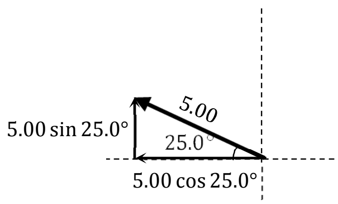
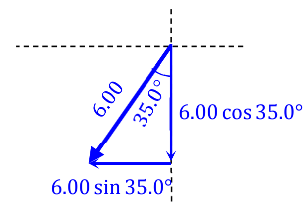
Ch3

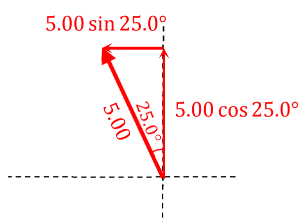
3.1



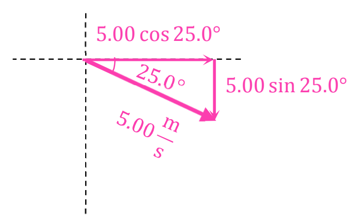
$$\vec{A} = (-5.00\cos 25.0°\,\hat{\imath} + 5.00\sin 25.0°\,\hat{\jmath})\,\frac{\mathrm{m}}{\mathrm{s}}$$
$$\vec{A} = \left(-4.5\underline{3}1\hat{\imath} + 2.1\underline{1}3\hat{\jmath}\right)\frac{\mathrm{m}}{\mathrm{s}}$$



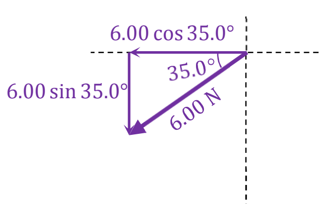
$$\vec{B} = (-6.00\sin 35.0°\,\hat{\imath} - 6.00\cos 35.0°\,\hat{\jmath})\ \mathrm{N}$$
$$\vec{B} = \left(-3.4\underline{4}1\hat{\imath} - 4.9\underline{1}5\hat{\jmath}\right)\ \mathrm{N}$$



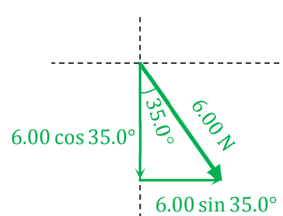
$$\vec{C} = (-5.00\sin 25.0°\,\hat{\imath} + 5.00\cos 25.0°\,\hat{\jmath})\,\frac{\mathrm{m}}{\mathrm{s}}$$
$$\vec{C} = \left(-2.1\underline{1}3\hat{\imath} + 4.5\underline{3}1\hat{\jmath}\right)\frac{\mathrm{m}}{\mathrm{s}}$$



$$\vec{D} = (+5.00\cos 25.0°\,\hat{\imath} - 5.00\sin 25.0°\,\hat{\jmath})\,\frac{\mathrm{m}}{\mathrm{s}}$$
$$\vec{D} = \left(+4.5\underline{3}1\hat{\imath} - 2.1\underline{1}3\hat{\jmath}\right)\frac{\mathrm{m}}{\mathrm{s}}$$



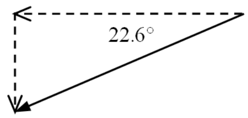
$$\vec{E} = (-6.00\cos 35.0°\,\hat{\imath} - 6.00\sin 35.0°\,\hat{\jmath})\ \mathrm{N}$$
$$\vec{E} = \left(-4.9\underline{1}5\hat{\imath} - 3.4\underline{4}1\hat{\jmath}\right)\ \mathrm{N}$$



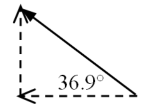
$$\vec{F} = (+6.00\sin 35.0°\,\hat{\imath} - 6.00\cos 35.0°\,\hat{\jmath})\ \mathrm{N}$$
$$\vec{F} = \left(+3.4\underline{4}1\hat{\imath} - 4.9\underline{1}5\hat{\jmath}\right)\ \mathrm{N}$$

3.2 Lazy with sig figs here.

$G = \|\vec{G}\| = 13\text{m}$

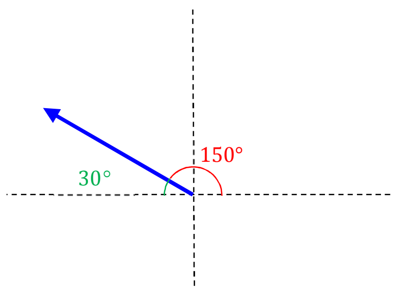


$H = \|\vec{H}\| = 5\text{m}$

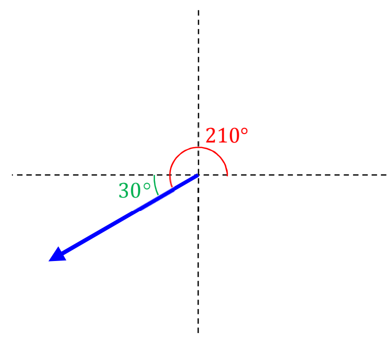


3.3 Lazy with sig figs here…more worried about getting plus and minus signs correct.

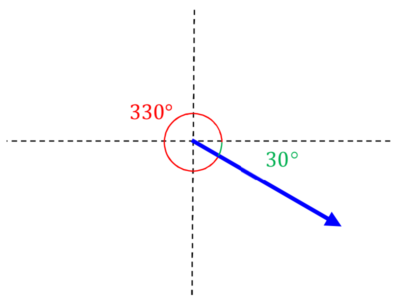
$\vec{A} = -26.0\dfrac{\text{m}}{s}\hat{\imath} + 15.0\dfrac{\text{m}}{s}\hat{\jmath}$



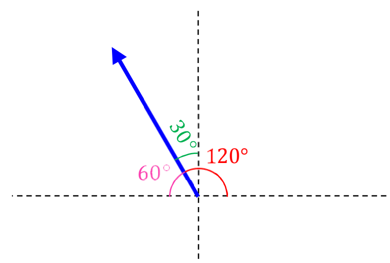
$\vec{D} = -26.0\dfrac{\text{m}}{s}\hat{\imath} - 15.0\dfrac{\text{m}}{s}\hat{\jmath}$



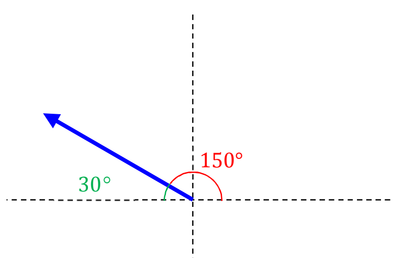
$\vec{B} = 26.0\dfrac{\text{m}}{s}\hat{\imath} - 15.0\dfrac{\text{m}}{s}\hat{\jmath}$



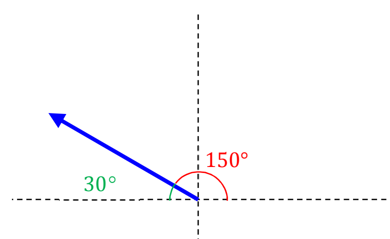
$\vec{E} = -15.0\dfrac{\text{m}}{s}\hat{\imath} + 26.0\dfrac{\text{m}}{s}\hat{\jmath}$



$\vec{C} = -26.0\dfrac{\text{m}}{s}\hat{\imath} + 15.0\dfrac{\text{m}}{s}\hat{\jmath}$



$\vec{F} = -26.0\dfrac{\text{m}}{s}\hat{\imath} + 15.0\dfrac{\text{m}}{s}\hat{\jmath}$



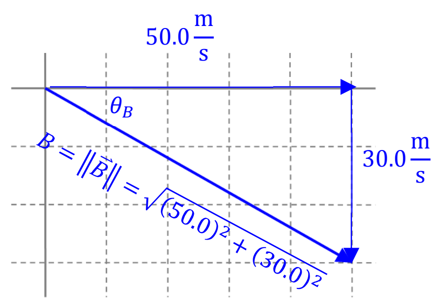
NOTICE $\vec{A} = \vec{C} = \vec{F}$…three different ways to say the same thing.

3.4 I was lazy with sig figs here as it is unclear how many you can get from picture. Unless otherwise specified, we typically try to use three sig figs for final answers.

a) $\vec{A} = -20.0\frac{\text{m}}{\text{s}}\hat{\jmath}$ and $\vec{B} = 50.0\frac{\text{m}}{\text{s}}\hat{\imath} - 30.0\frac{\text{m}}{\text{s}}\hat{\jmath}$

b) The first vector is fairly easy: $\vec{A} = 20.0\frac{\text{m}}{\text{s}}$ due south.

To understand $\vec{B}$, consider the sketch at right.



Notice you can get the magnitude of $\vec{B}$ using the Pythagorean theorem.

$$B = magnitude\ of\ \vec{B} = \|\vec{B}\| = \sqrt{\left(50.0\frac{\text{m}}{\text{s}}\right)^2 + \left(30.0\frac{\text{m}}{\text{s}}\right)^2} = 58.3\frac{\text{m}}{\text{s}}$$

One can get the angle using TOA (of SOH CAH TOA).

$$\tan\theta_B = \frac{opp}{adj}$$

$$\tan\theta_B = \frac{30.0\frac{\text{m}}{\text{s}}}{50.0\frac{\text{m}}{\text{s}}}$$

$$\theta_B = \tan^{-1}\left(\frac{30.0}{50.0}\right) = 31.0°$$

Now notice I can write the final answer as

$$\vec{B} = 58.3\frac{\text{m}}{\text{s}}\ \text{heading}\ 31.0°\ \text{S of E}$$

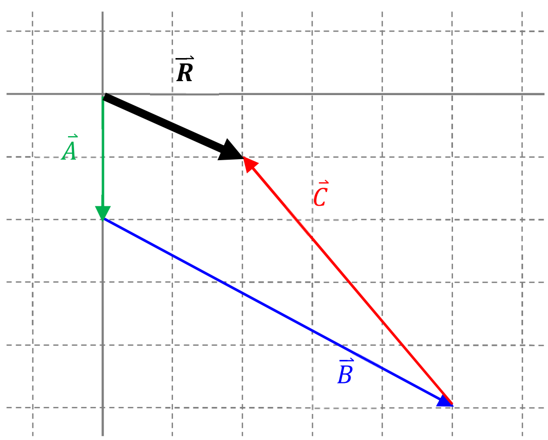
I could also write it like these other ways for full credit.

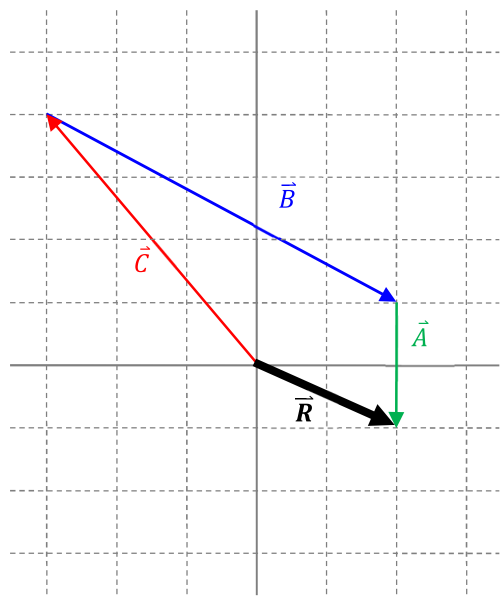
$$\vec{B} = 58.3\frac{\text{m}}{\text{s}}\ \text{heading}\ 59.0°\ \text{E of S} = 58.3\frac{\text{m}}{\text{s}}\ \text{heading}\ 329.0°\ \text{from the positive}\ x\ \text{axis}$$

c) $\vec{C} = -30.0\frac{\text{m}}{\text{s}}\hat{\imath} + 40.0\frac{\text{m}}{\text{s}}\hat{\jmath}$. See next step for sketch…

d) Two examples are shown at right.

