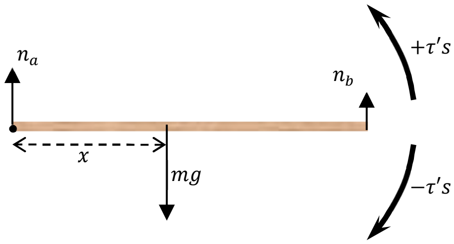
Ch12
12.1

a) I drew an FBD of the plank only at right. The plank exerts a *normal* force *upwards* on the zombie. By Newton's 3rd law the person must therefore exert a *normal* force *downwards* on the plank. All semester I have been telling you to beware…normal force (or tension) is *not necessarily* mg!!! Well, in this chapter only you get lucky (for the most part). Since most objects in this chapter are *not accelerating*, we *can* replace many of the tensions (or normal forces) with mg. That is why the zombie exerts force with magnitude mg downwards on the plank.



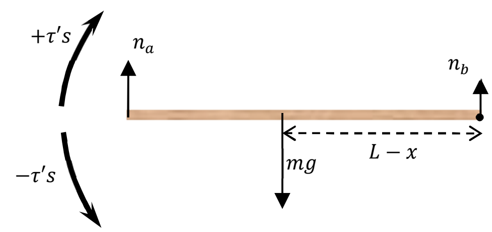
$$\Sigma\tau_{\mathrm{A}}: \quad -xmg + Ln_b = 0$$

b) Sum of forces in vertical direction gives $n_b + n_a = mg$

c) Combining the previous two results gives

$$x = L\frac{n_b}{n_b + n_a}$$

d) To check about point **B** I redrew the FBD and coordinates.



$$\Sigma\tau_{\mathrm{B}}: \quad -(L-x)mg + Ln_a = 0$$

Plugging in the result from part b) for mg gives

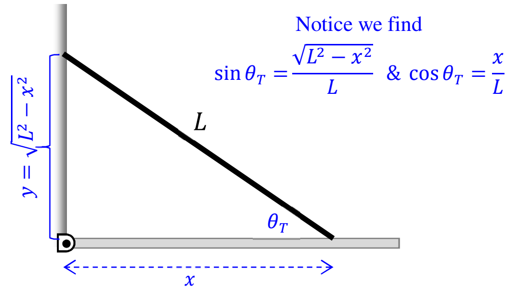
$$-(L-x)(n_b+n_a) + Ln_a = 0$$
$$(L-x)(n_b+n_a) = Ln_a$$
$$Ln_a - xn_a - xn_b + Ln_b = Ln_a$$
$$-xn_a - xn_b + Ln_b = 0$$

This gives the same result for x as before!

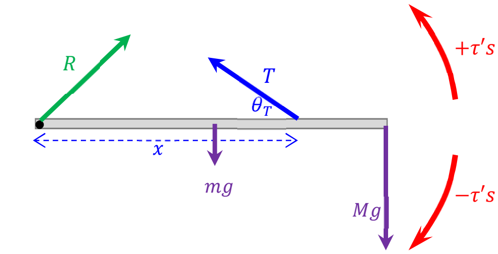
12.1½

a) Recall, tension (magnitude) in the string will equal Mg as long as the system is not accelerating. In this case, the rod is in equilibrium (which implies no acceleration).

b) I cannot emphasize enough the importance of drawing quality diagrams. Notice I first drew a picture to understand the geometry and figure out how the angle θ relates to the distances x & L.



Next I chose to make two FBD's so you could see some different styles. In the lower FBD I choose to split up forces into standard xy-components. Figure not to scale.

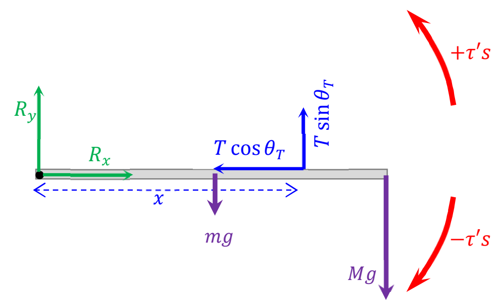


Last comment, do not be concerned if you don't know which way the reaction force $\vec{R}$ at the pivot points. Just do the math; if the answer for a component (R_x or R_y) is negative that component points *opposite the direction drawn*.

$$\Sigma\tau_{\text{Left End}}: \quad -\frac{L}{2}mg + xT\sin\theta - LMg = 0$$

$$\Sigma\tau_{\text{Left End}}: \quad xT\sin\theta - L\left(M + \frac{m}{2}\right)g = 0$$

Consider which force components contribute no torque.
If line of action runs through pivot then no torque.
Notice R_x, R_y, and $T\cos\theta$ contribute zero torque.



c) Horizontal forces gives

$$\Sigma F_x: \qquad R_x = T\cos\theta_T$$

d) Vertical forces gives

$$\Sigma F_y: \; R_y + T\sin\theta_T = (M+m)g$$

e) Notice it is easiest to learn about $\vec{T}$ (the tension in the support cable) using the torque equation and the top figure showing geometry. We get the *magnitude* T from solving the torque equation:

$$T = \frac{L\left(M+\frac{m}{2}\right)g}{x\sin\theta_T} = \frac{L\left(M+\frac{m}{2}\right)g}{x\left(\frac{\sqrt{L^2-x^2}}{L}\right)} = \frac{\boldsymbol{L^2\left(M+\frac{m}{2}\right)g}}{\boldsymbol{x\sqrt{L^2-x^2}}}$$

The *direction* is given by the geometry figure ($\boldsymbol{\theta_T = \cos^{-1}\left(\frac{x}{L}\right)}$ directed above the NEGATIVE x axis).
From there we can use SOH CAH TOA and the directions in the figure to show

$$\vec{T} = T_x\hat{\imath} + T_y\hat{\jmath}$$

$$\vec{T} = (-T\cos\theta_T)\hat{\imath} + (+T\sin\theta_T)\hat{\jmath}$$

The minus sign on the x-component is because that component points *to the left* in the figure!!!

$$\vec{T} = -\frac{L}{\sqrt{L^2-x^2}}\left(M+\frac{m}{2}\right)g\hat{\imath} + \frac{L}{x}\left(M+\frac{m}{2}\right)g\hat{\jmath}$$

$$\vec{T} = -\frac{L}{y}\left(M+\frac{m}{2}\right)g\hat{\imath} + \frac{L}{x}\left(M+\frac{m}{2}\right)g\hat{\jmath}$$

$$\boldsymbol{\vec{T} = \left(M+\frac{m}{2}\right)g\left(-\frac{L}{\sqrt{L^2-x^2}}\hat{\imath} + \frac{L}{x}\hat{\jmath}\right)}$$

MORE ON NEXT PAGE…

f) If $x = 0$, you are connecting the support cable at the *left* end of the rod. You are trying to hold up the rod with a support cable located at the pivot. This is impossible. That is why the tension goes to infinity when $x \to 0$.
If $x = L$, you are connecting the cable to the *right* end of the rod but we still see the tension going to infinity. This is because, in this special case, we are told the support cable and the rod have equal length. If they have equal length, the cable is horizontal (parallel to the rod) when it is connected to the right end. Since there is no vertical component of tension (when the cable is horizontal), it is again impossible to support the rod as $x \to L$!

g) To get the reaction forces, plug in the known value of T into the force equations and solve for R_x & R_y.

$$R_x = T\cos\theta = \frac{L}{\sqrt{L^2 - x^2}}\left(M + \frac{m}{2}\right)g = \frac{L}{y}\left(M + \frac{m}{2}\right)g$$

$$R_y = (M + m)g - T\sin\theta_T = (M + m)g - \frac{L}{x}\left(M + \frac{m}{2}\right)g$$

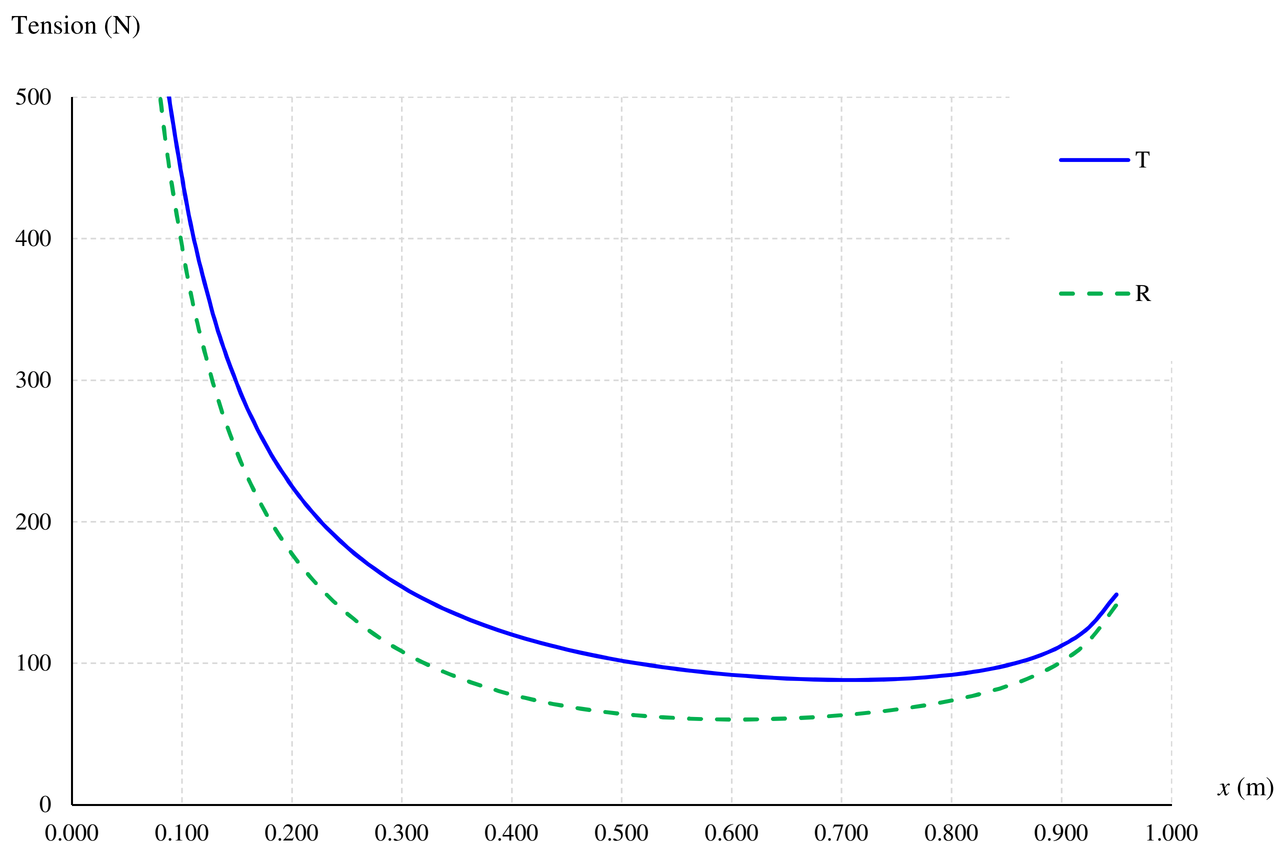
To get the *magnitude* use $R = \sqrt{R_x^2 + R_y^2}$. To get the direction use $\theta_R = \tan^{-1}\left(\frac{R_y}{R_x}\right)$.
There is no point in doing this as you've done enough work already.
NOTICE: if we use a small value for x, we expect $R_y < 0$!!!

