Physics Workbook Volume 1

Revised Summer 2022

The solutions are available for free as a pdf at http://www.robjorstad.com/Phys161/161Workbook.htm.

Use a laptop, tablet, or phone to access the solutions at my website above.

Find the chapter you want then open that link on your device.

The questions (this book) and the answers (on your device) ready to go at the same time!

Tip: you can search the solutions file for the problem number to skip directly to the solution you want (hit CTRL-F).

SIG FIGS, SCIENTIFIC NOTATION, UNITS, ETC

Why pay attention to sig figs?

Every time you write down a number, the number of sig figs implies the precision of that number. A number with 2 sigs figs has *about* 10% uncertainty while a number with 3 sig figs has *about* 1% uncertainty. Four sig figs imply approximately 0.1% uncertainty; five sig figs imply about 0.01% uncertainty…extremely precise.

- Writing numbers with too <u>few</u> sig figs causes errors in subsequent calculations.
- Writing numbers with too <u>many</u> sig figs implies a false sense of precision of the answer.

The best technique is to write one extra sig fig but indicate the uncertain digit. This allows you to avoid intermediate rounding errors while still properly communicating to others the true level of precision in your calculations.

To determine the number of sig figs in a number:

Rule 1: All non-zero digits are significant.
Rule 2: Zeros between other significant figures (bounded zeros) are significant.
Rule 3: Leading zeros are to the *left* of all non-zero numbers. They are never significant.
Rule 4: Trailing zeros are *right* of all non-zero numbers. If all trailing zeros are to the *left* of the decimal, none are significant. <u>**IF**</u> at least *one* trailing zero appears to the *right* of the decimal point, then ALL trailing zeros are significant!

Example 1: Determine the number of sig figs in 40300.
Notice, in this case, the trailing zeros are <u>NOT</u> significant since none of them lie to the right of the decimal.

Example 2: Determine the number of sig figs in 0.000003600 in scientific notation.
Notice, in this case, the trailing zeros <u>ARE</u> significant since they lie to the right of the decimal. Note: *leading* zeros are never significant.

Example 3: Determine the number of sig figs in 340.0.
Notice, in this case, the trailing zeros <u>ARE</u> significant. The trailing zero to the *right* of the decimal is significant. The trailing zero to the *left* of the decimal is between two significant figures and thus becomes significant! Trailing zeros are all significant if any one of them lies to the right of the decimal!

1.1 Identify the number of sig figs in each number using the methods described in class.

a	123	d	0.03	g	1.00
b	10.0	e	1.03	h	10$\underline{0}$0 (the underbar indicates that zero *is* significant)
c	103	f	0.030	i	100

1.2 Write the following numbers in scientific notation. If you have any trouble, try reading the next page.

a	203	b	200	c	0.03030

Scientific Notation

A number written in scientific notation is expressed with a number between 1 and 10 multiplied by 10 to some power. Essentially, one must write the number so there is only one non-zero number to the left of the decimal.

Example 1: Write 40300 in *scientific* notation.

$$\underleftarrow{40300} = 4.03 \times 10^{4}$$

Example 2: Write 0.000003600 in *scientific* notation.

$$0.\underrightarrow{000003}600 = 3.600 \times 10^{-6}$$

Example 3: Write 340.0 in *scientific* notation.

$$3\underleftarrow{40}.0 = 3.400 \times 10^{2}$$

1.3 Write the following numbers in *scientific* notation.

Number	# of sig figs	Scientific Notation
0.000354		
80.5		
80.0		
1234		
12		
0.000000030		
0.45600		
6700		
860		
860.0		

1.4 Write the following numbers *without scientific* notation.

Tip: use the EE &mode buttons on your scientific calculator to check your work…ask your instructor.

Scientific Notation	# of sig figs	Number
1.23×10^{-4}		
3.00×10^{0}		
1.000×10^{-1}		
2.0×10^{5}		

Engineering Notation

Engineering notation writes numbers like this

$$\# = a \times 10^{n}$$

Here a is a number between 1 and 1000 and n is some multiple of three. Engineering notation, at first glance, looks nearly identical to scientific notation. In fact, sometimes it is exactly the same.