The left example shows $\vec{A} + \vec{B} + \vec{C} = \vec{\boldsymbol{R}}$ while the right example shows $\vec{C} + \vec{B} + \vec{A} = \vec{\boldsymbol{R}}$





PROBLEM CONTINUES NEXT PAGE…

e) To perform component wise addition I like to stack the equations as shown below.

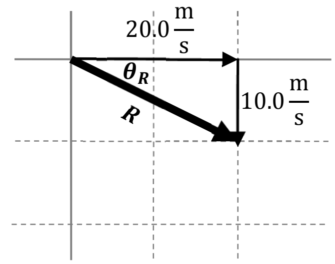
$$\vec{A} = \ (0.00\hat{\imath} - 20.0\hat{\jmath})\frac{\mathrm{m}}{\mathrm{s}}$$
$$\vec{B} = \ (50.0\hat{\imath} - 30.0\hat{\jmath})\frac{\mathrm{m}}{\mathrm{s}}$$
$$\vec{C} = (-30.0\hat{\imath} + 40.0\hat{\jmath})\frac{\mathrm{m}}{\mathrm{s}}$$

Then add the $\hat{\imath}$ column and $\hat{\jmath}$ column separately to find

$$\vec{R} = \ (20.0\hat{\imath} - 10.0\hat{\jmath})\frac{\mathrm{m}}{\mathrm{s}}$$

Immediately after finding the components of any vector MAKE A SKETCH!

I get the magnitude R of the result using the Pythagorean formula.



$$R = \sqrt{\left(20.0\frac{\mathrm{m}}{\mathrm{s}}\right)^2 + \left(10.0\frac{\mathrm{m}}{\mathrm{s}}\right)^2} = 22.4\frac{\mathrm{m}}{\mathrm{s}}$$

WATCH OUT! Units are required on final answers if units were given in the problem statement!!!

I get the angle using inverse tangent

$$\theta_R = \tan^{-1}\left(\frac{10.0\frac{\mathrm{m}}{\mathrm{s}}}{20.0\ \frac{\mathrm{m}}{\mathrm{s}}}\right) = 26.6°$$

Now I put it all together in one of the following forms

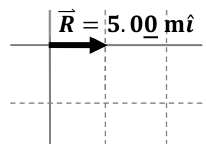
$$\vec{\boldsymbol{R}} = \mathbf{22.4}\frac{\mathbf{m}}{\mathbf{s}}\ @\ \mathbf{26.6°\ S\ of\ E}$$

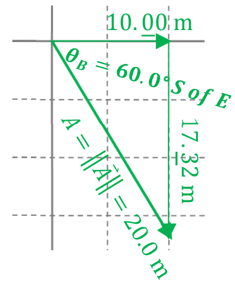
$$\vec{\boldsymbol{R}} = \mathbf{22.4}\frac{\mathbf{m}}{\mathbf{s}}\ @\ \mathbf{63.4°\ E\ of\ S}$$

$$\vec{\boldsymbol{R}} = \mathbf{22.4}\frac{\mathbf{m}}{\mathbf{s}}\ @\ \mathbf{333.4°\ from\ positive\ x\ axis}$$

$$\vec{\boldsymbol{R}} = \mathbf{22.4}\frac{\mathbf{m}}{\mathbf{s}}\ @ - \mathbf{26.6°\ from\ positive\ x\ axis}$$

3.5 Watch out for the wording in this question. We know $\vec{A}+\vec{B}+\vec{C}=\vec{R}$ but the problem statement does not give you $\vec{A}, \vec{B}, \& \vec{C}$. If you read carefully the problem statement gives you $\vec{A}, \vec{B}, \& \vec{\boldsymbol{R}}$.
First I would rearrange $\vec{A}+\vec{B}+\vec{C}=\vec{\boldsymbol{R}}$ to give $\vec{C}=\vec{\boldsymbol{R}}-(\vec{A}+\vec{B})$.





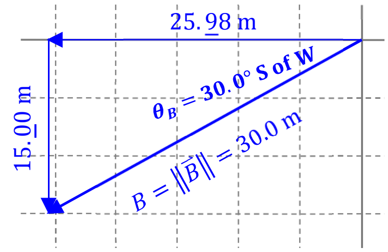
Then I would make the sketches shown at right.

Next I use my sketches to re-write the vectors in Cartesian form.
The Cartesian form for each vector is written below.

$$\vec{\boldsymbol{R}} = (\mathbf{5.0\underline{0}}\hat{\boldsymbol{\imath}} + \mathbf{0}\hat{\boldsymbol{\jmath}})\ \mathbf{m}$$

$$\vec{A} = (+10.\underline{0}0\hat{\imath} - 17.\underline{3}2\hat{\jmath})\ \mathrm{m}$$

$$\vec{B} = (-25.\underline{9}8\hat{\imath} - 15.\underline{0}0\hat{\jmath})\ \mathrm{m}$$



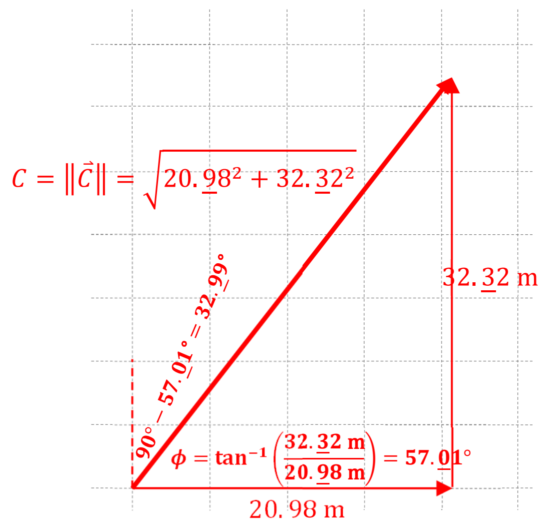
Finally, I plug in these Cartesian form vectors into $\vec{C}=\vec{\boldsymbol{R}}-(\vec{A}+\vec{B})$.

$$\vec{C} = \vec{\boldsymbol{R}} - (\vec{A}+\vec{B})$$
$$\vec{C} = (\mathbf{5.0\underline{0}}\hat{\boldsymbol{\imath}} + \mathbf{0}\hat{\boldsymbol{\jmath}})\mathrm{m} - \left((+10.\underline{0}0\hat{\imath} - 17.\underline{3}2\hat{\jmath})\mathrm{m} + (-25.\underline{9}8\hat{\imath} - 15.\underline{0}0\hat{\jmath})\mathrm{m}\right)$$

You should find

$$\vec{C} = (20.\underline{9}8\hat{\imath} + 32.\underline{3}2\hat{\jmath})\mathrm{m} = \begin{cases} 38.\underline{5}3\ \mathrm{m\ heading}\ 32.\underline{9}9°\ \textbf{E of N} \\ \textbf{OR} \\ 38.\underline{5}3\ \mathrm{m\ heading}\ 57.\underline{0}1°\ \textbf{N of E} \end{cases}$$

A sketch of $\vec{C}$ only is also worthwhile to get a feel for it.
This sketch is drawn at right.



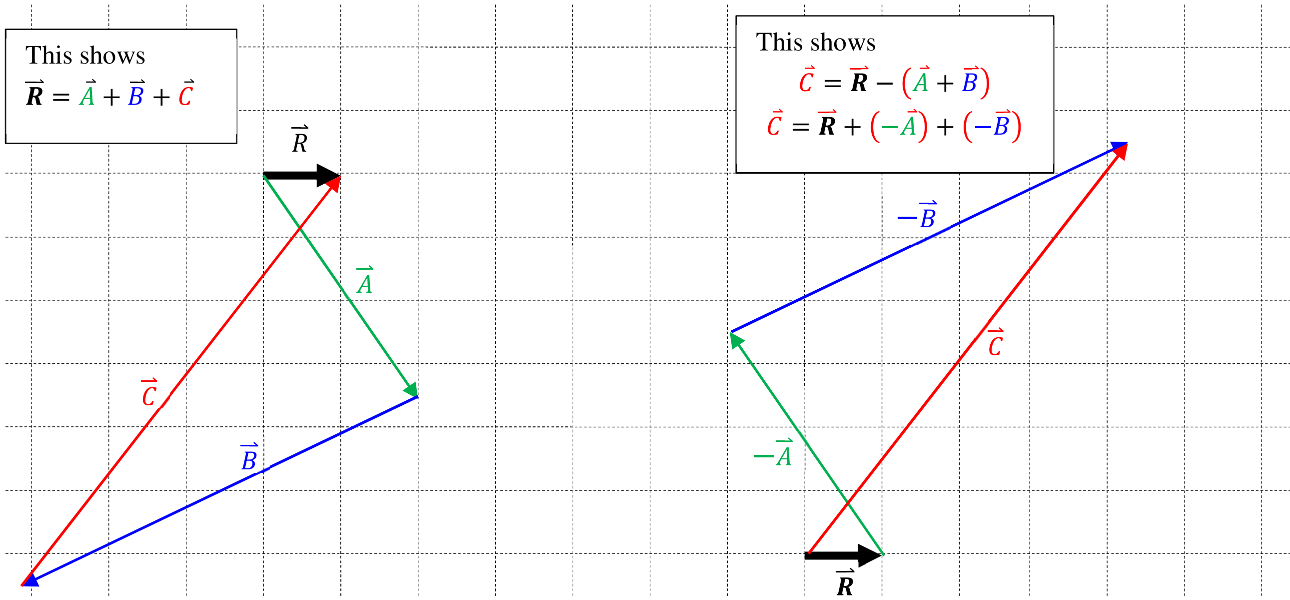
The full graphical vector addition is shown in two ways on the next page.

Graphical vector addition for 3.5

Watch out!

$-\vec{A}$ has same magnitude as $\vec{A}$ but points opposite direction.

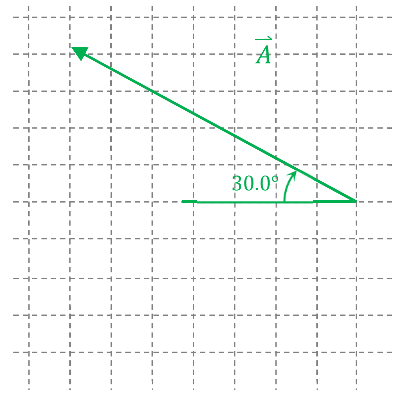
$-\vec{B}$ has same magnitude as $\vec{B}$ but points opposite direction.