h) When $x \to 0$ the support cable is straight up. As such it exerts no horizontal forces on the rod. If ther support cable exerts zero horizontal force on the rod, there is no need for the pivot to exert a horizontal force to balance a zero force. This is why $R_x \to 0$ when $x \to 0$.
Conversely, as $x \to L$ we required an infinitely large support cable tension which is applied almost entirely horizontally. We would thus require an infinitely large R_x.
Notice $x \to L$ the vertical component $R_y = \frac{1}{2}mg$! The pivot supports half the rod while the cable supports the other half of the rod's weight and the entirety of the hanging mass weight!
Notice $x \to 0$ the vertical component $R_y \to -\infty$. Because the support cable must pull essentially straight up with near infinite force, the pivot must pull down with near infinite force to balance the rod. Strictly speaking, the downwards force at the pivot does have slightly smaller magnitude as there is already $(M + m)g$ pulling down on the rod.

PART I on next page…

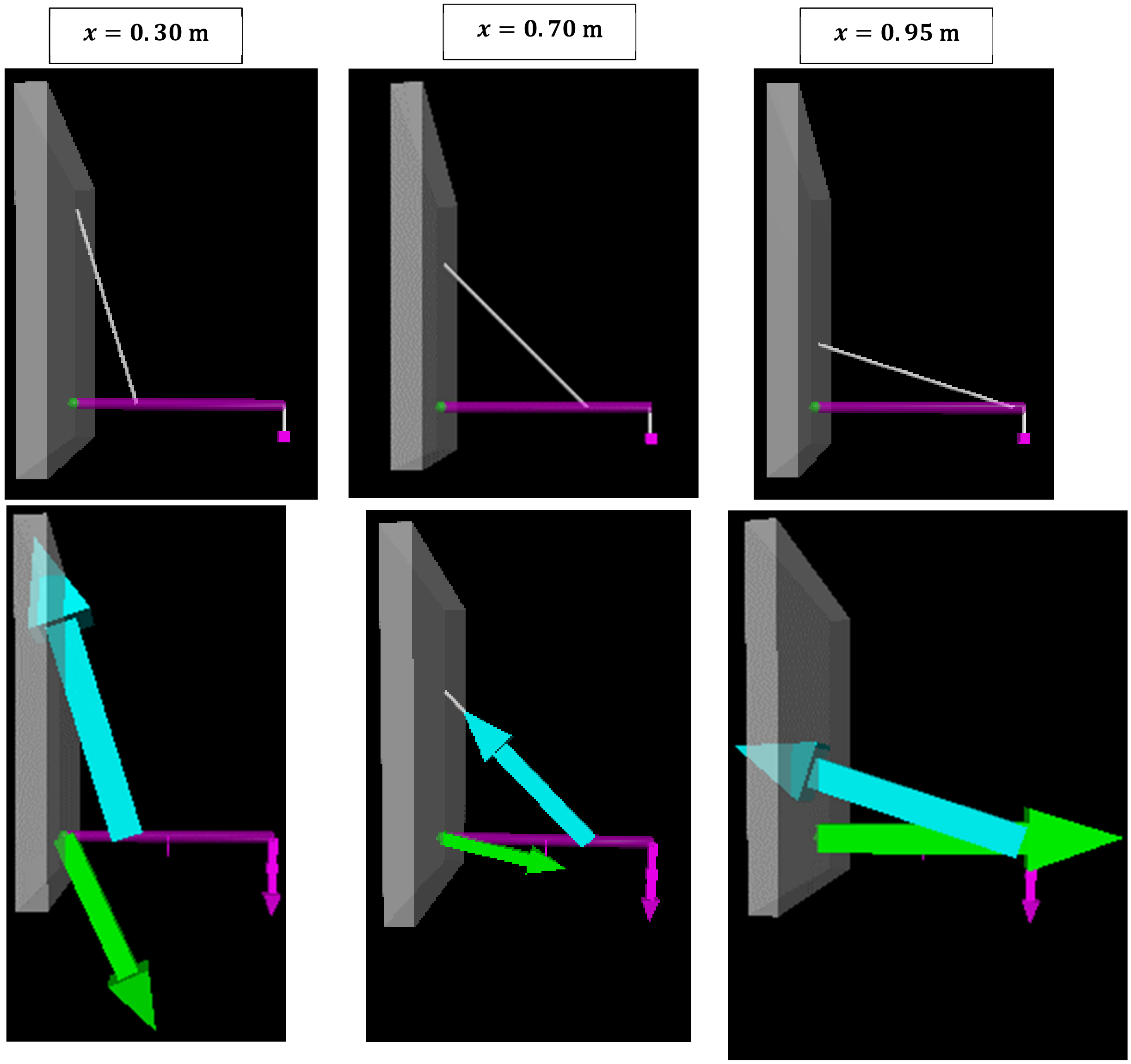
i) A partial table is shown below. Below that is a plot.

x (m)	T_x (N)	T_y (N)	T (N)	θ_T (rad)	θ_T (deg)	R_x (N)	R_y (N)	R (N)	θ_R (rad)	θ_R (deg)
0.050	-44	882	883	1.52	87	44	-833	834	1.52	87
0.075	-44	588	590	1.50	86	44	-539	541	1.49	85
0.100	-44	441	443	1.47	84	44	-392	394	1.46	84
0.125	-44	353	356	1.45	83	44	-304	307	1.43	82
0.150	-45	294	297	1.42	81	45	-245	249	1.39	80
0.175	-45	252	256	1.39	80	45	-203	208	1.35	78
0.200	-45	221	225	1.37	78	45	-172	177	1.31	75
0.225	-45	196	201	1.34	77	45	-147	154	1.27	73
0.250	-46	176	182	1.32	76	46	-127	135	1.23	70
0.275	-46	160	167	1.29	74	46	-111	120	1.18	68
0.300	-46	147	154	1.27	73	46	-98	108	1.13	65
0.325	-47	136	143	1.24	71	47	-87	98	1.08	62
0.350	-47	126	135	1.21	70	47	-77	90	1.02	59
0.375	-48	118	127	1.19	68	48	-69	83	0.96	55
0.400	-48	110	120	1.16	66	48	-61	78	0.90	52



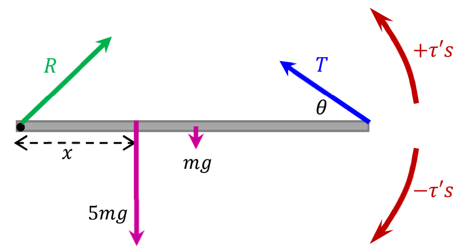
PART J ON NEXT PAGE

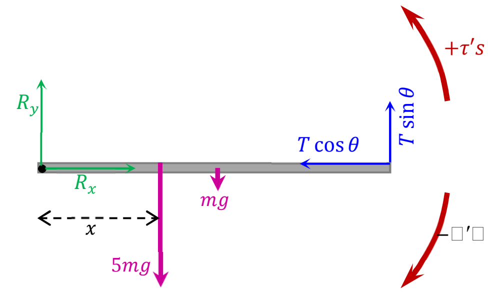
j) I made such a simulation. Some screen shots are shown below. First without forces drawn. Then with the forces arrows added in. Notice the reaction force is usually, but not always, below the horizontal.



12.2

a) I cannot emphasize enough the importance of drawing quality diagrams. Notice I chose to make two FBD's so you could see some different styles. In the lower FBD I choose to split up forces into standard xy-components. I tried to draw *roughly* to scale but it is not perfect by any means.
Last comment, do not be concerned if don't know which way the reaction force $\vec{R}$ at the pivot points. Just do the math; if the answer for a component (R_x or R_y) is negative that component points *opposite the direction drawn*.





b) Again, as long as acceleration is zero, we can use $5mg$ in lieu of the normal force exerted by the block on the rod.

c) Notice the lower figure makes it easy to do forces (and torques).

$$\Sigma F_x: \quad R_x = T\cos\theta$$
$$\Sigma F_y: \quad R_y + T\sin\theta = 6mg$$
$$\Sigma\tau_{\text{Left End}}: \quad -x(5mg) - \frac{L}{2}mg + LT\sin\theta = 0$$

Consider which force components contribute no torque.
If line of action runs through pivot then no torque.
Notice R_x, R_y, and $T\cos\theta$ contribute zero torque.

d) If we plug in $x = \frac{L}{4}$ The torque equation cleans up nicely to give

$$\frac{7}{4}mg = T\sin\theta$$

e) I used $T = 17.0\text{ N}$ and $\theta = 36.9°$. We are still assuming $x = \frac{L}{4}$.
Solving the previous result for m gives

$$m = \frac{4T\sin\theta}{7g} \approx 0.59\underline{5}2\text{ kg}$$

Watch out! In this problem m was the mass of the rod.
The bonus object has mass $5m \approx 2.9\underline{7}6$ kg!

Notice: total weight force is $6mg \approx 35.0\text{ N}$.
The rope, with a huge lever arm, can support this huge weight with a mere 17.0 N of tension?
Is something wrong?
No! The pivot point is also supporting the rod.
Go on to part f…

f) From the *horizontal* force equation we know

$$R_x = T\cos\theta$$
$$R_x = (17.0\text{ N})\cos 36.9°$$
$$\boldsymbol{R_x \approx 13.6\text{ N}}$$

From the *vertical* force equation we know

$$R_y + T\sin\theta = 6mg$$
$$R_y = 6mg - T\sin\theta$$
$$R_y = 6mg - \frac{7}{4}mg$$
$$\boldsymbol{R_y \approx 24.8\text{ N}}$$

The resultant force, in Cartesian form, is thus

$$\vec{\boldsymbol{R}} = (\boldsymbol{13.6\hat{\imath} + 24.8\hat{\jmath}})\text{N}$$

12.3

a) If we want to learn about the tension in the strings we should do sum of torques about some *other* point. Since the mass is known but the reaction forces at the pivot are not, it seems to make sense to do sum of torques about the pivot point. Then our torque equation will only involve $w = mg$ (known) and tension (unknown). In each case let us assume clockwise torques (about the pivot) are negative.
WATCH THE ANGLES!!! The angle between the $\vec{r}$ and tension vector is NOT 60° for cases **A** & **B**.