The difference between *scientific* & *engineering* notation:

When moving the decimal in *engr* notation, do it in groups *of three* and end with a number between 1 and 1000.

Example 1: Write 40300 in *engineering* notation. Move decimal over 3.

$$40300 = 40.3 \times 10^{3}$$

Example 2: Write 0.000003600 in *engineering* notation. Move decimal over $3 \times 2 = 6$.

$$0.000003600 = 3.600 \times 10^{-6}$$

Example 3: Write 4.01×10^{-2} in *engineering* notation. First write in normal mode, then move over 3 places.

$$4.01 \times 10^{-2} = 0.0401$$

$$0.0401 \quad = 40.1 \times 10^{-3}$$

Example 4: Write 1.230×10^{8} in *engineering* notation. Move decimal over $3 \times 2 = 6$.

$$1.230 \times 10^{8} = 123\underline{0}00000$$

$$123\underline{0}00000 = 123.0 \times 10^{6}$$

Example 5: Write 3.400×10^{2} in *engineering* notation. First write in normal mode; then notice you are already done!

$$3.400 \times 10^{2} = 340.0$$

$$340.0 \qquad = 340.0 \times 10^{0}$$

$$\textit{no need to shift}!$$

1.5 Write the following numbers in engineering notation.

Tip: use the EE &mode buttons on your scientific calculator to check your work…ask your instructor.

Number	**Engineering Notation**
0.000354	
80500	
0.00000003	
1234.0	
0.45600	
860.0	

Number	**Engineering Notation**
12	
9.09×10^{7}	
6.20×10^{-11}	
1.23×10^{-3}	
2.46×10^{4}	

Why bother with a prefix?

I prepared three data tables below. The numbers in each table are identical. The first uses scientific notation, the second uses engineering notation, the third uses engineering notation and an appropriate prefix. I think you will agree the third version just looks better.

x (m)
1.00E+03
2.00E+03
5.00E+03
1.000E+04
2.000E+04
5.000E+04
1.0000E+05
2.0000E+05
5.0000E+05

x (m)
1.00E+03
2.00E+03
5.00E+03
10.00E+03
20.00E+03
50.00E+03
100.00E+03
200.00E+03
500.00E+03

x (km)
1.00
2.00
5.00
10.00
20.00
50.00
100.00
200.00
500.00

In our society, scientists are expected to produce results AND communicate clearly to a wide range of audiences with non-technical backgrounds. The first two styles of results, while perfectly legitimate, will irritate non-technical audiences and make them stop listening to your message. Being able to write data in the third format makes you appear more professional. People will like you more. You will eventually get more money. Learn it! Oh yeah, your ability to choose an appropriate prefix will be tested on exams, too…

Prefix	**Abbreviation**	$\mathbf{10^?}$
Giga	G	10^9
Mega	M	10^6
kilo	k	10^3
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

Example 1: $1.23 \text{ nm} = 1.23 \times 10^{-9} \text{ m}$

Example 2: $0.456 \text{ kcal} = 0.456 \times 10^3 \text{ cal} = 456 \text{ cal}$

Example 3: $0.0297 \, \frac{\text{m}}{\text{s}} = 29.7 \times 10^{-3} \, \frac{\text{m}}{\text{s}} = 29.7 \, \frac{\text{mm}}{\text{s}}$

Example 4: $1{,}860{,}000 \text{ phones} = 1.86 \times 10^6 \text{ phones} = 1.86 \text{ Mphones}$

Example 5: $70000 \text{ m} = 70 \times 10^3 \text{ m} = 70 \text{ km}$

Example 6: $0.0350 \text{ s} = 35.0 \times 10^{-3} \text{ s} = 35.0 \text{ ms}$

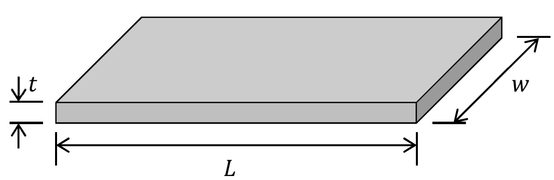
Example 7: $0.00012 \text{ A} = 120 \times 10^{-6} \text{ s} = 120 \text{ μA}$

1.6 Write the following numbers in engineering notation with units! Also write them with the appropriate prefix.
Tip: do these *without* a calculator first. Then type the number into calculator and change into engineering notation mode to check your work.