3.6

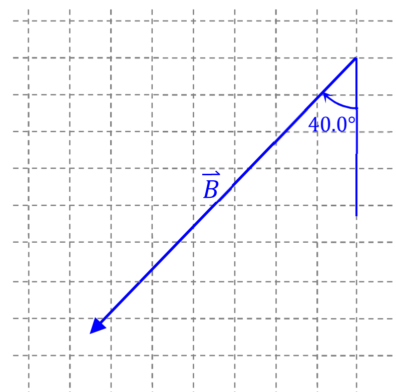
$$\vec{A} = -8.00\cos 30.0°\,\hat{\imath} + 8.00\sin 30.0°\,\hat{\jmath}$$

$$\vec{A} = -6.9\underline{2}8\,\hat{\imath} + 4.0\underline{0}0\,\hat{\jmath}$$



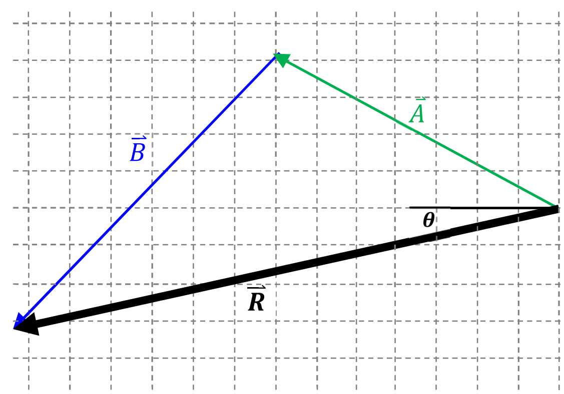
$$\vec{B} = -10.0\sin 40.0°\,\hat{\imath} - 10.0\cos 40.0°\,\hat{\jmath}$$

$$\vec{B} = -6.4\underline{2}8\hat{\imath} - 7.6\underline{6}0\,\hat{\jmath}$$



The bottom figure at left shows $\vec{A} + \vec{B} = \vec{\boldsymbol{R}}$.

$$\begin{array}{r} \vec{A} = -6.9\underline{2}8\,\hat{\imath} + 4.0\underline{0}0\,\hat{\jmath} \\ \vec{B} = -6.4\underline{2}8\hat{\imath} - 7.6\underline{6}0\,\hat{\jmath} \\ + \overline{} \\ \vec{\boldsymbol{R}} = -\mathbf{13.3\underline{5}6}\hat{\boldsymbol{\imath}} - \mathbf{3.6\underline{6}0}\hat{\boldsymbol{\jmath}} \end{array}$$



$$\boldsymbol{R} = \|\vec{\boldsymbol{R}}\| = \sqrt{(\mathbf{13.3\underline{5}6})^2 + (\mathbf{3.6\underline{6}0})^2}$$
$$\boldsymbol{R} = \sqrt{\mathbf{178.\underline{3}8 + 13.\underline{4}0}}$$
$$\boldsymbol{R} = \sqrt{\mathbf{191.\underline{7}8}}$$
$$\boldsymbol{R} = \mathbf{13.8\underline{4}8}$$

Notice the magnitude has *four* sig figs when following sig fig rules while each component only has *three*!!!

Calling the angle to the *horizontal* $\boldsymbol{\theta}$

$$\boldsymbol{\theta} = \mathbf{tan}^{-1}\left(\frac{\mathbf{3.6\underline{6}0}}{\mathbf{13.3\underline{5}6}}\right)$$
$$\boldsymbol{\theta} = \mathbf{tan}^{-1}(\mathbf{0.27\underline{4}0})$$
$$\boldsymbol{\theta} = \mathbf{15.\underline{3}2°}$$

Strictly speaking, there are better ways to determine the digit of uncertainty for functions. In general, this is discussed in lab courses.

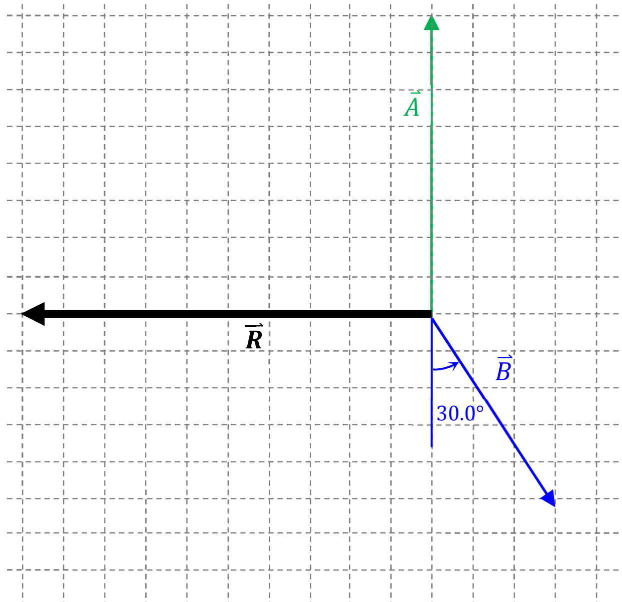
$$\vec{\boldsymbol{R}} \approx \mathbf{13.85\ heading\ 15.3°\ S\ of\ W}$$

3.7 Watch out for the wording in this question. We know $\vec{A} + \vec{B} + \vec{C} = \vec{R}$.
If you read carefully, the problem statement gives you $\vec{A}, \vec{B}$, & $\vec{R}$.
First I would rearrange

$$\vec{A} + \vec{B} + \vec{C} = \vec{R}$$
$$\vec{C} = \vec{R} - (\vec{A} + \vec{B})$$

Then I would find

$$\vec{R} = -10.\underline{0}\hat{\imath} + 0\hat{\jmath}$$
$$\vec{A} = 0\,\hat{\imath} + 8.0\underline{0}\,\hat{\jmath}$$
$$\vec{B} = 3.0\underline{0}0\,\hat{\imath} - 5.1\underline{9}6\,\hat{\jmath}$$



$$\vec{C} = (-10.\underline{0}\hat{\imath}) - \left((8.0\underline{0}\,\hat{\jmath}) + (3.0\underline{0}0\,\hat{\imath} - 5.1\underline{9}6\hat{\jmath})\right)$$
$$\vec{C} = (-10.\underline{0}\hat{\imath}) - (3.0\underline{0}0\,\hat{\imath} + 2.8\underline{0}4\,\hat{\jmath})$$
$$\boldsymbol{\vec{C} = -13.\underline{0}0\,\hat{\imath} - 2.8\underline{0}4\,\hat{\jmath}}$$

Pay attention to the sig fig rules and keep the left most column when adding terms…

$$\boldsymbol{C = \|\vec{C}\| = \sqrt{(13.\underline{0}0)^2 + (2.8\underline{0}4)^2}}$$
$$\boldsymbol{C = \sqrt{16\underline{9}.0 + 7.8\underline{6}2}}$$
$$\boldsymbol{C = \sqrt{17\underline{6}.9}}$$
$$\boldsymbol{C = 13.\underline{3}0}$$

Calling the angle to the *horizontal* $\boldsymbol{\theta}$

$$\boldsymbol{\theta = \tan^{-1}\left(\frac{2.8\underline{0}4}{13.\underline{0}0}\right)}$$
$$\boldsymbol{\theta = \tan^{-1}(0.215\underline{7})}$$
$$\boldsymbol{\theta = 12.\underline{1}7°}$$

$$\boldsymbol{\vec{C} \approx 13.3 \text{ heading } 12.2° \text{ S of W}}$$

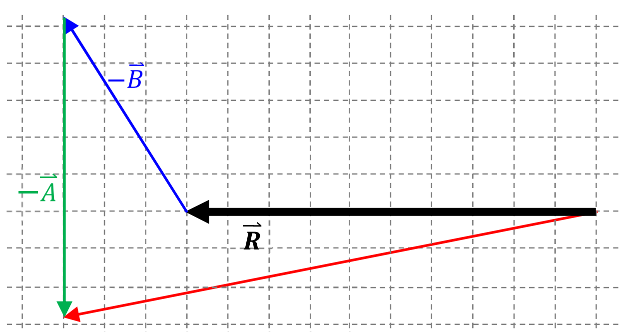
Figure at right shows the graphical vector addition of

$$\boldsymbol{\vec{C} = \vec{R} - (\vec{A} + \vec{B})}$$
$$\boldsymbol{\vec{C} = \vec{R} + (-\vec{A}) + (-\vec{B})}$$

Watch out!

$-\vec{A}$ has same magnitude as $\vec{A}$ but points opposite direction.

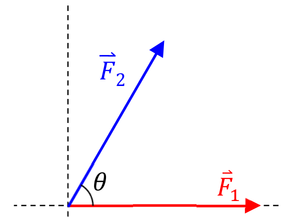
$-\vec{B}$ has same magnitude as $\vec{B}$ but points opposite direction.



$$\boldsymbol{\vec{C} = \vec{R} + (-\vec{A}) + (-\vec{B})}$$

3.7½

a) Consider the figure at right. The net force is given by



$$\vec{F}_{NET} = \vec{F}_1 + \vec{F}_2$$

Recall the problem statement told us each force had the same magnitude F.

We say $\|\vec{F}_1\| = \|\vec{F}_2\| = F$.

We can split both forces into components

$$\vec{F}_{NET} = (F\hat{\imath}) + (F\cos\theta\,\hat{\imath} + F\sin\theta\,\hat{\jmath})$$

Now group the $\hat{\imath}$ terms together.

$$\vec{F}_{NET} = (F + F\cos\theta)\hat{\imath} + (F\sin\theta)\hat{\jmath}$$

Now get the magnitude.

$$F_{NET} = \|\vec{F}_{NET}\| = \sqrt{(F + F\cos\theta)^2 + (F\sin\theta)^2}$$

Clean this up…

$$F_{NET} = \sqrt{F^2 + 2F(F\cos\theta) + F^2\cos^2\theta + F^2\sin^2\theta}$$
$$F_{NET} = \sqrt{F^2 + 2F^2\cos\theta + F^2(\cos^2\theta + \sin^2\theta)}$$
$$F_{NET} = \sqrt{F^2 + 2F^2\cos\theta + F^2}$$
$$F_{NET} = \sqrt{2F^2 + 2F^2\cos\theta}$$
$$F_{NET} = \sqrt{2F^2(1 + \cos\theta)}$$
$$F_{NET} = \sqrt{2}F\sqrt{(1 + \cos\theta)}$$

At this point, recall we were told $F_{NET} = 1.75F$ and we want to find θ. Before plugging in for F_{NET} I will first solve algebraically for θ.

$$F_{NET}^2 = 2F^2(1 + \cos\theta)$$
$$1 + \cos\theta = \frac{F_{NET}^2}{2F^2}$$
$$\cos\theta = \frac{F_{NET}^2}{2F^2} - 1$$
$$\theta = \cos^{-1}\left(\frac{F_{NET}^2}{2F^2} - 1\right)$$

Now plug in $F_{NET} = 1.75F$

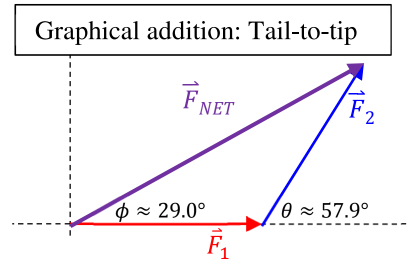
$$\theta = \cos^{-1}\left(\frac{(1.75F)^2}{2F^2} - 1\right) = \cos^{-1}\left(\frac{3.0625F^2}{2F^2} - 1\right) = \cos^{-1}(0.53125) \approx 57.91°$$

Note: perhaps you noticed we essentially derived the law of cosines in grinding out this calculation! If you happened to have that memorized (and used that instead) good for you!