CASE A	**CASE B**	**CASE C**	**CASE D**
$-\frac{L}{2}w + LT\sin 30° = 0$ $T = w$	$\frac{L}{2}T\sin 30° - Lw = 0$ $T = 4w$	$\frac{L}{2}T - Lw = 0$ $T = 2w$	$-\frac{L}{2}w + LT = 0$ $T = \frac{w}{2}$

I think you can rank them pretty easily from there.

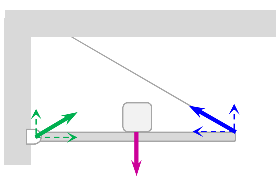
b) For this part consider horizontal force components only. In Case **A** the string is pulling the rod towards the left (pulling it into the wall). The pivot pushes the opposite way on the rod. Similarly the pivot exerts a force to the left in opposition the string pulling the rod away from the wall. In the other two cases the string exerts no horizontal force. This means the pivot will not exert a horizontal force either!

Part c on the next page…

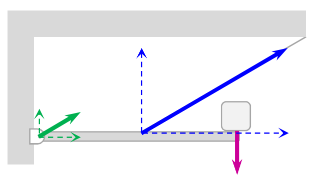
c) Now we want to learn about the reaction force instead of the tension. A clever move is to redo the sum of torques for each case using the point where the string attaches instead of the pivot point. Why is this convenient? The tension force will not appear in the torque equations. Any horizontal force exerted by the pivot will not be in the equation either (because any R_x from the pivot has a line of action that runs through the chosen pivot). This means only the weight w (known) and R_y (unknown), the thing the question asked about, appear in the torque equation and R_y is easy to find.

If you didn't think of this technique, you could've also used your results from part a and then do sum of forces in the vertical direction. If you do see the clever way, you then could use the sum of forces in the vertical direction as a check on your answers.

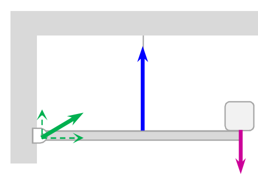
- I choose to do the sum of torques about the point where the string meets the rod.
- I will assume clockwise (about this pivot) is a negative torque.
- I will assume R_y is *upwards* for each case.
    - If I get a negative sign in my answer for R_y, that means R_y points *downwards*.
- I will assume R_x is *to the right* for each case.
    - If I get a negative sign in my answer for R_x, that means R_x points to the *left*.



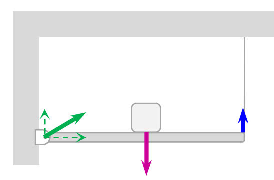
A



B



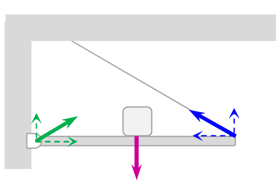
C



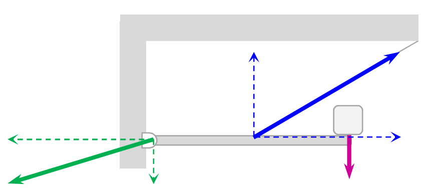
D

CASE A	CASE B	CASE C	CASE D
$-LR_y + \frac{L}{2}w = 0$ $R_y = \frac{w}{2}$	$-\frac{L}{2}R_y - \frac{L}{2}w = 0$ $R_y = -w$	$-\frac{L}{2}R_y - \frac{L}{2}w = 0$ $R_y = -w$	$-LR_y + \frac{L}{2}w = 0$ $R_y = \frac{w}{2}$

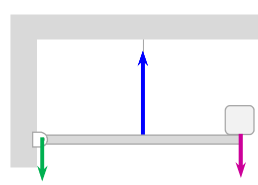
Notice in Cases **B** & **C** the pivot exerts a *downwards* force component.
Figures below show forces approximately to scale with reaction forces in correct directions.



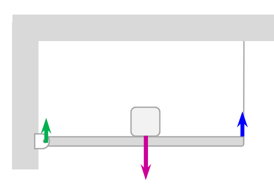
A



B



C



D

12.4 The point of this problem is to learn about torques in the human body.

a) The torque magnitude is given by

$$dm_2 g + \frac{d}{2} m_1 g = 5\underline{4}.1\ \mathrm{N \cdot m}$$

Notice this is ~1/3 of the torque caused by head, torso, abdomen, and arms for a medium adult.

b) We want

$$\tau_{weights} = \tau_{e.spinae}$$

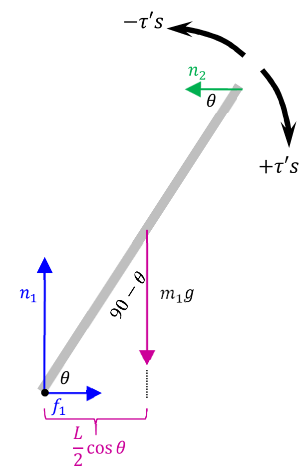
$$5\underline{4}.1\ \mathrm{N \cdot m} = Tx \sin 12°$$

$$T = \frac{5\underline{4}.1\ \mathrm{N \cdot m}}{x \sin 12°} = \frac{5\underline{4}.1\ \mathrm{N \cdot m}}{(0.42\ \mathrm{m}) \sin 12°} \approx 620\ \mathrm{N} \approx 140\ \mathrm{lbs}$$

This implies $M \approx 63$ kg. Since the problem scales linearly, expect > 400 lbs of tension for adult!

12.5 Ladder Part I

a) Figure shown at right. I'm assuming pivot at bottom of ladder.



b)

ΣF_y: $n_1 = mg$	ΣF_x: $f_1 = n_2$	$\Sigma\tau_{bot}$: $mg\frac{L}{2}\cos\theta = n_2 L \sin\theta$

Notice I used the trig identity $\sin(90-\theta) = \cos\theta$ in the torque equation.

c) **IN THIS PART NOT AT ONSET OF SLIPPING.** $f_1 \neq \mu n_1$.

Instead use above equations to find $f_1 = \frac{mg}{2}\cot\theta$.

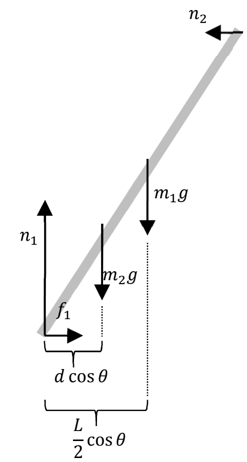
d) **At onset of slipping,** the previous result still valid.

In addition, we can now use $f_1 = \mu_s n_1$.

We find $\boldsymbol{\mu_s = \frac{1}{2}\cot\theta}$.

12.6 Ladder part II

a) See figure at right. Assume same pivot and coordinate system as previous problem.



b)

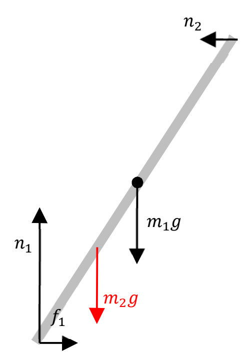
$n_1 = (m_1 + m_2)g$	$f_1 = n_2$	$m_1 g \frac{L}{2}\cos\theta + m_2 g d\cos\theta = n_2 L \sin\theta$

c) If just about to slip $f_1 = \mu n_1 = \mu(m_1 + m_2)g = \frac{1}{2}\cot\theta\,(m_1 + m_2)g = n_2$.
We find $d = \frac{L}{2}$. This makes sense if we recall mass dropped out in the previous problem!

d) If we step on the *first rung* of a ladder, we dramatically increase the normal force n_1.
Maximum *possible* frictional force is $f_{max} = \mu n_1$.
Stepping on the ladder dramatically increases the maximum *possible* friction force.
The ladder is less likely to slip with extra weight on the *bottom* step.

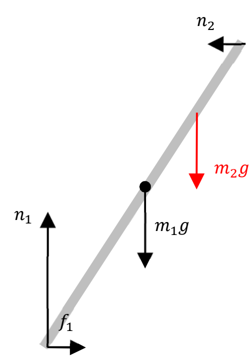
To really think it through, however, consider sum of torques about the center of the ladder.
In particular, consider what happens when the zombie with mass m_2 gets high on the ladder

Zombie *low* on ladder	**Zombie *high* on ladder**



Notice the torque caused by $m_2 g$ about the *center* opposes the torque caused by n_2.

We do not require the maximum *possible* amount of friction $f_{max} = \mu n_1$ to keep the ladder in static equilibrium.



Notice the torque caused by $m_2 g$ about the *center* opposes the torque caused by f_1.

Maximum *possible* friction is limited to $f_{max} = \mu n_1$.

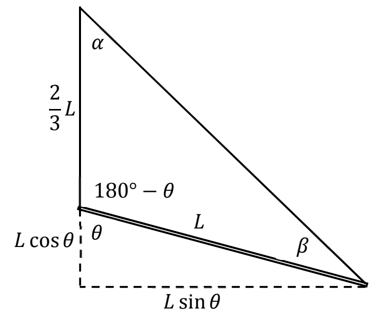
There is insufficient friction to keep the ladder in static equilibrium…

12.7 Draw Bridge

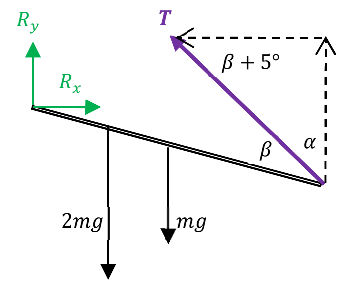
a) The upper figure at right shows only the geometry and makes it easier to determine the angles. Notice from the large right triangle that $\tan\alpha = \frac{L\sin\theta}{\frac{2}{3}L + L\cos\theta}$.

Then use the fact the angles of the upper triangle sum to 180°.

Note: figure not to scale.



b) See the second figure. Note: figure not to scale.



c) The equations are

$$R_x = T\cos(\beta + 5°) = T\sin(\alpha)$$
$$3mg = R_y + T\sin(\beta + 5°) = R_y + T\cos(\alpha)$$
$$2mgx\sin\theta + mg\frac{L}{2}\sin\theta = LT\sin\beta$$

d) We find

$$\boldsymbol{T = \frac{mg\sin\theta}{\sin\beta}\left[2\frac{x}{L} + \frac{1}{2}\right]}$$

e) Assuming it can only withstand $T = 4mg$, the cable snaps when $x = 0.818L$.

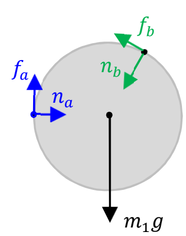
f) The string is most likely to break when the block is at $x = L$. Set $T = 4mg$ and re-solve for the block mass M using the torque eqt'n.

