$A_{end} = A_{outer} - A_{inner} = \frac{\pi}{4}(D^2 - d^2)$$

- c) Density is given by  $\rho = \frac{m}{V}$  which gives  $m = \rho V$ . The mass is given by

$$m = \rho V = \rho A_{end} h = \frac{\pi}{4} \rho h (D^2 - d^2)$$

It is tempting to use the thin-walled trick like we used on the table tennis ball in 1.14. The thin-walled trick was

$$V = (\text{Surface Area}) \times \text{thickness} = \pi Dh t = \pi Dh \left( \frac{D - d}{2} \right)$$

**Unfortunately that approach would give huge errors since our thickness is about 25% of the outer diameter.**

- d) Notice I asked for mass per unit length, not mass. Mass *per unit length* is

$$\frac{m}{h} = \frac{\pi}{4} \rho (D^2 - d^2) = \frac{\pi}{4} \left( 1295 \frac{\text{kg}}{\text{m}^3} \right) (1.315^2 \text{ in}^2 - 1.029^2 \text{ in}^2) = 681.84 \frac{\text{kg} \cdot \text{in}^2}{\text{m}^3}$$

Watch out for the sig figs on the term in parentheses! Upon squaring both numbers still have four sig figs.

During the subtraction you keep the column of sig figs (not the number of sig figs).

Finally, let's convert to the requested units

$$681.84 \frac{\text{kg} \cdot \text{in}^2}{\text{m}^3} \times \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^3 \times \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right)^2 \times \frac{1000 \text{ g}}{1 \text{ kg}} = 4.399 \frac{\text{g}}{\text{cm}} = 4.40 \frac{\text{g}}{\text{cm}}$$

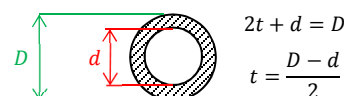
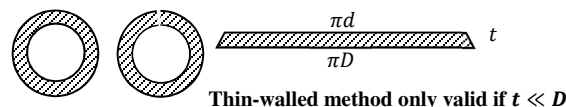
- e) The question asks, "When does  $2A_{end} = (1\%)A_{side}$ ?" One finds

$$2 \left( \frac{\pi}{4} (D^2 - d^2) \right) = 0.01 \pi Dh$$

which gives

$$h = \frac{50}{D} (D^2 - d^2) \approx 25 \text{ in}$$

Note I didn't care about sig figs on this one.



## 1.51

- a) I assume the following: typical heart rate is 60 beats per minute (on average), typical life is perhaps 75 years long. With these assumptions, we find

$$\frac{60 \text{ beats}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{24 \text{ hrs}}{1 \text{ day}} \times \frac{365.25 \text{ days}}{1 \text{ yr}} \times \frac{75 \text{ yrs}}{1 \text{ lifetime}} = 2.37 \times 10^9 \approx 2 \text{ billion}$$

Note, since we have no way of knowing if our assumptions are even close we round to only one sig fig. The power of ten is about the only thing we think might be in the correct ballpark. That's why this is called an order of magnitude calculation. If your answer is within a factor of 10 (between  $2 \times 10^8$  and  $2 \times 10^{10}$ ) consider it reasonable.

- b) I estimated that a snail moves an inch in perhaps 5 seconds. That gives

$$v = \frac{1 \text{ in}}{5 \text{ sec}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{10^{-2} \text{ m}}{1 \text{ cm}} = 0.00508 \text{ m/s}$$

Using a web search I found speeds ranged between 0.003 m/s and 0.013 m/s. My estimate seems as good as any as it falls in that range. Again, since this estimate is crude at best we expect to keep only one sig fig on the final answer. Again, if you are within a factor of 10, consider your result as reasonable.

Now to convert we use

$$v = 0.00508 \frac{\text{m}}{\text{sec}} \times \frac{3600 \text{ s}}{1 \text{ hr}} \times \frac{24 \text{ hr}}{1 \text{ day}} \times \frac{14 \text{ days}}{1 \text{ fortnight}} \times \frac{1 \text{ mi}}{1609 \text{ m}} \times \frac{1 \text{ furlong}}{0.125 \text{ mi}} \approx 30 \frac{\text{furlongs}}{\text{fortnight}}$$

- c) I suppose a car might get  $\frac{30 \text{ mi}}{\text{gal}}$  and the distance across the country is perhaps 4000 miles depending on the route you take. Notice we want gallons as an answer so we need gallons on top and miles to cancel out. From that you get

$$\frac{1 \text{ gal}}{30 \text{ mi}} \times 4000 \text{ mi} = 133 \text{ gal} \approx 100 \text{ gal}$$

Depending on the mpg mileage you assumed we might expect a large range of numbers. A wide range of answers are acceptable in this type of question.

- d) Have fun...you're on your own.