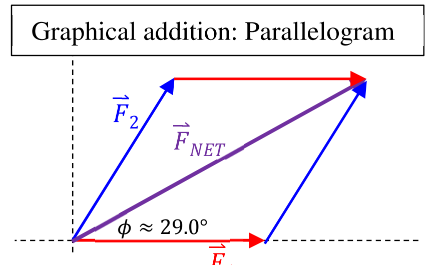
MORE ON NEXT PAGE…

b) I drew the vector addition at right. To get the *angle* of the net force use

$$\phi = \tan^{-1}\left(\frac{F_{NETy}}{F_{NETx}}\right)$$
$$\phi = \tan^{-1}\left(\frac{F\sin\theta}{F + F\cos\theta}\right)$$
$$\phi = \tan^{-1}\left(\frac{\sin\theta}{1 + \cos\theta}\right)$$
$$\phi = \tan^{-1}\left(\frac{\sin 57.91°}{1 + \cos 57.91°}\right) \approx 29.0°$$

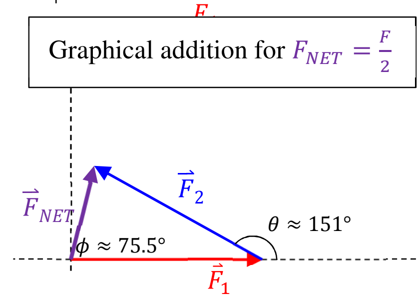


Side note: in this problem we are always adding two forces of equal magnitude. As such, we expect the net force will always point in direction $\phi = \frac{\theta}{2}$. That said, the procedure shown works even when the force magnitudes are *not* equal.



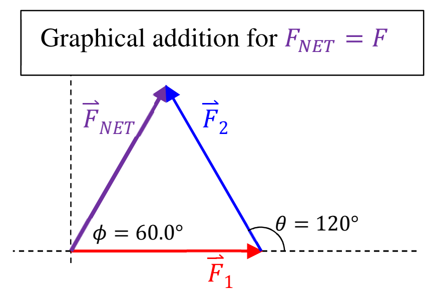
c) Now we see the advantage of doing everything algebraically. Plug in $F_{NET} = \frac{F}{2}$

$$\theta = \cos^{-1}\left(\frac{\left(\frac{1}{2}F\right)^2}{2F^2} - 1\right) = \cos^{-1}\left(\frac{\frac{1}{4}F^2}{2F^2} - 1\right) = \cos^{-1}\left(\frac{1}{8} - 1\right) \approx 151°$$



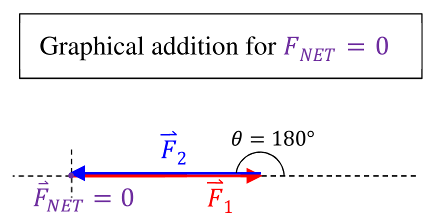
d) Plug in $F_{NET} = F$

$$\theta = \cos^{-1}\left(\frac{(F)^2}{2F^2} - 1\right) = \cos^{-1}\left(\frac{1}{2} - 1\right) = 120°$$

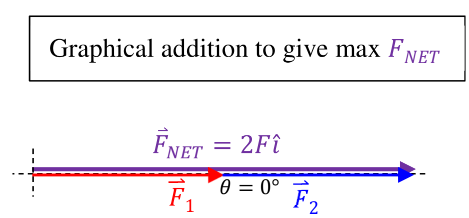


e) For no net force you could plug in zero and follow the same procedure as before. Alternatively, you could use reasoning: if the forces point opposite directions you should get no net force. Either way one finds

$$\theta = 180°$$



f) To get maximum net force, point the two vectors in the same direction. This implies $\theta = 0°$. In this special case, when both vectors are aligned together, the net force magnitude is actually equal to the sum of the two force magnitudes. In general this is *not* true…look at the other parts of this solution.



3.8 If you are clever you might use law of sines (or law of cosines). The following method is also fine.
First get equations for each vector in the *x* and *y* directions.
For the second leg of the trip use $B_x = 80\cos\theta$ and $B_y = 80\sin\theta$.
For the third leg use $C_x = -C\sin 20$ and $C_y = -C\cos 20$.

Since $\vec{A} + \vec{B} + \vec{C} = 0$ we find that $A_x + B_x + C_x = 0$ and $A_y + B_y + C_y = 0$.

One way to proceed from here is to rearrange these equations to $B_x = -A_x - C_x$ and $B_y = -A_y - C_y$.

Then we find $80\cos\theta = -A_x - C_x$ and $80\sin\theta = -A_y - C_y$.
By squaring each equation and adding them together one can eliminate the unknown angle!
This leaves only one unknown, C, in the equation. In particular we find

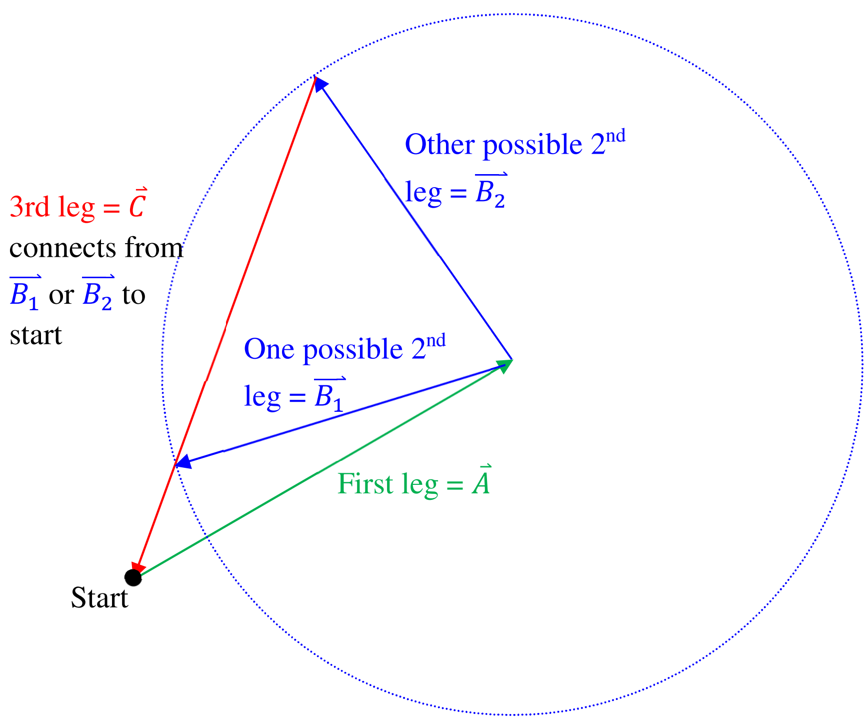
$$(80\cos\theta)^2 + (80\sin\theta)^2 = (-A_x - C_x)^2 + \left(-A_y - C_y\right)^2$$

$$80^2 = (A_x + C_x)^2 + \left(A_y + C_y\right)^2$$

$$80^2 = \left(80 + (-C\sin 20)\right)^2 + \left(60 + (-C\cos 20)\right)^2$$

From here it is a lot of algebra. Notice that you will get a quadratic formula for C. This means you could have 0, 1, or 2 solutions (see below for how this might work out). In this case I believe the math should give you two solutions for C (one short and one long) but I haven't checked it yet... Each of these solutions gives a different choice of angle for the second leg of the trip.

I thought about this geometrically as well. The first leg is nothing unusual. For the second leg, we know only how far Boanventur walks. That means he could end up anywhere on the circle that has a radius equal to the distance he walked. Finally, we know the last leg must connect from somewhere on that circle to the starting point AND be along the heading stated. In this case that gives rise to two possible answers.



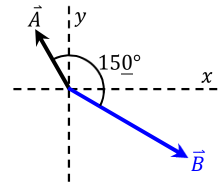
If the size of the circle was just perfect for the final heading, there would only be one answer. This occurs when the final vector lies tangent to the circle.

Finally, if the 2nd leg distance is too short, the 3rd leg never intersects the circle. Remember, we were told the heading <u>and</u> final position of the 3rd leg so we are not free to slide it around…only to change the length.

In this case the problem makes no sense…or the weed was <u>really</u> good.

3.9 **I assumed three sig figs on all given numbers and tracked the sig figs.**

a) This first case is solved quickly with a picture and equation 3.2. Consider the figure at right. One finds



$$\vec{A}\cdot\vec{B} = (10.0)(20.0)\cos 15\underline{0}°$$
$$\boldsymbol{\vec{A}\cdot\vec{B} = -17\underline{3}.2}$$

b) The case is more easily solved using equation 3.1. I found

$$\vec{C}\cdot\vec{D} = C_xD_x + C_yD_y + C_zD_z$$
$$\vec{C}\cdot\vec{D} = (1.00)(-3.00) + (-2.00)(1.00) + (3.00)(-2.00)$$
$$\vec{C}\cdot\vec{D} = (-3.0\underline{0}) + (-2.0\underline{0}) + (-6.0\underline{0})$$
$$\boldsymbol{\vec{C}\cdot\vec{D} = -11.0\underline{0}}$$

c) First get the vector $\vec{A}$ in Cartesian form (i-hats and j-hats) *then divide by the magnitude*.

$$\vec{A} = -10.0\sin 30.0°\,\hat{\imath} + 10.0\cos 30.0°\,\hat{\jmath}$$
$$\vec{A} = -5.0\underline{0}0\hat{\imath} + 8.6\underline{6}0\hat{\jmath}$$

Don' t forget I asked for $\hat{A}$…not $\vec{A}$!!!

$$\hat{A} = \frac{\vec{A}}{A}$$
$$\hat{A} = \frac{-5.0\underline{0}0\hat{\imath} + 8.6\underline{6}0\hat{\jmath}}{10.0}$$
$$\hat{A} = \frac{-5.0\underline{0}0\hat{\imath}}{10.0} + \frac{8.6\underline{6}0\hat{\jmath}}{10.0}$$
$$\boldsymbol{\hat{A} = -0.50\underline{0}0\hat{\imath} + 0.86\underline{6}0\hat{\jmath}}$$

d) To get $\hat{C} = \frac{\vec{C}}{C}$ we first need $C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(1.00)^2 + (-2.00)^2 + (3.00)^2} = \sqrt{14.0\underline{0}} \approx 3.74\underline{1}7$

$$\boldsymbol{\hat{C} = 0.26\underline{7}3\hat{\imath} - 0.53\underline{4}5\hat{\jmath} + 0.80\underline{1}8\hat{k}}$$

e) I used my sketch to find the angle between $\vec{A}$ & $\vec{B}$ was $\mathbf{15\underline{0}.0°}$.
That said, notice the technique shown below gives consistent results.

$$\theta_{AB} = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right) = \cos^{-1}\left(\frac{-17\underline{3}.2}{(10.0)(20.0)}\right) = \mathbf{15\underline{0}.0°}$$

Note: if you didn't know the angle between the two vectors, the numerator inside the parentheses is found using $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$.

f) I used

$$\theta_{CD} = \cos^{-1}\left(\frac{\vec{C}\cdot\vec{D}}{CD}\right)$$
$$\theta_{CD} = \cos^{-1}\left(\frac{-11.0\underline{0}}{\sqrt{14.0\underline{0}}\sqrt{14.0\underline{0}}}\right)$$
$$\boldsymbol{\theta_{CD} = 14\underline{1}.8°}$$

Note: I got the numerator using the result of part b. The denominator terms were found using the process similar to part d. Rather than *memorizing* this equation, hopefully you remember the *procedure* and feel comfortable with the equations.