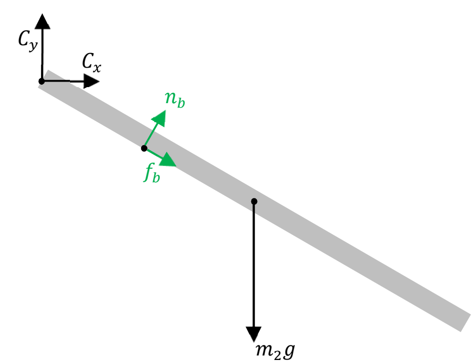
$$MgL\sin\theta + mg\frac{L}{2}\sin\theta = L(4mg)\sin\beta$$

One finds block mass is $\boldsymbol{M = 1.63m}$.

12.8 Ball and Plank

a) & b) See at right.





c) Doing torques at the center of the ball we find

$$Rf_a = Rf_b$$

d) Doing forces on the ball gives

$$\Sigma F_x: n_a = f_b\sin\theta + n_b\cos\theta$$
$$\Sigma F_y: f_a + f_b\cos\theta = m_1 g + n_b\sin\theta$$

e) For the plank we find

$$\frac{3}{2}Rn_b = 3Rm_2 g\sin\theta$$

f) Doing forces on the plank gives

$$\Sigma F_x: C_x + f_b\sin\theta + n_b\cos\theta = 0$$
$$\Sigma F_y: C_y + n_b\sin\theta = f_b\cos\theta + m_2 g$$

Notice ΣF_x implies C_x is a negative number! Thus the hinge actually exerts a force *to the left* on the plank! Magnitude $|C_x|$ is correct but vector $\vec{C}_x$ points <u>opposite the direction drawn</u> (to the left).

g) <u>Perhaps</u> the ball *rotates* out such that there is an instantaneous pivot at point **A**.

<u>Perhaps</u> <u>not</u>.

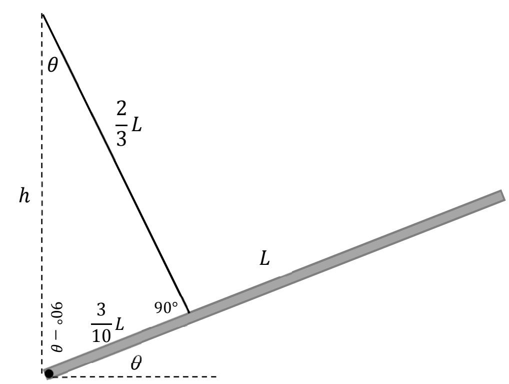
If yes, there is slipping at **B** ($f_b = \mu n_b$) but not necessarily at **A** ($f_a < \mu n_a$).

Notice we still require $f_a = f_b$ from torques about the center of the ball.

This in turn implies $n_a > n_b$.

I suppose one thing you could do to figure out what actually happens is to get a plank and take a video of a ball as it slips out.

12.9 Note: arrows in this FBD are not even close to scale.



a) The upper figure at right shows only geometry (no forces). I like to do it this way to figure out all the distances and angles before bothering to get the forces. If you can't do the geometry, you can't get the forces. Notice we find

$$h = \sqrt{\left(\frac{2}{3}L\right)^2 + \left(\frac{3}{10}L\right)^2} = \frac{\sqrt{481}}{30}L \approx 0.7311L$$

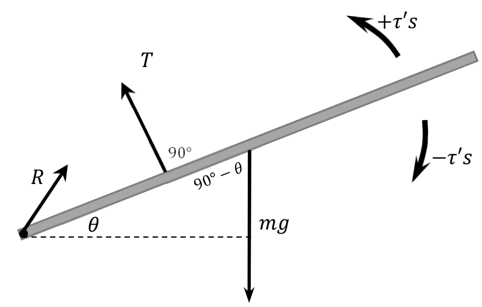
b) I used SOH CAH TOA to get

$$\theta = \tan^{-1}\left(\frac{\frac{3}{10}L}{\frac{2}{3}L}\right) = \tan^{-1}\left(\frac{9}{20}\right) \approx 24.23°$$

$$\sin\theta = \frac{\frac{3}{10}L}{\frac{\sqrt{481}}{30}L} = \frac{9}{\sqrt{481}} \approx 0.4104$$

$$\cos\theta = \frac{\frac{2}{3}L}{\frac{\sqrt{481}}{30}L} = \frac{20}{\sqrt{481}} \approx 0.9119$$

Why didn't I just plug in the angle into my calculator to get the second two parts? I could've, but the method shown above is good practice for your engineering statics courses.



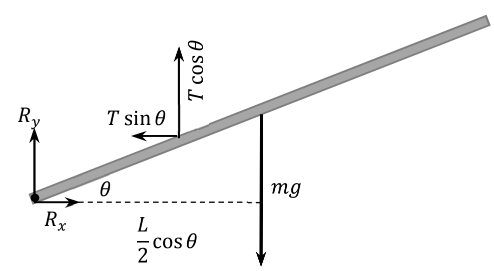
c) I'll use the second figure (first FBD) for doing $\Sigma\tau_{\text{Left End}}$:

$$-\frac{L}{2}mg\sin(90-\theta) + \frac{3}{10}LT\sin 90° = 0$$

$$\sin(90-\theta) = \cos\theta$$

$$\frac{1}{2}mg\cos\theta = \frac{3}{10}T$$

$$T = \frac{5}{3}mg\left(\frac{20}{\sqrt{481}}\right) = \frac{100}{3\sqrt{481}}mg \approx 1.520mg$$



d) Doing ΣF_x:

$$R_x = T\sin\theta \approx 0.6237mg$$

Doing ΣF_y:

$$R_y + T\cos\theta = mg$$

$$R_y = mg - T\cos\theta \approx -0.3861mg$$

Don't worry about minus sign! The *magnitude* of R_y is $0.3861mg$ but the *direction* is *OPPOSITE THE DIRECTION DRAWN*. In this case, opposite the direction drawn is downwards.

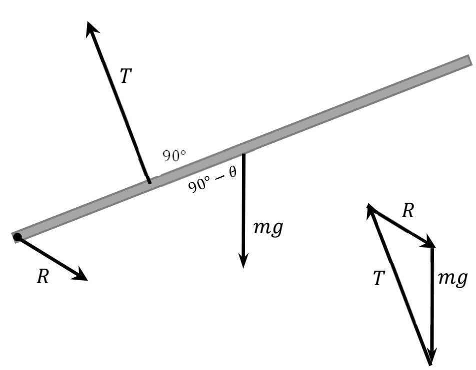
e) Using

$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{(0.6237mg)^2 + (0.3861mg)^2} \approx 0.7335mg$$

The bottom figure at right shows the FBD corrected for scale and direction. Notice I also did the graphical vector addition (tail-to-tip) for the three force vectors acting on the rod to show they sum to zero. This is as good as I can do.

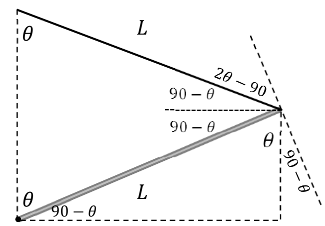
FBD CORRECTED FOR SCALE AND DIRECTION

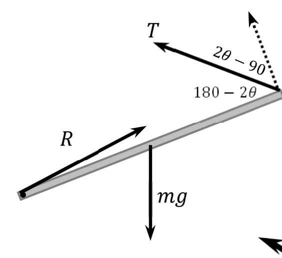


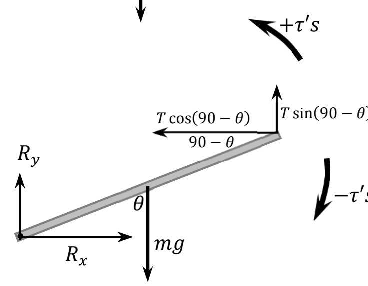
12.10 Assume θ is known.

Note: the clever way to do this problem is to use lever arms (shown on the next page)…

The upper figure has no forces. I use it just to get a handle on the geometry and figure out how all the different angles relate to θ. I probably won't need all these angles…but the geometry practice is worth it at this point. The lower two figures are FBDs. The second FBD has the forces split into components. It is getting a bit challenging to draw these to scale without already knowing the answer. My reaction force arrows (size and direction) are a guess.







a) The torque equation can be found from either of the FBDs easily enough (once you know the angles). If you are unable to get the angles…nice knowing you. One finds $\Sigma\tau_{\text{Left End}}$:

$$-\frac{L}{2}mg\sin\theta + LT\sin(180-2\theta) = 0$$
$$\frac{1}{2}mg\sin\theta = T\sin(180-2\theta)$$
$$\frac{mg}{2}\sin\theta = T\sin(180-2\theta)$$

Using

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\sin(180-2\theta) = \sin 180\cos 2\theta - \cos 180\sin 2\theta$$
$$\sin(180-2\theta) = \sin 2\theta$$
$$\frac{mg}{2}\sin\theta = T\sin 2\theta$$

b) Doing ΣF_x:

$$R_x = T\cos(90-\theta)$$
$$R_x = T\sin\theta$$

Doing ΣF_y:

$$R_y + T\sin(90-\theta) = mg$$
$$R_y + T\cos\theta = mg$$

c) Rearranging the result of part a) gives

$$T = \frac{mg\sin\theta}{2\sin 2\theta} = \frac{mg\sin\theta}{4\sin\theta\cos\theta} = \frac{mg}{4\cos\theta}$$

You might be worried about that minus sign. From the figure it appears $\theta > 45°$. That implies $\cos 2\theta < 0$ which will cancel the minus sign.

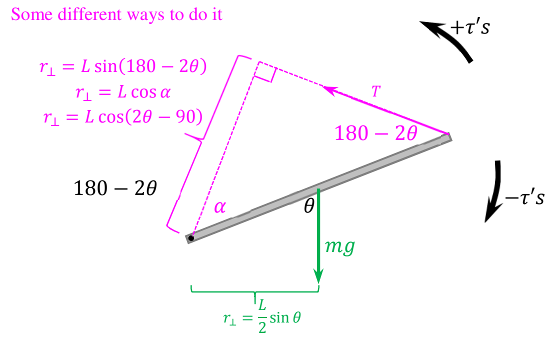
$$R_x = T\sin\theta$$
$$\boldsymbol{R_x = mg\frac{\tan\theta}{4}}$$
$$R_y = mg - T\cos\theta$$
$$\boldsymbol{R_y = mg\left(1-\frac{1}{4}\right) = \frac{3}{4}mg}$$
$$R = \sqrt{R_x^2 + R_y^2}$$
$$\boldsymbol{R = \frac{mg}{4}\sqrt{\tan^2\theta + 9}}$$

To paraphrase Samuel L. Jackson's character in <u>Changing Lanes</u>: I want my time back!