Number	**Engineering Notation**	**Eng. Not. With Prefix**
0.0434 m		
501000 V		
0.00000000020 C		
90000000 F		
0.77 g		
120000000000 Bq		
12 ft		
8.08×10^{8} J		
1.50×10^{-13} N		
1.23×10^{-8} bel		
2.46×10^{3} cd		

Why not always do this process with a calculator? When you write code, you might need to understand this process and you won't be able to use a calculator! As an example: suppose you obtain distance data (y) in mm but a calculation uses $g = 9.8\,\frac{\text{m}}{\text{s}^2}$. Perhaps the equation for time is $t = \sqrt{\frac{2y}{g}}$. In your computer code you would need to convert the prefix and write it as $t = \sqrt{\frac{2\left(\frac{y}{1000}\right)}{g}}$. **Don't worry about this paragraph now if it makes no sense.**

1.7 A slab of metal has length $L = 0.40$ m, width $w = 0.0808$ m, and thickness $t = 750 \times 10^{-7}$ m. Figure not to scale.



a) Write down L in *scientific* notation (correct sig figs and units).
b) Write down L in *engineering* notation with correct sig figs *and appropriate prefix*!
c) Write down w in *scientific* notation (correct sig figs and units).
d) Write down w in *engineering* notation with correct sig figs *and appropriate prefix*!
e) Write down t in *scientific* notation (correct sig figs and units).
f) Write down t in *engineering* notation with correct sig figs *and appropriate prefix*!

Math with sig figs

1. When multiplying or dividing, the crappiest *number* of sig figs is kept.
2. When adding or subtracting, the crappiest *column* of sig figs is kept.
   WARNING: Addition & subtraction can change the number of sig figs.
3. Unless otherwise specified, use *three* sig figs for everything.
   Exception: sometimes people default to *four* sig figs if the first digit of a number is 1.
4. In *scientific* notation, all numbers are always significant!
5. Keep at least one *extra* sig fig for all math work. Then, round you final results to the appropriate number of sig figs in the last step.

Avoiding intermediate rounding error

When doing computations, it is important to track sig figs as you go through the problem. The best method is to keep at least one extra sig fig on all numbers during your calculations and only round at the final step.

If you round your numbers *before* the final step, you introduce *intermediate rounding error*. By rounding too soon, your final answer can differ dramatically from the correct method.

I keep track of my sig figs using a small underline in each number. For example, the number $2.4\underline{3}6$ shows I have a 3 sig fig number (perhaps my calculator gave the extra digit after a computation).

Done Correctly Keep one extra sig fig, round after final answer	**Done Incorrectly** Round after each step
$x = \left[\frac{9.8}{6.2} - \frac{8.5}{7.5}\right]^2 - \frac{6.2}{8.5}$	$x = \left[\frac{9.8}{6.2} - \frac{8.5}{7.5}\right]^2 - \frac{6.2}{8.5}$
$x = \left[1.\underline{5}8 - (1.\underline{1}3)\right]^2 - 0.7\underline{2}9$	$x = \left[1.\underline{6} - (1.\underline{1})\right]^2 - 0.7\underline{3}$
$x = \left[0.\underline{4}5\right]^2 - 0.7\underline{2}9$	$x = \left[0.\underline{5}\right]^2 - 0.7\underline{3}$
$x = 0.\underline{2}03 - 0.7\underline{2}9$	$x = 0.\underline{3} - 0.7\underline{3}$
$x = -0.\underline{5}26$	$x = -0.\underline{4}$
$x = -0.\underline{5}$	Final answer differs by 20% from correct method!!!