Solution continues on next page…

g) I usually go straight to this memorized result (derivation to follow):

$$\theta_{\pm z} = \cos^{-1}\left(\pm\frac{C_z}{C}\right)$$

In this case, we are going to the *positive* z-axis so I use the $+$ symbol.

$$\boldsymbol{\theta_z = 36.\underline{7}0°}$$

Following the step-by-step procedure outlined in the workbook, I will derive the above result:

$$\vec{C}\cdot\hat{k} = \vec{C}\cdot\hat{k}$$

Here it is useful to remember that $+\hat{k}$ means one unit in the $+z$-direction. Said another way

$$\hat{k} = 0\hat{\imath} + 0\hat{\jmath} + 1\hat{k}$$

Notice the magnitude of $\hat{k}$ is $\|\hat{k}\| = 1$.

$$\|\vec{C}\|\,\|\hat{k}\|\cos\theta_{\vec{C}\text{ to }+z\text{ axis}} = C_x(0) + C_y(0) + C_z(+1)$$

$$(C)(1)\cos\theta_{\vec{C}\text{ to }+z\text{ axis}} = +C_z$$

$$\cos\theta_{\vec{C}\text{ to }+z\text{ axis}} = +\frac{C_z}{C}$$

$$\theta_{\vec{C}\text{ to }+z\text{ axis}} = \cos^{-1}\left(+\frac{C_z}{C}\right)$$

Notice: had we wanted the angle to the *negative* z-axis, the pink plus signs would change to minus signs…

h) The question asked for $\vec{A}\cdot\vec{C}$. Since $\vec{C}$ is 3D, it seems easiest to me to use the Cartesian form of $\vec{A}$.

$$\vec{A}\cdot\vec{C} = A_xC_x + A_yC_y + A_zC_z$$

Note: Because the $\hat{k}$ term of $\vec{A}$ is zero we know $A_z = 0$.

$$\vec{A}\cdot\vec{C} = (-5.0\underline{0}0)(1.00) + (8.6\underline{6}0)(-2.00) + (0)(3.00)$$

$$\vec{A}\cdot\vec{C} = -5.0\underline{0}0 - 17.\underline{3}2$$

$$\boldsymbol{\vec{A}\cdot\vec{C} = -22.\underline{3}2}$$

i) To get the angle between $\vec{B}$ & $\vec{D}$:

$$\theta_{BD} = \cos^{-1}\left(\frac{\vec{B}\cdot\vec{D}}{BD}\right)$$

$$\theta_{BD} = \cos^{-1}\left(\frac{B_xD_x + B_yD_y + B_zD_z}{BD}\right)$$

We were given $\vec{B}$ in polar form, so we must first convert it to Cartesian.

$$\vec{B} = 20.0\cos 30.0°\,\hat{\imath} - 20.0\sin 30.0°\,\hat{\jmath}$$

$$\vec{B} = 17.\underline{3}2\hat{\imath} - 10.\underline{0}0\hat{\jmath}$$

Now we can plug in. Because the $\hat{k}$ term of $\vec{B}$ is zero we know $B_z = 0$.

$$\theta_{BD} = \cos^{-1}\left(\frac{(17.\underline{3}2)(-3.00) + (-10.\underline{0}0)(1.00) + (0)(-2.00)}{(20.0)\sqrt{14.0\underline{0}}}\right)$$

$$\theta_{BD} = \cos^{-1}\left(\frac{-51.\underline{9}6 - 10.\underline{0}0}{74.\underline{8}3}\right)$$

$$\boldsymbol{\theta_{BD} = 14\underline{5}.9°}$$

Solution continues on the next page…

j) To get the angle between $\vec{D}$ and the *negative* y-axis I use a similar procedure as in part g:

$$\theta_{\vec{D}\text{ to}-y\text{ axis}} = \cos^{-1}\left(-\frac{D_y}{D}\right)$$

$$\theta_{\vec{D}\text{ to}-y\text{ axis}} = \cos^{-1}\left(-\frac{1.00}{\sqrt{14.0\underline{0}}}\right)$$

$$\boldsymbol{\theta_{\vec{D}\text{ to}-y\text{ axis}} = 10\underline{5}.5°}$$

k) Trick question suckas!

We know that $\vec{B}$ is in the xy-plane.

Therefore the angle from $\vec{B}$ to the z-axis is $90°$!

You could do all the work to figure that out or just know the answer.

3.10

a) System A **<u>is</u> right handed**.
b) System B **<u>is not</u> right handed**.
c) System C **<u>is</u> right handed**.
d) System D **<u>is</u> right handed**.
e) System E **<u>is</u> right handed**.
f) System F **<u>is not</u> right handed**.
g) System G **<u>is</u> right handed**.
h) System H **<u>is</u> right handed**.

3.11

a) The *magnitude* of the cross-product is given by

$$\left\|\vec{A}\times\vec{B}\right\| = (10.0)(20.0)\sin 15\underline{0}^\circ = 10\underline{0}$$

According to the coordinate system, *out of the page* is $+\hat{k}$.

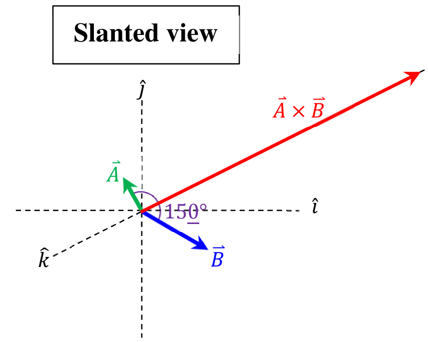
Using the right hand rule (see below), the *direction* of $\vec{A}\times\vec{B}$ is *into* the page.

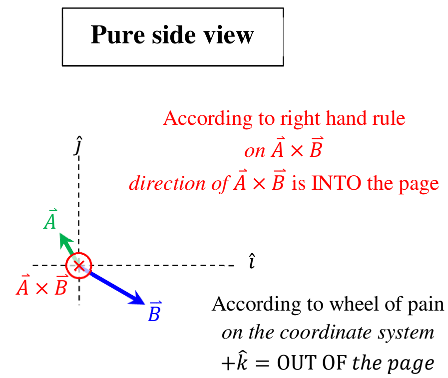
Therefore $\vec{A}\times\vec{B} = -10\underline{0}\hat{k}$.

Right hand rule for this case:

1. Stick out your right hand.
2. Swivel the picture around until $\vec{A}$ lines up with your right hand.
3. Try curling the fingers of your right hand towards $\vec{B}$.
   You may need to try flipping you right hand to a couple of different orientations until you find one where it is easy to curl your fingers to $\vec{B}$.
4. Finally, straighten out your right thumb. It points in the direction of $\vec{A}\times\vec{B}$.

Therefore $\vec{A}\times\vec{B} = -10\underline{0}\hat{k}$.





If you didn't see the above trick, you can always go with the sledgehammer (equation 3.9)…

$$\vec{A} = (-10.0\sin 30.0^\circ\,\hat{\imath} + 10.0\cos 30.0^\circ\,\hat{\jmath})$$

$$\vec{A} = \left(-5.0\underline{0}\hat{\imath} + 8.66\underline{0}\hat{\jmath}\right)$$

$$\vec{B} = (20.0\cos 30.0^\circ\,\hat{\imath} - 20.0\sin 30.0^\circ\,\hat{\jmath})$$

$$\vec{B} = \left(17.\underline{3}2\hat{\imath} - 10.\underline{0}\hat{\jmath}\right)$$

$$\vec{A}\times\vec{B} = \left(-5.0\underline{0}\hat{\imath} + 8.66\underline{0}\hat{\jmath}\right)\times\left(17.\underline{3}2\hat{\imath} - 10.\underline{0}\hat{\jmath}\right)$$

TIP: when doing cross products, $\hat{\imath}\times\hat{\imath} = 0$ and $\hat{\jmath}\times\hat{\jmath} = 0$. IGNORE THESE TERMS!!!

$$\vec{A}\times\vec{B} = \left(-5.0\underline{0}\hat{\imath}\right)\times\left(-10.\underline{0}\hat{\jmath}\right) + \left(8.66\underline{0}\hat{\jmath}\right)\times\left(17.\underline{3}2\hat{\imath}\right)$$

$$\vec{A}\times\vec{B} = 50.\underline{0}(\hat{\imath})\times(\hat{\jmath}) + 15\underline{0}.0(\hat{\jmath})\times(\hat{\imath})$$

$$\vec{A}\times\vec{B} = 50.\underline{0}\left(+\hat{k}\right)\times +15\underline{0}.0\left(-\hat{k}\right)$$

$$\boldsymbol{\vec{A}\times\vec{B} = -10\underline{0}.0\hat{k}}$$

b) Using equation 3.9 for cross-products gives

$$\vec{C}\times\vec{B} = \left(-40.\underline{0}\hat{\jmath} + 30.\underline{0}\hat{k}\right)\times\left(17.\underline{3}2\hat{\imath} - 10.\underline{0}\hat{\jmath}\right)$$

$$\vec{C}\times\vec{B} = -69\underline{2}.8(\hat{\jmath}\times\hat{\imath}) + 40\underline{0}(\hat{\jmath}\times\hat{\jmath}) + 51\underline{9}.6\left(\hat{k}\times\hat{\imath}\right) - 30\underline{0}\left(\hat{k}\times\hat{\jmath}\right)$$

$$\vec{C}\times\vec{B} = -69\underline{2}.8\left(-\hat{k}\right) + 0 + 51\underline{9}.6(\hat{\jmath}) - 30\underline{0}(-\hat{\imath})$$

Now put it in standard order and clean up the minus signs.

$$\boldsymbol{\vec{C}}\times\boldsymbol{\vec{B}} = \boldsymbol{30\underline{0}\hat{\imath} + 51\underline{9}.6\hat{\jmath} + 69\underline{2}.8\hat{k}}$$

In the third line above I used the wheel of pain trick and dropped $\hat{\jmath}\times\hat{\jmath}$. With experience this can sometimes be a real time saver. Notice it is important to keep everything in the proper right-to-left order or you will introduce minus sign errors when doing cross-products.

Going further: you could express this as a magnitude times a unit vector like this

$$\boldsymbol{\vec{C}}\times\boldsymbol{\vec{B}} = \mathbf{91\underline{6}.5}\left(\mathbf{0.32\underline{7}3}\boldsymbol{\hat{\imath}} + \mathbf{0.56\underline{6}9}\boldsymbol{\hat{\jmath}} + \mathbf{0.75\underline{5}9}\boldsymbol{\hat{k}}\right)$$

Solution continues on next page…

c) When we perform order of operations on $\vec{C} \times (\vec{A} \cdot \vec{B})$ we would first take the dot product. The output of the dot product is a *scalar*. One cannot proceed to cross vector $\vec{C}$ with a scalar! In $\vec{C} \cdot (\vec{A} \times \vec{B})$ the output of the parentheses is a *vector* which then works as an input for the dot product.

d) Trick question! We already did all the heavy lifting.
In part a) we determined

$$\boldsymbol{\vec{A} \times \vec{B} = -10\underline{0}.0\hat{k}}$$

This question asked for $\boldsymbol{\vec{B} \times \vec{A}}$.