Wait a minute…what is the direction/angle of the reaction force?

$$\phi = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{\boldsymbol{\frac{3}{4}mg}}{\boldsymbol{mg\frac{\tan\theta}{4}}}\right) = \tan^{-1}\left(\frac{\boldsymbol{3}}{\boldsymbol{\tan\theta}}\right)$$

12.10 using lever arms…
I will redo sum of torques using the lever arms.
Since R_x and R_y cause no torque aout the pivot I will leave them off (they clutter the geometry).

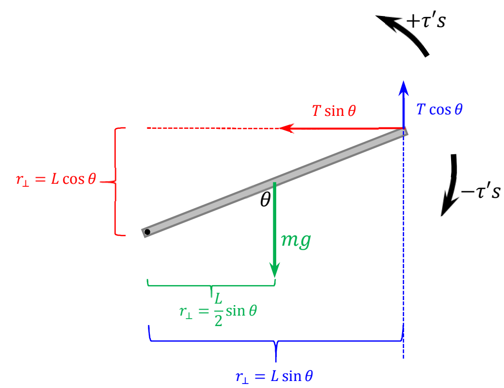


The $180-2\theta$ angle adjacent to the tension force shown in pink can be figured out from the *original isosceles triangle* (angles in triangle sum to 180).

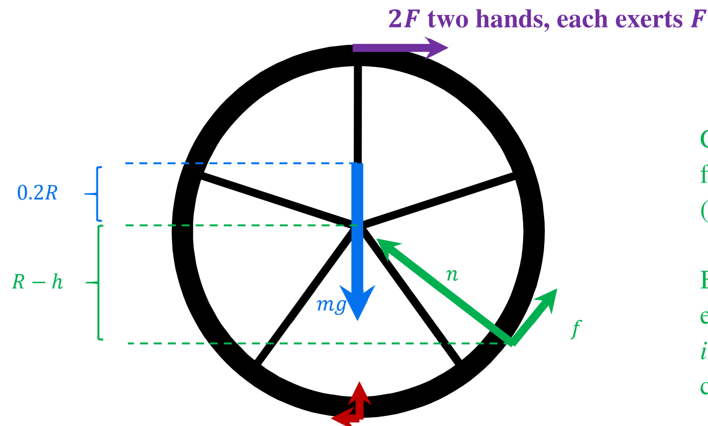
From there angles in the *pink triangle sum* to 180 as well.

$$\alpha + 90 + 180 - 2\theta = 180$$
$$\alpha + 90 - 2\theta = 0$$
$$\alpha = 2\theta - 90$$

Alternate 12.10 using lever arms on components…
I am using the 3rd FBD shown at the bottom of the previous page.



12.11 There is a lot to internalize with this FBD. Study carefully before proceeding. Presumably, the wheel is rolling over curb without slipping. This implies we may use the instantaneous pivot. Furthermore, at minimum force to make it over the curb, the angular acceleration about the instantaneous pivot is essentially zero. That is why I put this problem in the statics chapter. Hope this makes sense.



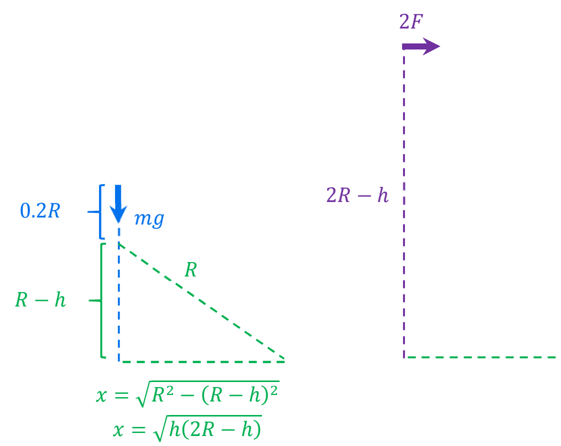
Circular wheel exerts normal force on curb radially *outwards* (and friction tangent to wheel).

By Newton's third law the curb exerts normal force radially *inwards* (and friction tangent to curve in opposite direction).

TRICKY! The forces at the bottom of the wheel are ZERO! The problem ask for min force to get over the curb. If getting over the curb you are not making contact with the bottom!!!!

Sum of torques about instantaneous pivot (contact point with the curb) requires a bit of geometry to figure out lever arms. See figure at right. Lastly, I will assume the weight is a negative torque (implying the hand force exerts a positive torque)

$$-mgr_\perp + 2Fr_\perp = 0$$
$$mgx = 2F(2R-h)$$
$$mg\sqrt{h(2R-h)} = 2F(2R-h)$$
$$m^2g^2h(2R-h) = 4F^2(2R-h)^2$$
$$h = \left(\frac{2F}{mg}\right)^2(2R-h)$$
$$h = R\frac{2k}{1+k}$$
where $k = \left(\frac{2F}{mg}\right)^2$.

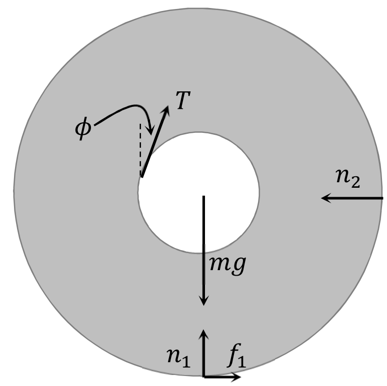


Note: we could do sum of torques about the center of the *wheel* (because this is effectively a statics problem). Note, center of *mass* not at center of *wheel* but mg still causes no torque about center of *wheel* because line of action through pivot. Doing this gives a fairly simple equation relating friction f to hand force F. I used this to determine the direction of friction in the FBD shown above.

Finally, even though this problem says max curb height we should <u>not</u> assume $f = \mu_s n$. Nothing indicates wheel is on verge of slipping.

12.12

a) The FBD can be seen at right *roughly* to scale. I will assume standard coordinates (up is positive, to the right is positive). Furthermore, when I do torques I will choose the center of the spool and assume CW is positive (so tension, the force causing the twist, is causing a positive torque), CCW is negative. Other choices of coordinates and/or signs on torques should give identical final results.



Vertical Forces	$n_1 + T\cos\phi = mg$ Ups = downs
Horizontal Forces	$f_1 + T\sin\phi = n_2$ lefts = rights
Torques about center	$\frac{R}{3}T = Rf_1$ CW's = CCW's

Key assumptions: spool doesn't lift off and friction is negligible between wall and spool. We are told on the verge of slipping so between floor and spool expect $f_1 = \mu_s n_1$. Inner radius is $R/3$ while outer radius is R.

We might need n_1 because on the verge of slipping. From vertical forces we know $n_1 = mg - T\cos\phi$.
Since on verge of slipping $f_1 = \mu_s n_1 = \mu_s(mg - T\cos\phi) = \mu_s mg - \mu_s T\cos\phi$.
Using torque equations gives

$$\frac{R}{3}T = Rf_1$$
$$\frac{1}{3}T = \mu_s mg - \mu_s T\cos\phi_1$$
$$T\left(\frac{1}{3} + \mu_s\cos\phi\right) = \mu_s mg$$
$$T = \frac{\mu_s mg}{\frac{1}{3} + \mu_s\cos\phi}$$
$$\boldsymbol{T = \frac{mg}{\frac{1}{3\mu_s} + \cos\phi}}$$

Parts b and c are discussed at length on the next page.

Parts b and c of 12.12 If you are concerned about lift off think about n_1. If $n_1 = 0$, we know the block lifts off. Therefore lift off will occur when $mg = T\cos\phi$ or $T = \frac{mg}{\cos\phi}$. Notice the *denominator* in lift-off force *is smaller than* result for minimum force to cause rotation for *any* value of μ_s. This implies lift off force required is greater than min force to cause rotation for *any* value of μ_s. That means it won't lift off. Notice as well, for $\phi < 0$ we still have $\cos\phi > 0$.

I suppose if you got to angles greater than 90° or less than −90° we might want to think more about it…but I guess I assumed we have some positive vertical component of T. With a negative vertical component on T it may never slip depending on the coefficients of friction…I don't know…haven't thought it through!!!

The entire discussion above I was concerned with lift-off. For $\phi < 0$ we know the spool might *slide away from the wall* (rather than spin). It loses contact with the wall if $n_2 = 0$ or when $f_1 = -T\sin\phi$. The negative sign makes sense when $\phi < 0$ because, for instance, $\sin(-12°) = -\sin(12°)$…the 12 is some number I pulled out of nowhere just to use as an example. To learn if it will slide *away from the wall* before rotating we then analyze

$$\mu_s mg - \mu_s T\cos\phi_1 = -T\sin\phi$$
$$\mu_s mg = T(\mu_s\cos\phi - \sin\phi)$$
$$T = \frac{\mu_s mg}{\mu_s\cos\phi - \sin\phi}$$
$$T = \frac{mg}{\cos\phi - \frac{1}{\mu_s}\sin\phi}$$

Let us know compare the min tension to *spin* to the min tension to *slide away* from the wall (remembering the discussion only makes sense if $\phi < 0$).

$$\frac{T_{slide}}{T_{spin}} = \frac{\dfrac{mg}{\cos\phi - \frac{1}{\mu_s}\sin\phi}}{\dfrac{mg}{\frac{1}{3\mu_s} + \cos\phi}} = \frac{\frac{1}{3\mu_s} + \cos\phi}{\cos\phi - \frac{1}{\mu_s}\sin\phi}$$

Any time $\frac{T_{slide}}{T_{spin}} < 1$ it says the minimum force to cause sliding away from the wall is less than the minimum force to start rotating. If this occurs we suspect it will slide away from the wall before spinning.

$$\frac{\frac{1}{3\mu_s} + \cos\phi}{\cos\phi - \frac{1}{\mu_s}\sin\phi} < 1$$
$$\frac{1}{3\mu_s} + \cos\phi < \cos\phi - \frac{1}{\mu_s}\sin\phi$$
$$\frac{1}{3} < -\sin\phi$$
$$-\frac{1}{3} > \sin\phi$$

WATCH OUT! In inequalities you must flip the inequality when multiplying by -1!!!

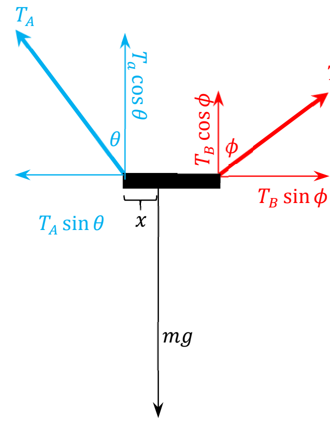
$$-19.5° \gtrsim \phi$$
$$\phi \lesssim -19.5°$$

This means, regardless of the frictional coefficients we should expect we need to be concerned about sliding away from the wall for angles of 19.5° or more *to the left of the vertical*. Note: the $\frac{1}{3}$ came from the ratio of the inner radius to the outer radius…evidently that ratio is the key factor in determining the range of angles for which the spool will spin instead of slide away from the wall!!!

I hope someone bothers to read all this…I thought it was fun to think about.

12.13

a) I drew an FBD at right to help me explain things. This figure is roughly to scale. I choose to first consider horizontal sum of forces. We know, in equilibrium, the leftwards force from point **A** must balance the rightwards force from point **B**. If this occurs we know



$$\boldsymbol{T_A \sin\theta = T_B \sin\phi}$$

The tension with smaller angle must be larger for this equation to be valid. Said another way, since $\theta < \phi$ we expect $T_A > T_B$.

b) The vertical force equation is

$$\boldsymbol{T_A \cos\theta + T_B \cos\phi = mg}$$

From part a) we know $T_A > T_B$. Furthermore, since $\theta < \phi$ we know $\cos\theta > \cos\phi$. This implies $T_A \cos\theta > T_B \cos\phi$. Just like the person lying on two scales (problem **12.1**), the center of mass is closer to the larger vertical support force (closer to **A**). If not, torques about the center of mass would not balance. Think through the torques about the center of mass equation in your mind if that helps.

c) Notice the above two equations are sufficient to determine the tensions (2 equations, 2 unknowns).

$$\boldsymbol{T_A \sin\theta = T_B \sin\phi}$$

$$T_A = T_B \frac{\sin\phi}{\sin\theta}$$

$$\boldsymbol{T_A \cos\theta + T_B \cos\phi = mg}$$

$$\left(T_B \frac{\sin\phi}{\sin\theta}\right)\cos\theta + T_B \cos\phi = mg$$

$$T_B = \frac{mg}{\left(\frac{\sin\phi}{\tan\theta} + \cos\phi\right)} = 0.600\underline{0}0mg$$

$$T_A = T_B \frac{\sin\phi}{\sin\theta} = 0.800\underline{0}0mg$$

d) *Now* do sum of torques. I chose to do torques about point **A** (left end). For my sign convention, I assumed mg exerts a negative torque while T_B exerts a positive torque.

$$-mgx + LT_B \cos\phi = 0$$

$$mgx = LT_B \cos\phi$$

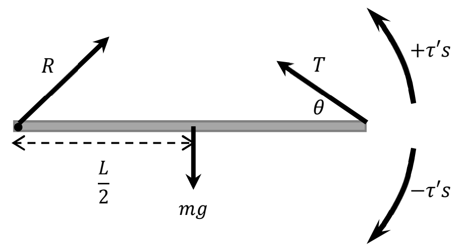
$$mgx = L(0.600\underline{0}0mg)(0.600\underline{0}0)$$

$$x = 0.360\underline{0}0L$$

THINK! The string on the left side is more vertical. From life experience, you might realize the center of mass should be closer to the more vertical string (in this case closer to the left end). Seems to make sense with intuition!

12.14 I decided this problem is more instructive if we assume $x = 7.071$ m.

a) The angles aren't given but we do know dimensions. In torque and force problems we don't really need the angles…we only need the sine and cosine of the angles. It turns out these are written in terms of the dimensions in an easy way. One finds $\sin\theta = \frac{x}{\sqrt{x^2+L^2}}$ and $\cos\theta = \frac{L}{\sqrt{x^2+L^2}}$. For the rest of the problem you can use θ as if it is a known. Then, in the last step of your work, plug in the formulas for $\sin\theta$ and $\cos\theta$ so your final answers are given in terms of the known variables.

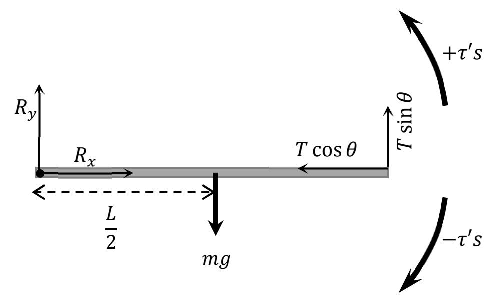


b) See FBD at right. Either one is fine. Might be other useful styles as well. Torque equation about the pivot becomes

$$-\frac{L}{2}mg + LT\sin\theta = 0$$

$$\frac{1}{2}mg = T\sin\theta$$

Since mass is not given, use density to determine the mass. We know the density of the rod is $\boldsymbol{\lambda = 250\,\frac{kg}{m}}$. Therefore rewrite the mass as $\boldsymbol{m = \lambda L}$. Finally, we are told not to answer in terms of θ so use $\sin\theta = \frac{x}{\sqrt{x^2+L^2}}$.



Torques about pivot	**Vertical forces**	**Horizontal**
$\frac{\lambda Lg}{2} = T\frac{x}{\sqrt{x^2+L^2}}$ Explained above.	$R_y + T\frac{x}{\sqrt{x^2+L^2}} = \lambda Lg$ The Ups = The Downs	$R_x = T\frac{L}{\sqrt{x^2+L^2}}$ The Rights = The Lefts

c) Working with the torque equation we notice everything is given except for L. Because we want the longest rod I am assuming $T = T_{max} = 15.0$ kN. It should be possible determine L. When I see something like this, the first thing I think to do is reduce the number of fractions and square root mess. A good trick to know is to isolate the square root term, then square both sides, then try to simplify.

$$\frac{\lambda Lg}{2} = T\frac{x}{\sqrt{x^2+L^2}}$$

$$L\sqrt{x^2+L^2} = \frac{2T}{\lambda g}x$$

In this case we know $\frac{2T}{\lambda g}x$. To reduce clutter in the math, it would make sense to introduce a new constant. For example, let $k = \frac{2T}{\lambda g} = 12.245$ m. I like this since $k, x, \& \, L$ all have the same units! This gives

$$L\sqrt{x^2+L^2} = kx$$

$$L^2(x^2+L^2) = k^2x^2$$

$$L^4 + L^2x^2 - k^2x^2 = 0$$

Notice this is quadratic in L^2. Using the quadratic formula I found $L^2 \approx 65.122\ \text{m}^2$ giving $L \approx 8.07$ m.

d) Once you know L you can figure out all the other pieces by plugging in. I found $R_x = 11.3$ kN and also $R_y = 9.89$ kN. Don't forget, if a question asks for the *magnitude* of the reaction force you must do the Pythagorean theorem on these components.

More on next page regarding tension…

12.14 Going further: Using the sum of torques equation at the pivot and solving for T we can think about how the tension changes. I found

$$T = \frac{\lambda L g \sqrt{x^2 + L^2}}{2x}$$

$$T = \frac{\lambda L g}{2} \sqrt{1 + \frac{L^2}{x^2}}$$

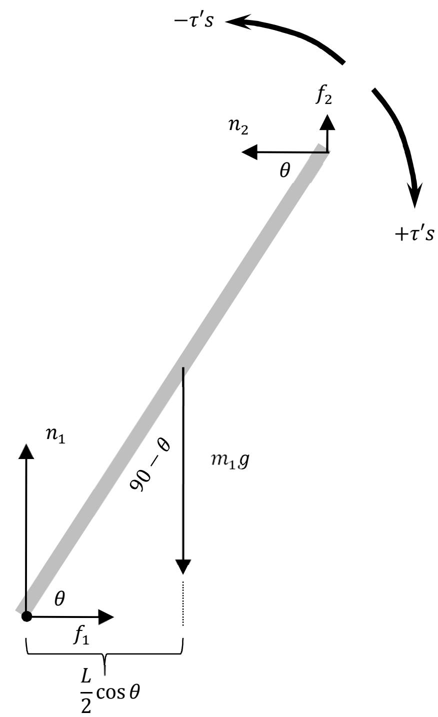
$$T = \left(1.225 \frac{\text{kN}}{\text{m}}\right) L \sqrt{1 + \frac{L^2}{(50 \text{ m}^2)}}$$

Notice in this last formula the input for L must be in meters while the output for T is in kN. From here it is pretty straightforward to make a plot of T vs L using Excel or some similar software.

12.15

Key assumptions:

1) Friction between wall and ladder has same coefficient as friction between floor and ladder.
2) On the verge of slipping if we have the *minimum* coefficient of friction.



Vert. forces	Horiz. forces	Torques about bottom left
$n_1 + f_2 = mg$	$f_1 = n_2$	$mg\frac{L}{2}\cos\theta = n_2(L\sin\theta) + f_2(L\cos\theta)$ Divide all by $L\cos\theta$ to clean up $\frac{mg}{2} = n_2\tan\theta + f_2$

Since on verge of slipping, we can use $f_2 = \mu n_2$ and $f_1 = \mu n_1$. Note: to simplify the problem I'm assuming $\mu_{s1} = \mu_{s2} = \mu$. Rewriting gives

Vert. forces	Horiz. forces	Torques about bottom left
$n_1 + \mu n_2 = mg$	$\mu n_1 = n_2$	$\frac{mg}{2} = n_2\tan\theta + \mu n_2$ $\frac{mg}{2} = n_2(\tan\theta + \mu)$

Use the vertical force equation to eliminate n_1 in the horizontal force equation.

Vert. forces	Horiz. forces
$n_1 + \mu n_2 = mg$ $n_1 = mg - \mu n_2$	$\mu n_1 = n_2$ $n_2 = \mu mg - \mu^2 n_2$ $n_2 = \frac{\mu}{1+\mu^2}mg$

Now plug that result for n_2 into the torque equation:

$$\frac{mg}{2} = n_2(\tan\theta + \mu)$$

$$\frac{mg}{2} = \left(\frac{\mu}{1+\mu^2}mg\right)(\tan\theta + \mu)$$

$$\frac{1}{2}(1+\mu^2) = \mu\tan\theta + \mu^2$$

$$1 + \mu^2 = 2\mu\tan\theta + 2\mu^2$$

$$\mu^2 + 2\mu\tan\theta - 1 = 0$$

$$\mu = \frac{-(2\tan\theta) \pm \sqrt{(2\tan\theta)^2 - 4(1)(-1)}}{2(1)}$$

$$\mu = \frac{-2\tan\theta \pm \sqrt{4(\tan^2\theta + 1)}}{2(1)}$$

Note: $\tan^2\theta + 1 = \sec^2\theta$. Rather than memorize this, divide $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$.

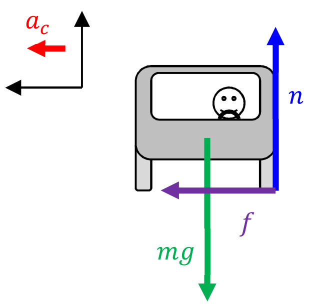
$$\boldsymbol{\mu = \sec\theta - \tan\theta}$$

I bet there was a more clever way to do this…maybe you will find it!