In the list of facts about cross products in the workbook, you were supposed to notice flipping the order of terms in a cross product flips the sign.

$$\boldsymbol{\vec{B} \times \vec{A} = -(\vec{A} \times \vec{B}) = +10\underline{0}.0\hat{k}}$$

If you didn't notice this, you should get this result by following the standard procedure shown in part a.

Another way to think about it:
When you switch the terms in the cross product, you switch the order of all $\hat{\imath}$'s & $\hat{\jmath}$'s in the computation.
This means you would switch the direction you go in the wheel of pain for each term.
This flips the sign of each term.
When you flip the sign of each term, that is the same as flipping the sign of the result.

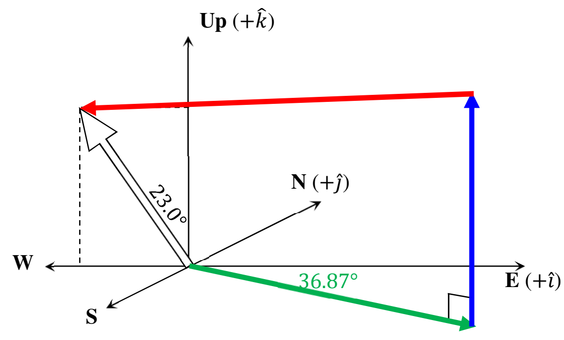
3.12 Some color coding probably will help with this one.

$\vec{A}$ = First displacement = 10.00 m heading 36.87° south of east.

$\vec{B}$ = Second displacement = upwards 7.25 m = 7.25 m $\hat{k}$

$\vec{C}$ = Third displacement is unknown.

$\vec{R}$ = Final position = resultant = 5.00 m angled 23.0° west of up.



a) $\vec{A} = 10.00\text{ m}(\cos 36.87°)\hat{\imath} - 10.00\text{ m}(\sin 36.87°)\hat{\jmath}$

$\vec{A} = 8.00\underline{0}0\text{ m }\hat{\imath} - 6.00\underline{0}0\text{ m }\hat{\jmath}$

WATCH OUT! I expect units on *this* answer because you were initially given numbers with units.

b) $\vec{R} = -5.00\text{ m}(\sin 23.0)\,\hat{\imath} + 5.00\text{ m}(\cos 23.0)\,\hat{k}$

$\vec{R} = -1.9\underline{5}4\text{ m }\hat{\imath} + 4.6\underline{0}3\text{ m }\hat{k}$

Typically these numbers should only have three sig figs as the given numbers were only stated to three sig figs. That said, I told you in the problem statement to write them to four sig figs.

The *percent* uncertainty is more accurately represented when we keep the extra digit for numbers with first digit one.

c) First I would rearrange $\vec{A} + \vec{B} + \vec{C} = \vec{R}$ to give $\vec{C} = \vec{R} - (\vec{A} + \vec{B})$. Then I would find

$$\vec{C} = \vec{R} - (\vec{A} + \vec{B})$$
$$\vec{C} = \vec{R} + (-\vec{A}) + (-\vec{B})$$

$$\vec{R} = \; -1.9\underline{5}4\text{ m }\hat{\imath} + 0.000\text{ m }\hat{\jmath} + 4.6\underline{0}3\text{ m }\hat{k}$$
$$-\vec{A} = -8.00\underline{0}0\text{ m }\hat{\imath} + 6.00\underline{0}0\text{ m }\hat{\jmath} + 0.0000\text{ m }\hat{k}$$
$$\underline{-\vec{B} = 0.0000\text{ m }\hat{\imath} + 0.0000\text{ m }\hat{\jmath} - 7.2\underline{5}0\text{ m }\hat{k}}$$
$$\vec{C} = -9.9\underline{5}4\text{ m }\hat{\imath} + 6.00\underline{0}0\text{ m }\hat{\jmath} - 2.6\underline{4}7\text{ m }\hat{k}$$

The problem statement then said to round all numbers to three sig figs. I will also factor out the units of m.

$$\vec{C} = (-9.95\hat{\imath} + 6.00\hat{\jmath} - 2.65\hat{k})\text{ m}$$

If you did all the vector stuff right but left off the units you should expect to lose points.

Generally, if a problem statement gives you decimal numbers with units, your answers should have decimal numbers with units.

If a problem statement gives you letters for variables (without units) you should answer with variables (without units). Note: in engineering and physics it is standard to simplify fractions or compute and leading fraction and write it as a single decimal number (in the numerator) with at 3-4 sig figs.

Note: often in engineering we make special exceptions for numbers whose first digit is a 1. **If the first digit of a final result is 1, we often write an extra sig fig in your answers.** If you are curious, consider the numbers 9.9 versus 10.0. They are both off by about 0.1 out of about 10 (1% uncertainty). This gives an intimation as to why keeping the extra sig fig on numbers with first digit 1.

3.13

a) You are told $\vec{L} = \vec{r} \times \vec{p}$. The units of $\vec{r}$ are m and the units of $\vec{p}$ are $\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}$. Units of $\vec{L}$ are $\mathbf{kg \cdot \frac{m^2}{s}}$.

$$[\vec{L}] = [\vec{r}] \times [\vec{p}] = \mathrm{m} \times \mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}} = \mathbf{kg \cdot \frac{m^2}{s}}$$

b) Use the procedure outlined in the workbook

$$\vec{r} \cdot \vec{p} = \vec{r} \cdot \vec{p}$$
$$r_x p_x + r_y p_y + r_z p_z = \|\vec{r}\| \|\vec{p}\| \cos\theta$$

The magnitudes of each vector are

$$r = \|\vec{r}\| = \sqrt{(-3.00)^2 + (2.00)^2} = 3.6\underline{0}6 \text{ m}$$
$$p = \|\vec{p}\| = \sqrt{(5.00)^2 + (-4.00)^2} = 6.4\underline{0}3 \text{ kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}$$

Going back to our earlier premise

$$\vec{r} \cdot \vec{p} = \vec{r} \cdot \vec{p}$$
$$r_x p_x + r_y p_y + r_z p_z = \|\vec{r}\| \|\vec{p}\| \cos\theta$$

Notice $r_z = 0$ and $p_y = 0$. Therefore the only surviving term on the left hand side should be $r_x p_x$!!!

$$(-3.00 \text{ m})\left(5.00 \text{ kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}\right) + (2.00 \text{ m})(0) + (0)\left(-4.00 \text{ kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}\right) = (3.6\underline{0}6 \text{ m})\left(6.4\underline{0}3 \text{ kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}\right) \cos\theta$$

$$-15.\underline{0}0 \text{ kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}} = 23.\underline{0}9 \text{ kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}} \cos\theta$$

$$\cos\theta = \frac{-15.\underline{0}0}{23.\underline{0}9} \quad \textit{UNITS CANCEL!!!}$$

$$\boldsymbol{\theta = \cos^{-1}(-0.64\underline{9}7) \approx 13\underline{0}.5°}$$

c) Now we are asked to compute the angular momentum. This means compute the cross product. I would go straight to the wheel of pain here. It is easy to determine the *magnitude* of the cross-product since we now know both the magnitudes as well as the angle between…but that doesn't help that much when trying to express the final result in Cartesian form. I will ignore units until the last step for clarity.

$$\vec{L} = \vec{r} \times \vec{p}$$
$$\vec{L} = (-3.00\hat{\imath} + 2.00\hat{\jmath}) \times (5.00\hat{\imath} - 4.00\hat{k})$$

Ignore terms with $\hat{\imath} \times \hat{\imath} = 0$

Order matters…

$$\vec{L} = (-3.00)(-4.00)(\hat{\imath} \times \hat{k}) + (2.00)(5.00)(\hat{\jmath} \times \hat{\imath}) + (2.00)(-4.00)(\hat{\jmath} \times \hat{k})$$
$$\vec{L} = (12.0)(-\hat{\jmath}) + (10.0)(-\hat{k}) + (-8.00)(\hat{\imath})$$

I will now rewrite in the standard order ***with units***!!!

$$\boldsymbol{\vec{L} = (-8.00\hat{\imath} - 12.0\hat{\jmath} - 10.0\hat{k})\ \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}}$$

If you forgot to include the units on your final answer you should expect to lose some points…not much but some.

Final comment: problem statement said to write in Cartesian form with each component having three sig figs. If you had only wanted the *magnitude* one could've done this

$$L = \|\vec{L}\| = \|\vec{r} \times \vec{p}\|$$
$$L = rp \sin\theta$$
$$L = (3.6\underline{0}6 \text{ m})\left(6.4\underline{0}3 \text{ kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}\right) \sin 13\underline{0}.5° \approx 17.\underline{5}5 \text{ kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}}$$

The only trouble? We don't have a direction…this in not a vector (it is just a vector *magnitude*). One could work at it from here…but much easier to just do the wheel of pain.

3.14 Problem says to ignore units. The vector is given as $\vec{E} = -2.00\hat{\imath} + 3.00\hat{\jmath} - 4.00\hat{k}$. The magnitude is thus

$$E = \|\vec{E}\| = \sqrt{(-2.00)^2 + (3.00)^2 + (-4.00)^2} = 5.3\underline{8}5$$

The unit vector is obtained using

$$\hat{E} = \frac{\vec{E}}{E}$$

$$\hat{E} = \frac{-2.00}{5.3\underline{8}5}\hat{\imath} + \frac{3.00}{5.3\underline{8}5}\hat{\jmath} - \frac{4.00}{5.3\underline{8}5}\hat{k}$$

$$\hat{E} = -0.37\underline{1}4\,\hat{\imath} + 0.55\underline{7}1\,\hat{\jmath} - 0.74\underline{2}8\,\hat{k}$$

The angle to the positive z-axis is given by

$$\theta_z = \cos^{-1}\left(\frac{\vec{E}\cdot\hat{k}}{E}\right) = \cos^{-1}(-0.74\underline{2}8\,) \approx 13\underline{8}.0°$$

3.15

a) $\hat{r} = 0.371\hat{\imath} - 0.557\hat{\jmath} - 0.743\hat{k}$ with **NO UNITS!!!** Think: $[\hat{r}] = \frac{[\vec{r}]}{[r]} = \frac{\mathrm{m}}{\mathrm{m}} = no\ units$!

Why three sig figs? Because I asked you to do it that way in problem statement.

b) The quickest way is to memorize

$$\theta_{+z} = \cos^{-1}\left(\left(+\hat{k}\right)\cdot\hat{r}\right) = \cos^{-1}(-0.7428) = 13\underline{8}.0°$$

Notice I used the <u>unrounded</u> answer to ensure I got my angle correct to 3 sig figs.
Avoid the cross-product and use the dot product for this. It always works with minimal mental effort.
Perhaps you thought of doing this style

$$\vec{r}\cdot\hat{k} = \vec{r}\cdot\hat{k}$$

$$\|\vec{r}\|\,\|\hat{k}\|\cos\theta_z = \left(r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{k}\right)\cdot\hat{k}$$

On the left hand side we know the magnitude of a unit vector is 1.
On the right hand side we know $\hat{\imath}\cdot\hat{k} = 0$, $\hat{\jmath}\cdot\hat{k} = 0$, and $\hat{k}\cdot\hat{k} = 1$.

$$r\,(1)\cos\theta_z = r_z$$

$$\theta_z = \cos^{-1}\left(\frac{r_z}{r}\right) = \cos^{-1}(-0.7428) = 13\underline{8}.0°$$

c) $\vec{\tau} = \left(-18\hat{\imath} + 8\hat{\jmath} - 15\hat{k}\right)\,\mathrm{N}\cdot\mathrm{m}$

Side note: I don't know why some we say N · m for torque (as opposed to m · N).
Perhaps you have heard of ft · lbs? Why these particular orders…I don't really care but it is curious.