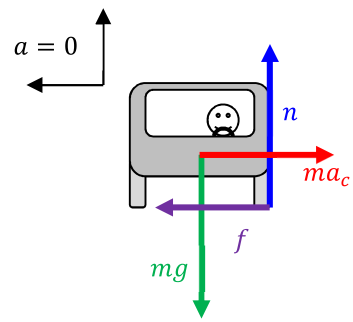
12.16 Part a) See FBD at left below. Because $f = ma_c$ one finds $v = \sqrt{\mu_s Rg}$.

Part b) The trick for turning a problem with acceleration into a static problem is to convert ma into a fictitious force *which points opposite the acceleration*. See the figures below. Notice, once you have converted the acceleration into a fictitious force, you assume zero acceleration in the coordinate system. I hate doing problems this way in general, but in this special case I love it. Arrows not to scale.

Normal FBD



Normal FBD drawn by replacing acceleration with fictitious force



Notice, once you have drawn the *fictitious* FBD, it is obvious the fictitious ma_c force will cause the car to tip over about the outside wheel (the wheel farthest from the center of the circle). This indicates the inner wheel is about to lose contact with the ground. This, in turn, indicates the normal force acts entirely at the outer wheel when the car is about to tip over.

In the fictitious FBD we can use $\Sigma\tau = 0$ about the outer wheel to give

$$\frac{w}{2}mg = yma_c$$

$$\frac{wg}{2} = \frac{yv^2}{R}$$

$$v = \sqrt{\frac{wRg}{2y}}$$

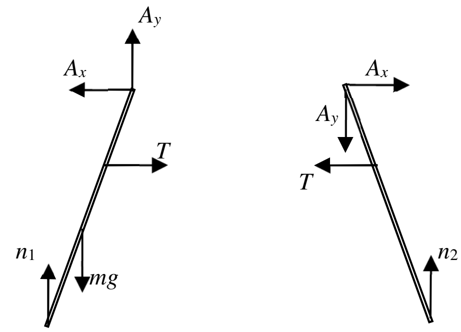
Notice a bigger w (wider wheel base) implies greater speed possible before onset of tipping.
Also, a smaller y (keeping center of mass low to the ground) implies greater speed possible before onset of tipping.

Part c) The mass of the car is unimportant for both parts!

Part d) No.

Part e) Yes. See comments in red above.

12.17 Zombie Ladder Revisited The angle is given by $\tan\theta = \frac{2h}{w}$. The FBDs are shown at right. Notice the five unknowns: n_1, n_2, A_x, A_y, and T.



a) For the left half of the ladder we find
ΣF_x: $T = A_x$
ΣF_y: $n_1 + A_y = mg$

b) For the right half we get the same x equation plus
ΣF_y: $n_2 = A_y$

c) Doing torques on the left half (pivot at top) we get
$\Sigma\tau$: $Ln_1\cos\theta = \frac{L}{3}T\sin\theta + mg(L-d)\cos\theta$

Or

$\Sigma\tau$: $\frac{w}{2}n_1 = \frac{h}{3}T + mg\frac{w}{2}\left(1 - \frac{d}{L}\right)$

Here L is the length of the rod given by $L = \sqrt{\frac{w^2}{4} + h^2}$ **or** $\frac{h}{\sin\theta}$.

d) Doing torques on the right half (pivot at bottom) we get
$\Sigma\tau$: $\frac{w}{2}A_y + \frac{2h}{3}T = hA_x$

e) We find the tension is given by $T = \frac{3wd\sin\theta}{4h^2}mg$.

f) The normal forces are given by $n_1 = mg\left(1 - \frac{d\sin\theta}{2h}\right)$and $n_2 = mg\left(\frac{d\sin\theta}{2h}\right)$

g) When $d = 0$ we find $n_2 = 0$, $n_1 = mg$ and $T = 0$. Reasonable: all the weight is directly over the left support.
When $d = L$ we find $n_2 = n_1 = mg/2$ and $T = \frac{3w}{4h}mg$. Reasonable: weight splits equally, T is max.

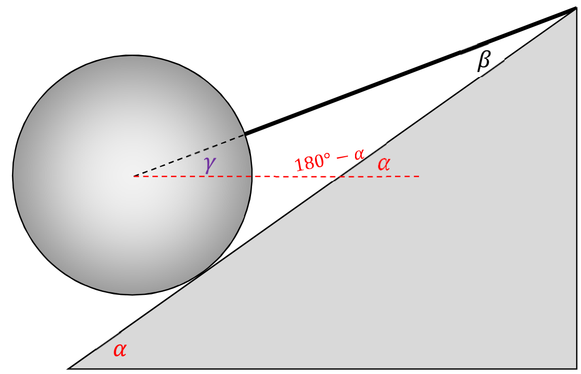
h) Do you ever get tired? I do.

i) See part h.

12.18

a) The figure is shown at right. Since the interior angles of a triangle sum to 180° we find

$$\gamma + \beta + (180° - \alpha) = 180°$$
$$\gamma + \beta - \alpha = 0$$
$$\boldsymbol{\gamma = \alpha - \beta}$$



b) The second figure is probably the easiest way to determine the torques. Notice I used the lever arm method to determine

$$r_\perp = R\cos\beta$$

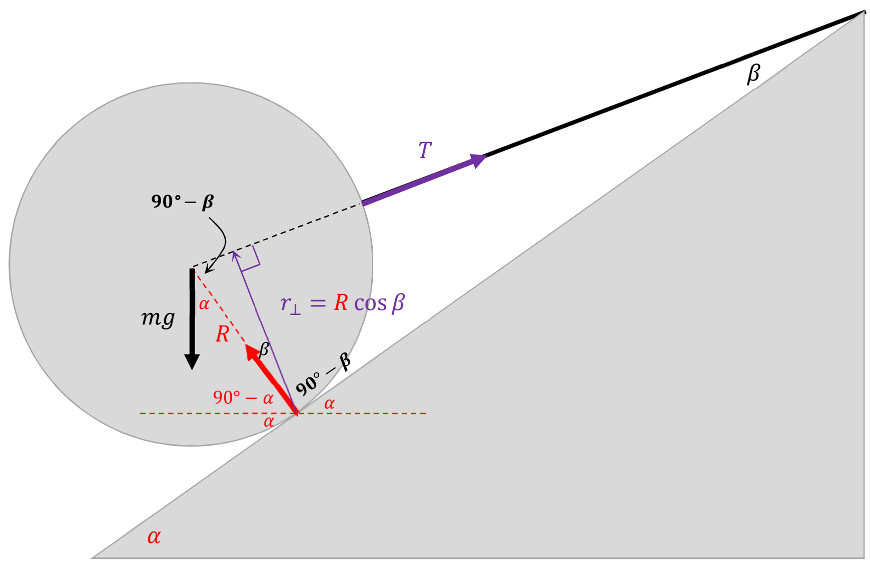
I labeled a bunch of angles so you could hopefully follow the geometry better. The red force arrow is the normal force but I had no room to label it in the figure.

Normal force exerts zero torque about contact point.

I assumed tension exerts a negative torque and mg a positive torque.

$$\Sigma\vec{\tau} = 0$$
$$-r_\perp T + Rmg\sin\alpha = 0$$
$$-RT\cos\beta + Rmg\sin\alpha = 0$$

$$\boldsymbol{T = mg\frac{\sin\alpha}{\cos\beta}}$$



c) Forces in the standard horizontal direction gives

$$T\cos\gamma - n\cos(90° - \alpha) = 0$$
$$T\cos\gamma = n\sin\alpha$$

d) Forces in the standard vertical direction gives

$$T\sin\gamma + n\sin(90° - \alpha) - mg = 0$$
$$T\sin\gamma + n\cos\alpha = mg$$

e) Using the torque equation was easiest for me. I found the answer as $\boldsymbol{T = mg\frac{\sin\alpha}{\cos\beta}}$.

Using forces you could also find it! From part c I found

$$n = T\frac{\cos\gamma}{\sin\alpha}$$

Plugging into part d gives

$$T\sin\gamma + \left(T\frac{\cos\gamma}{\sin\alpha}\right)\cos\alpha = mg$$
$$T\left(\sin\gamma + \frac{\cos\gamma}{\sin\alpha}\cos\alpha\right) = mg$$
$$T = \frac{mg}{\sin\gamma + \frac{\cos\gamma}{\sin\alpha}\cos\alpha}$$

$$T = \frac{mg}{\sin\gamma\frac{\sin\alpha}{\sin\alpha} + \frac{\cos\gamma}{\sin\alpha}\cos\alpha} = \frac{mg\sin\alpha}{\sin\gamma\sin\alpha + \cos\gamma\cos\alpha} = \frac{mg\sin\alpha}{\cos(\gamma - \alpha)} = \frac{mg\sin\alpha}{\cos\left((\boldsymbol{\alpha - \beta}) - \alpha\right)} = \frac{mg\sin\alpha}{\cos\beta}$$

12.19

a) The lower right corner of the block.

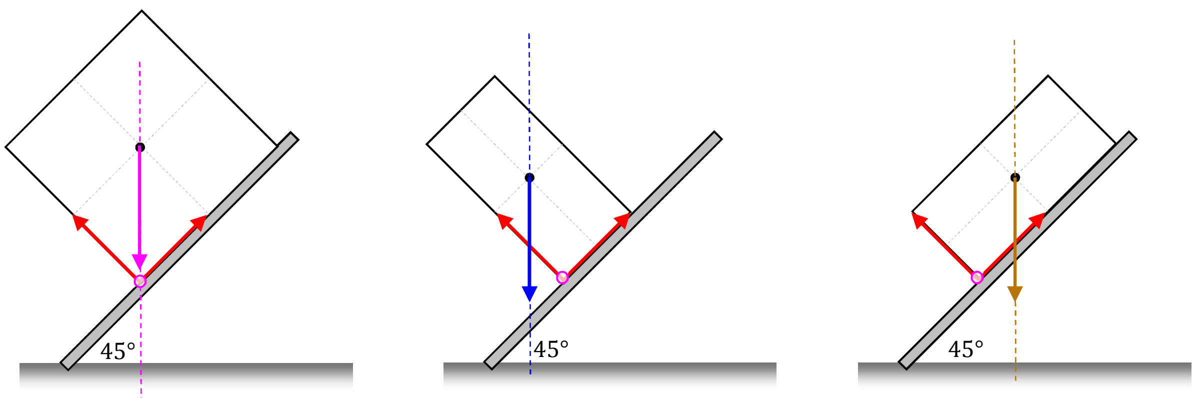
b) If the block *is not* tipping, the line of action for weight is to the *right* of the pivot. If it *is* tipping, the line of action for weight is to the *left* of the pivot. **At the onset of tipping, the line of action for weight runs through the pivot.**

c) With some geometry I found $\theta = \tan^{-1}\left(\frac{0.75}{1.05}\right) \approx 35.3°$. From chapter 6 this requires $\mu_s \gtrsim 0.714$.

d) For a cube, the line of action runs through the pivot when $\theta = 45°$ (se leftmost figure below).
If you recall from Chapter 6, if the block begins to slide at $\theta = 45°$ we know $\mu_s = \tan 45° = 1$.
Typically, coefficients of static friction are *less* than 1.
We expect a cube is more likely to slip well before $\theta = 45°$ **unless** you have a very tacky block made out of rubber or something. Note: sometimes hockey pucks on lacquered boards used as shelves stick to 45°.

e) Consider the figures below.
The tall block probably tips instead of slides.
Its mg lever arm extends to the *left* of the pivot causing a *counter-clockwise* torque.
The short block probably slides instead of tips.
Its mg lever arm extends to the *right* of the pivot causing a *clockwise* torque.