3.16 I got lazy with sig figs. Your final answers might differ in 3rd digit since I rounded to 3 early on.

a) From the wording of the problem statement we find $\vec{A}+\vec{B}+\vec{C}=0$ or $\vec{\boldsymbol{C}}=-\vec{\boldsymbol{A}}-\vec{\boldsymbol{B}}$.

WATCH OUT! Don't screw up the signs on $\vec{A}$ (or forget to change $\vec{A}\to-\vec{A}$). If you look in the picture you will see $\vec{A}$ points up and to the *left*. According to the coordinate system shown in the figure we know

$$\vec{A}= \quad 0\hat{\imath} \quad -\ 10.0\sin 20.0°\,\hat{\jmath} \ +\ 10.0\cos 20.0°\,\hat{k}$$
$$\vec{A}= \quad 0\hat{\imath} \quad -\quad 3.42\,\hat{\jmath} \quad +\quad 9.40\ \hat{k}$$

Therefore $-\vec{A}$ is given by

$$-\vec{\boldsymbol{A}}= \quad \boldsymbol{0\hat{\imath}} \quad +\quad \boldsymbol{3.42\,\hat{\jmath}} \quad -\quad \boldsymbol{9.40\ \hat{k}}$$

$$\vec{B}=20\sin 30°\,\hat{\imath}\ +\ 20\cos 30°\,\hat{\jmath}+\qquad 0\hat{k}$$
$$\vec{B}=\quad 10.0\hat{\imath}\ +\quad 17.32\,\hat{\jmath}\quad +\qquad 0\hat{k}$$

Therefore $-\vec{B}$ is given by

$$-\vec{\boldsymbol{B}}= \quad -\boldsymbol{10.0\hat{\imath}} - \quad \boldsymbol{17.32\,\hat{\jmath}} \quad + \qquad \boldsymbol{0\hat{k}}$$

Doing the vector addition $\vec{\boldsymbol{C}}=-\vec{\boldsymbol{A}}-\vec{\boldsymbol{B}}$

$$-\vec{\boldsymbol{A}}= \quad \boldsymbol{0\hat{\imath}} \ +\boldsymbol{3.42\,\hat{\jmath}} \ -\boldsymbol{9.40\hat{k}}$$
$$-\vec{\boldsymbol{B}}=-\boldsymbol{10.0\hat{\imath}}-\boldsymbol{17.32\,\hat{\jmath}}+\boldsymbol{0\hat{k}}$$
$$\vec{\boldsymbol{C}}=-\boldsymbol{10.0\hat{\imath}}-\boldsymbol{13.90\hat{\jmath}}-\boldsymbol{9.40\hat{k}}$$

It is worth comparing this result to the initial figure. The third displacement is supposed to bring the moth back to the origin. From the figure we can tell from both the $\hat{\imath}$ and $\hat{k}$ terms *must* be negative and *probably* the $\hat{\jmath}$, too. Because the figure is not to scale I choose not to trust it completely for my $\hat{\jmath}$ estimate. Our result agrees qualitatively with this observation.

b) The magnitude is $C=\sqrt{(-10.0)^2+(-13.9)^2+(-9.40)^2}\approx 19.5$.

c) The angle to the negative z-axis is given by $\hat{C}\cdot(-\hat{k})=\cos\theta_z$. One finds $\theta_z=\cos^{-1}\left(\frac{-(-9.4)}{19.5}\right)\approx 61.2°$. Derivation of this result is below.

Watch out! That extra minus sign inside the inverse cosine function comes from the wording of the problem (to the *negative* axis…not the *positive* axis).

How to Determine the Angle Between Two Known Vectors

STYLE 1	**STYLE 2**
$\vec{A}\cdot\vec{B}=\vec{A}\cdot\vec{B}$	$\vec{A}\cdot\vec{B}=\vec{A}\cdot\vec{B}$
$AB\cos\theta_{AB}=A_xB_x+A_yB_y+A_zB_z$	$AB\cos\theta_{AB}=\vec{A}\cdot\vec{B}$
$\cos\theta_{AB}=\dfrac{A_xB_x+A_yB_y+A_zB_z}{AB}$	$\cos\theta_{AB}=\dfrac{\vec{A}\cdot\vec{B}}{AB}$ **OR** $\hat{A}\cdot\hat{B}$
$\theta_{AB}=\cos^{-1}\left(\dfrac{A_xB_x+A_yB_y+A_zB_z}{AB}\right)$	$\theta_{AB}=\cos^{-1}\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right)$ **OR** $\cos^{-1}(\hat{A}\cdot\hat{B})$

Angle between a vector and an axis

The above procedure can be used to determine the angle between a vector and any axis.

To determine the angle between $\vec{C}$ and the *negative* z-axis let $\vec{A}=\vec{C}$ & $\vec{B}=-\hat{k}$ and do the procedure above.

$$\theta_{\boldsymbol{neg}\ z\ axis}=\cos^{-1}\left(\frac{\vec{C}\cdot(-\hat{k})}{C(1)}\right)=\cos^{-1}\left(\frac{-C_z}{C}\right)$$

Notice: $\frac{C_z}{C}$ is simply the $\hat{k}$ part of $\hat{C}$. We could instead write the above result as

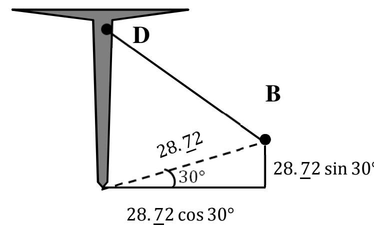
$$\theta_{\boldsymbol{neg}\ z\ axis}=\cos^{-1}(-1\text{ times the }\hat{k}\text{ part of }\hat{C})$$

3.17

a) I used the Pythagorean theorem to determine the distance from the origin to the point where the guy lines touch the ground ($28.\underline{7}2$ m). The vector from **D** to **A** goes down ($-\hat{k}$) and out of the page ($+\hat{\imath}$) giving

$$\vec{A} = 28.\underline{7}2\hat{\imath} - 20.\underline{0}\hat{k}$$
$$\hat{A} = 0.82\underline{0}6\hat{\imath} - 0.57\underline{1}4\hat{k}$$

b) From **D** to point **B** we expect the vector must go down ($-\hat{k}$), to the right ($+\hat{\jmath}$), and into the page ($-\hat{\imath}$).



$$\vec{B} = -28.\underline{7}2 \sin 30° \,\hat{\imath} + 28.\underline{7}2 \cos 30° \,\hat{\jmath} - 20.\underline{0}\hat{k}$$
$$\hat{B} = -0.41\underline{0}3\hat{\imath} + 0.71\underline{0}6\hat{\jmath} - 0.57\underline{1}4\hat{k}$$

c) Same as $\vec{B}$ except go left instead of right.

$$\hat{C} = -0.41\underline{0}3\hat{\imath} - 0.71\underline{0}6\hat{\jmath} - 0.57\underline{1}4\hat{k}$$

d) The angle should be the same between any pair of wires by symmetry. The angle is given by

$$\theta = \cos^{-1}\left(\frac{\hat{A}\cdot\hat{B}}{1\cdot 1}\right) = \cos^{-1}(-0.01\underline{0}19) \approx 9\underline{0}.6°$$

WOW! They turned out *nearly* perpendicular! That's interesting.

e) For some reason I neglected to include the weight of the structure in the problem statement (a bad mistake). That said, we can still do the math as if there only force vectors from each guy line and the wind. Perhaps the weight of the structure is negligible somehow; maybe it is made out of aerogel?

I will assume there is a force with magnitude Z from the ground acting straight up (in the $\hat{k}$ direction).

$$\vec{A} + \vec{B} + Z\hat{k} + \vec{C} + 1000\hat{\jmath} = 0$$
$$A\hat{A} + B\hat{B} + Z\hat{k} + 6000\hat{C} + 1000\hat{\jmath} = 0$$
$$A(0.82\underline{0}6\hat{\imath} - 0.57\underline{1}4\hat{k}) + B(-0.41\underline{0}3\hat{\imath} + 0.71\underline{0}6\hat{\jmath} - 0.57\underline{1}4\hat{k}) + Z\hat{k} + (-24\underline{6}2\hat{\imath} - 42\underline{6}4\hat{\jmath} - 34\underline{2}8\hat{k}) + 1000\hat{\jmath} = 0$$
$$(0.82\underline{0}6A - 0.41\underline{0}3B)\hat{\imath} + (0.71\underline{0}6B)\hat{\jmath} + (-0.57\underline{1}4A - 0.57\underline{1}4B + Z)\hat{k} = 24\underline{6}2\hat{\imath} + 32\underline{6}4\hat{\jmath} + 34\underline{2}8\hat{k}$$

Comparing $\hat{\imath}$'s to $\hat{\imath}$'s, $\hat{\jmath}$'s to $\hat{\jmath}$'s, and $\hat{k}$'s to $\hat{k}$'s gives three equations.

$$\begin{aligned} &\textit{From the } \hat{\imath}'s\text{:} & 0.82\underline{0}6A - 0.41\underline{0}3B &= 24\underline{6}2 \\ &\textit{From the } \hat{\jmath}'s\text{:} & 0.71\underline{0}6B &= 32\underline{6}4 \\ &\textit{From the } \hat{k}'s\text{:} & -0.57\underline{1}4A - 0.57\underline{1}4B + Z &= 34\underline{2}8 \end{aligned}$$

The $\hat{\jmath}$ equation gives $B = 80\underline{2}2$ N ≈ 8.02 kN.

Next I used the $\hat{\imath}$ equation to find $A = 70\underline{1}1$ N ≈ 7.01 kN.

Finally I used the $\hat{k}$ equation to find $Z = 120\underline{1}8$ N ≈ 12.02 kN.

3.18

a) $\vec{d}_{1to2} = 2R\hat{\jmath}$

b) $\vec{d}_{1to3} = -2R\hat{\imath}$

c) We know the orange is centered above the gap in the lower set of oranges. We know this gap should occur halfway towards the orange to the right and halfway to the orange behind. This tells us

$$\vec{d}_{1to4} = -R\hat{\imath} + R\hat{\jmath} + z\hat{k}$$

Unfortunately we don't know how far up the orange is. We do know, however, the distance between the centers of the oranges is $2R$. This tells us

$$\left\|\vec{d}_{1to4}\right\| = 2R = \sqrt{R^2 + R^2 + z^2}$$

Square both sides and solve for z to show $z = \sqrt{2}R$. Therefore the vector that describes the distance between these two oranges is given by

$$\vec{d}_{1to4} = -R\hat{\imath} + R\hat{\jmath} + 2R\hat{k}$$

$$\vec{d}_{1to4} = 2R\left(-\frac{1}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath} + \frac{\sqrt{2}}{2}\hat{k}\right)$$

Notice I wrote the last line as a magnitude times a unit vector. This style is useful in other classes.