.



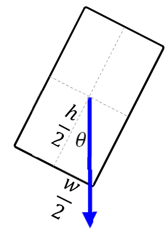
Note: it is common in lab to see objects *seemingly* disobey the laws of physics when doing critical angle experiments. It often appears one gets critical angles larger than 45° which, in turn, implies $\mu_s > 1$!!!

If this puzzles you, consider doing a web search for “gauge blocks”. Gauge blocks are precision machined pieces of metal. Interestingly, because the surfaces are so perfectly machined, a process called wringing can be used to attach the blocks to each other without any adhesives, magnets, or fastening of any kind! The first time I saw this it blew my mind. Then I thought, “This seems similar to what happens when I put a hockey puck on a board and get a critical angle over 45°.”

12.20

a) From the critical angle problem in chapter 6, **6.14** I believe, we know $\theta_c = \tan^{-1} \mu_s$.

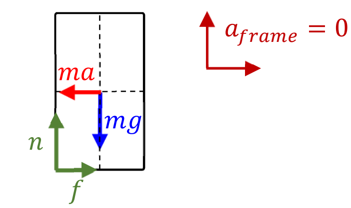
b) For this scenario, the box will tip any time the line of action of the weight force falls to the right of the bottom right corner.



$$\tan\theta = \frac{\frac{w}{2}}{\frac{h}{2}}$$

$$\boldsymbol{\tan\theta = \frac{w}{h}}$$

c) Instead of saying the block accelerates to the right at rate a, assume a ficticous force acts at the center of mass pointing the opposite way (to the left) with magnitude ma.



Doing torques about the bottom left corner gives

$$\frac{h}{2}ma = \frac{w}{2}mg$$

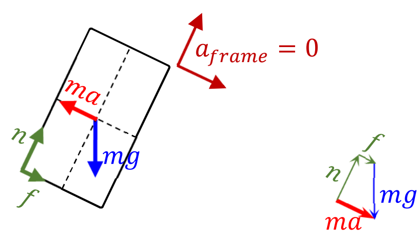
Note: on the verge of tipping I assume CCW torque magnitude equals CW torque magnitude.

In the problem statemnet for part c it said to assume the aspect ratio determined in part b.

Therefore, one finds the box should tip any time acceleration is larger than

$$\boldsymbol{a = \frac{w}{h}g = g\tan\theta}$$

d) Consider the FBD at right.



Notice the lever arm for mg is now slightly *larger* (when compared to the box on the flat portion of the conveyor belt).

The box should NOT tip.

Furthermore, notice the frictional force required to cause such an acceleration is slightly lower (compared to the level belt FBD).

The box will neither tip nor slip during positive acceleration.

Be careful! While accelerating down the ramp the box is <u>not</u> on the verge of slipping!

$$f + mg\sin\theta = ma$$

$$f = mg\tan\theta - mg\sin\theta$$

To give you a better feel for this result, in my figure I used $\theta = 27°$ giving $f = 0.0555mg$.

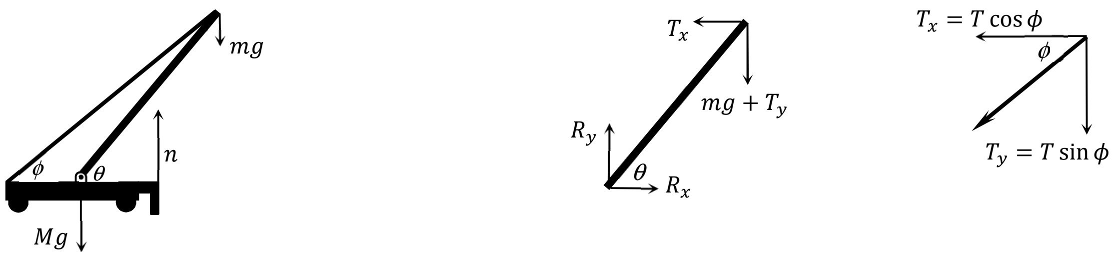
12.21 Crane problem

a) $\cos\theta = \frac{L/5}{L}$ which gives 78.5°

b) $\tan\phi = \frac{L\sin\theta}{\frac{L}{5}+L\cos\theta}$ which gives $\phi = \tan^{-1}\left(\frac{\sin\theta}{\frac{1}{5}+\cos\theta}\right)$

While this formula is a pain in the ass, we in theory know ϕ once we have θ. Since cranes often change angles it seemed worthwhile to derive this result.

c) FBD for the entire crane and the boom only are shown. Notice I chose to split up the tension at the point where it is applied to the boom. This is a tricky part because the component of the tension depends on the angle ϕ, not the boom angle θ as shown in the far right of the figure.



Entire Crane	**Boom Only**
ΣF: $n = (m+M)g$ Note: we assumed the wheels are barely in contact with the ground. This implies the outrigger is supporting the entire crane.	ΣF_y: $R_y = mg + T\sin\phi$ ΣF_y: $R_x = T\cos\phi$
$\Sigma\tau$: $\frac{L}{5}Mg = \left(L\cos\theta - \frac{L}{5}\right)mg$	$\Sigma\tau$: $L\cos\theta\,(mg + T\sin\phi) = L\sin\theta\,(T\cos\phi)$ Notice I split the force up in to components and then used a lever arm for each component!

The torque equation for the entire crane tells us

$$M = (5\cos\theta - 1)m$$

The value of M is determined purely by m and θ! Fortunately the torques from the boom gives

$$m = \frac{T}{g}(\cos\phi\tan\theta - \sin\phi)$$

Subbing in gives the sick result

$$M = \frac{T}{g}(\cos\phi\tan\theta - \sin\phi)(5\cos\theta - 1)$$

Plugging in values we find $\phi = \tan^{-1}\left(\frac{\sin 40°}{\frac{1}{5}+\cos 40°}\right) = 33.64°$ and

$$M_{min} = \frac{20\text{ N}}{9.8\ \frac{\text{m}}{\text{s}^2}}(\cos 33.64°\tan 40° - \sin 33.64°)(5\cos 40° - 1) = 0.835m$$

d) Notice there is no friction required to keep the crane in equilibrium. All of the external forces on the crane are vertical!

e) If the mass is greater there is more than enough counter mass to keep the crane in contact with the ground. The wheels will remain in contact with the ground and actually support some of the crane's weight. If less mass is used, the crane will tip over! Notice this is why I labeled it M_{min} in the equation above. Another way to look at it: one limit for the max safe load for this crane is $m < \frac{M}{(5\cos\theta - 1)}$.

f) Solving the boom torque equation for T gives

$$T = \frac{mg}{(\cos\phi\tan\theta - \sin\phi)}$$

As the angle goes up the required tension goes down since $\tan\theta$ decreases as θ increases

g) If moving at constant speed, the equations are unchanged. If the mass is accelerating up the tension increases slightly. If the mass is accelerating down the tension is slightly lower. If the crane starts to tip you can let out cable while raising the boom angle which reduces the torque from mg on the crane.

h) If you use a longer outrigger less counterbalance mass is required. Lengthening the outrigger will also force you to use lower angles for the boom. Lower boom angles handle lower loads.

i) Shorter booms will require greater angles. Notice, however, for a shorter the boom T is more perpendicular to the boom (ϕ is closer to 90°). For a given guy line strength more torque can be exerted on the shorter boom. Typically larger load masses can be lifted!

j) The guy line in the first design and hydraulic piston in the second design exert the torque to balance the load. The angles from the boom for the guy line and hydraulic piston are not exactly the same but fairly close. The hydraulic connects much closer and will probably be exposed to greater forces.

I asked a friend who works with cranes about this problem and here is his reply:

> *Most mobile truck cranes (ones with rubber wheels) use some kind of hydraulic piston to support the crane boom, as do some mobile crawler cranes (ones with tank style treads).*
>
> *The advantage to a mobile crane with a hydraulic multi-part boom and a hydraulic raising cylinder is speed of setup. You can be lifting loads with minimal setup and teardown time on a variety of surfaces. The disadvantage is reach (boom length and height) and lifting capacity. However, there are some very capable mobile cranes that have hydraulic booms.*
>
> *Fixed boom (non-telescopic) cranes (most all with luffing jib) take longer to set up but can ultimately lift more weight to a greater height.*
>
> *The most important element in most modern cranes is the steel cable, which actually does most of the heavy lifting, raising, and lowering of loads. They hydraulic cylinder or luffing cable does not so much to actually raise or lower the load as it supports the boom or mast of the crane.*
>
> *The real interesting math/physics problem is the mechanical advantage when using multiple parts of line in a block and tackle system (as found in crane blocks).*

Random comment related to a link I removed…If you are like me, you didn't think of Indiana as having a major shipping port but Lake Michigan connects to the ocean via the Mississippi river & the Illinois Waterway or through the Saint Lawrence Seaway, Great Lakes Waterway & the Great Lakes.

12.22 The point of this problem is to show we may not want to always assume the normal force between two objects will always act towards the center of the block! Answers in problems statement.

12.23 Not solved yet.

12.24 Not solved yet.

12.25 $A_x = B_x = mg$, $A_y = mg$. $\vec{A}$ points up and to the left, 108.4° CCW from +*x*-axis. If a hinge was used instead of a roller, we would have an additional reaction force B_y. With *four* unknowns the system is *underdetermined*. There are an infinite number of possibilities for A_y and B_y as long $A_y + B_y = mg$. If you try to find an extra torque equation, you will find it is linearly dependent (opposite of linearly independent) on your previous three equations!

12.26 Typically done in class.
12.27 Typically discussed in class.
12.28 Typically discussed in class.
12.29 Typically discussed in class.
12.30 Typically done in class.
12.31 Typically done in class.