3.19

a) If $\vec{\mu}$ is anti-aligned with $\vec{B}$, (or aligned with $-\hat{\jmath}$), it has max energy. Think: if you try to purposefully misalign magnets they have a lot of stored energy and will flip over as soon as you release them!

b) If $\vec{\mu}$ is aligned with $\vec{B}$, (or aligned with $+\hat{\jmath}$), it has min energy.

c) $\Delta U = U_{max} - U_{min} = [-\mu(-\hat{\jmath}) \cdot B\hat{\jmath}] - [-\mu(\hat{\jmath}) \cdot B\hat{\jmath}] = 2\mu B = 10.0 \times 10^{-27}\mathrm{J}$

d) The torque is given by $\vec{\tau} = \vec{\mu} \times B\hat{\jmath}$. The cross product is zero whenever $\vec{\mu}$ is aligned with $\pm\hat{\jmath}$.

e) If $\vec{\mu} = \mu\hat{k}$ then $U = 0$ and $\vec{\tau} = \mu\hat{k} \times B\hat{\jmath} = -\mu B\hat{\imath}$.

f) $U = 0$ any time $\vec{\mu} \perp \vec{B}$. Whenever $\vec{\mu}$ lies in the *xz*-plane.

3.20

a) $\vec{r} = 1.00\text{m}\hat{\imath} + 6.00\text{m}\hat{k}$

b) The force is $\vec{F} = 26\underline{9}0\text{N}\hat{\jmath} - 57\underline{1}.8\text{N}\hat{k}$

c) $\vec{\tau} = \vec{r} \times \vec{F} = (1.00\text{m}\hat{\imath} + 6.00\text{m}\hat{k}) \times (26\underline{9}0\text{N}\hat{\jmath} - 57\underline{1}.8\text{N}\hat{k}) = (-16\underline{1}40\hat{\imath} + 57\underline{1}.8\hat{\jmath} + 26\underline{9}0\hat{k})\text{N}\cdot\text{m}$

3.21

a) $\vec{r}_1 = (0.750\hat{\imath} + 0.64\hat{\jmath} + 7.88\hat{k})\text{m}$ and $\vec{r}_2 = (-0.750\hat{\imath} + 0.64\hat{\jmath} + 7.88\hat{k})\text{m}$

b) $\vec{\tau}_1 = \vec{r}_1 \times \vec{F} = (0.750\hat{\imath} + 0.64\hat{\jmath} + 7.88\hat{k})\text{m} \times (-3000\text{N}\hat{k}) = (-1920\hat{\imath} + 2250\hat{\jmath})\text{N}\cdot\text{m}$

$\vec{\tau}_2 = \vec{r}_2 \times \vec{F} = (-0.750\hat{\imath} + 0.64\hat{\jmath} + 7.88\hat{k})\text{m} \times (-3000\text{N}\hat{k}) = (-1920\hat{\imath} - 2250\hat{\jmath})\text{N}\cdot\text{m}$

3.22 Let $A_x = 20$ and $B_x = -30$. We find the following equations

From equation 3.1	$\vec{A}\cdot\vec{B} = -1000 = -600 + A_yB_y$ or $A_y = \frac{-400}{B_y}$
From equation 3.2	$\vec{A}\cdot\vec{B} = -1000 = AB\cos 153.44°$ or $AB = 111\underline{8}.0$
From equation 3.4	$A = \sqrt{400 + A_y^2}$ $B = \sqrt{900 + B_y^2}$

Plugging in the last two equations into the second gives

$$\sqrt{400 + A_y^2}\sqrt{900 + B_y^2} = 111\underline{8}.0$$

Squaring both sides gives

$$(400 + A_y^2)(900 + B_y^2) = 124\underline{9}900$$

Plugging the result of the first equation for B_y gives

$$\left(400 + \frac{160000}{B_y^2}\right)(900 + B_y^2) = 124\underline{9}900$$

Here I choose to multiply both sides by B_y^2 to get rid of fractions. I bring the B_y^2 inside the first paren's to clean things up.

$$B_y^2\left(400 + \frac{160000}{B_y^2}\right)(900 + B_y^2) = 124\underline{9}900B_y^2$$

$$(400B_y^2 + 160000)(900 + B_y^2) = 124\underline{9}900B_y^2$$

Doing some algebra simplifies this down to a quadratic equation in B_y^2:

$$B_y^4 - 182\underline{4}.75B_y^2 + 360000 = 0$$

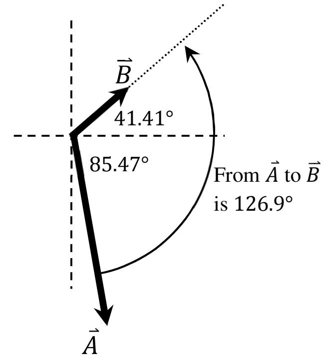
$$B_y^2 = \frac{-(-182\underline{4}.75) \pm \sqrt{(-182\underline{4}.75)^2 - 4(1)(360000)}}{2(1)}$$

From there you find $B_y = \pm 40$ or ± 15. I made an Excel spreadsheet to check all four cases and found the problem works when $B_y = \pm 40$ giving $A_y = \mp 10$. To be clear, when B_y is positive, A_y is negative. The magnitudes in each case are $A = 22.36$ and $B = 50$. Similarly when $B_y = \pm 15$ giving $A_y = \mp 26.67$ giving $A = 33.33$ and $B = 33.54$. I sketched all four vectors roughly to scale to verify the angle between appeared to be approximately $153.44°$ and all four cases looked good.

3.23 **Lazy with sig figs on this one…**Only one solution is possible! From the problem statement we have

From equation 3.2	$\vec{A}\cdot\vec{B} = -6 = 5B\cos\theta$
From equation 3.10	$\lVert\vec{A}\times\vec{B}\rVert = 8 = 5B\sin\theta$

Taking the ratio of the first two equations gives $\tan\theta = -1.333$. Plugging into your calculator gives -53.1. WATCH OUT! Your calculator only gives an answer in the first or fourth quadrants. Another possible solution occurs in the 2nd quadrant by adding 180°. The two possible angles are -53.13° or 126.9°. Using the either of the equations above to solve for B shows -53.13° gives a *negative* value for the *magnitude* of $\vec{B}$ which is impossible! Therefore only 126.9° is a valid solution with $B = 2$. You should find $B_y = \pm 1.323$. If $B_y = 1.323$ then $A_y = -4.984$ and $A_x = 0.395$ which agrees with $A = 5$ form the problem statement.



Now we know $\vec{A} = -4.984\hat{\imath} + 0.395\hat{\jmath}$ and $\vec{B} = 1.5\hat{\imath} + 1.323\hat{\jmath}$ which looks like the figure shown at right. Note: using the right hand rule the cross-product is out of the page as expected.

3.24

a) We know the vector points to the right and up. It will be of the form $\vec{r} = \frac{d}{2}\hat{\jmath} + z\hat{k}$. To figure out z in terms of the givens, use the Pythagorean Thm.

$$\left(\frac{d}{2}\right)^2 + z^2 = L^2$$

$$z = \sqrt{L^2 - \left(\frac{d}{2}\right)^2}$$

Therefore the final result is

$$\vec{r} = \frac{d}{2}\hat{\jmath} + \sqrt{L^2 - \left(\frac{d}{2}\right)^2}\,\hat{k}$$

b) The $\hat{\jmath}$ part flips sign but the $\hat{k}$ part remains the same.

3.25

a) Consider the triangle at right. Notice

$$\cos 30° = \frac{\frac{s}{2}}{d}$$

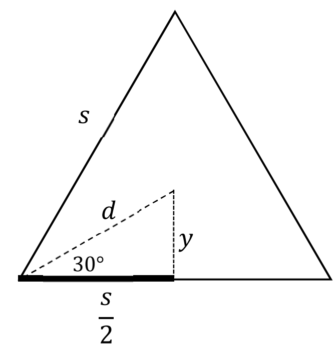
$$d = \frac{s}{2\cos 30°} = \frac{s}{\sqrt{3}}$$

While we're at it, notice

$$\sin 30° = \frac{y}{d}$$

$$y = d\sin 30° = \frac{s}{2\sqrt{3}}$$

This shows us that the center of the triangle is at a *third* of the height…



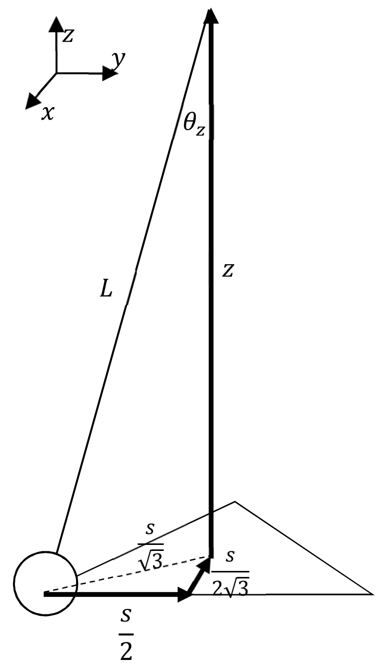
b) To travel from the front left ball to the origin we must move $\frac{s}{2}$ to the right, $\frac{s}{2\sqrt{3}}$ into the page, and some other distance up. Because the string is angled we cannot say we move distance L upwards. Notice a right triangle is formed between the sides with lengths z, L, and $\frac{s}{\sqrt{3}}$.

$$z^2 + \left(\frac{s}{\sqrt{3}}\right)^2 = L^2$$

$$z = \sqrt{L^2 - \frac{s^2}{3}}$$

Make note of the coordinate system I chose. In this case $\hat{\jmath}$ is to the right and into the page is $-\hat{\imath}$. The displacement vector is thus

$$\vec{r} = -\frac{s}{2\sqrt{3}}\hat{\imath} + \frac{s}{2}\hat{\jmath} + \sqrt{L^2 - \frac{s^2}{3}}\hat{k}$$



c) To get the angle from the vertical axis I choose to do

$$\vec{r}\cdot\hat{k} = \|\vec{r}\|\|\hat{k}\|\cos\theta_z$$

Don't forget, even though $\vec{r}$ looks ugly, we already know $\|\vec{r}\| = L$!!!
Also, the magnitude of any unit vector is 1 by definition ($\|\hat{k}\| = 1$).
Finally $\hat{\imath}\cdot\hat{k} = 0$ and $\hat{\jmath}\cdot\hat{k} = 0$. Plug it all in to find

$$\vec{r}\cdot\hat{k} = \|\vec{r}\|\|\hat{k}\|\cos\theta_z$$

$$\sqrt{L^2 - \frac{s^2}{\sqrt{3}}} = L(1)\cos\theta_z$$

$$\cos\theta_z = \frac{\sqrt{L^2 - \frac{s^2}{3}}}{L} = \sqrt{1 - \frac{s^2}{3L^2}}$$

$$\theta_z = \cos^{-1}\left(\sqrt{1 - \frac{s^2}{3L^2}}\right)$$

You could also look at the picture and use

$$\boldsymbol{\theta_z = \sin^{-1}\left(\frac{s}{\sqrt{3}L}\right)}$$