1.8 Perform the following mathematical operations while keeping track of the correct number of sig figs. Write your final answer in scientific notation.

a) $\frac{120.0}{3} + 70 =$
b) $13 + 0.741 =$
c) $65.02 - 64.99 =$
d) $(12.0 - 9.99) \times (8.00 \times 10^6)$
e) $\frac{(13.10-13.00)^2}{(800+300)} =$

1.9 Explain the difference between 10E3 and 10^3. Try typing both numbers into your calculator now so you don't mess this up on an exam!

1.10 Answer the following subtraction problem with **correct significant figures** and **correct scientific notation**.

$$1.012 \times 10^4 \text{ km} - 9943.0 \text{ km} =?$$

1.11 You are told $= 8769.8\underline{9}7$ N, $B = 8770.3\underline{2}4$ N, $C = 0.00083897$ m and $D = 0.000842\underline{2}4$ m.

a) Compute $B + A$. Answer in *engineering* notation *with appropriate prefix and correct sig figs.*
b) Compute $B - A$. Answer in *engineering* notation *with appropriate prefix and correct sig figs.*
c) Compute $D + C$. Answer in *engineering* notation *with appropriate prefix and correct sig figs.*
d) Compute $D - C$. Answer in *engineering* notation *with appropriate prefix and correct sig figs.*

1.12 A very rare physics unit is the mockingbird. $E = 4.97 \times 10^3$ mockingbird & $F = 20$ mockingbird. Compute $R = 346.5(F) - E$ with correct sig figs, engineering notation, & appropriate prefix.

1.13 Suppose you have a rectangular fence with sides of 4.0 m and 6.0 m. Determine the perimeter and the area of the fence. Answer with proper units, sig figs, and scientific notation.

1.14 You are given the data table shown at right and the formula below.

$$\Delta x = \frac{1}{2} a_x t^2 + v_{0x} t$$

$a_x \left(\frac{\text{m}}{\text{s}^2}\right)$	$v_{0x} \left(\frac{\text{m}}{\text{s}}\right)$	t (s)
-1.23×10^{-1}	5.85×10^{-3}	0.0987

Compute Δx while correctly keeping track of sig figs. Write your final answer in engineering notation with appropriate prefix. Correctly round (or indicate rounding column with the underbar).

- We will learn later that the ½ has infinite sig figs. For now, assume $\frac{1}{2} = 0.500\underline{0}$. By having more sig figs than any other number in the problem it will not affect the sig figs in the calculation.
- Carry along the units at each step and cancel out the units of seconds (s) as appropriate.

1.15 You are given the data table shown at right and the formula shown.

$$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$$

$a_y \left(\frac{\text{m}}{\text{s}^2}\right)$	$v_{fy} \left(\frac{\text{m}}{\text{s}}\right)$	Δy (m)
-9.8	9.2×10^1	8.7×10^1

a) Solve <u>algebraically</u> for v_{iy}.
b) Next, plug in numbers form the data table to compute v_i. Write your final answer in engineering notation with appropriate prefix. Correctly round (or indicate rounding column with the underbar). Assume 2 has infinite sig figs.

Notice there are two questions at the bottom of this page.

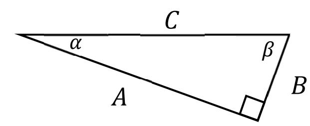
STUFF YOU SHOULD KNOW ALREADY

Example 1: SOH CAH TOA as it applies to the triangle at right.

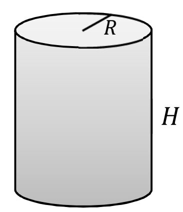
$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{B}{C}$$

Example 2:

$$\tan\beta = \frac{\text{opposite}}{\text{adjacent}} = \frac{A}{B}$$



Volume of a sphere: $V_{sphere} = \frac{4}{3}\pi R^3$
Surface Area of a Sphere: $A_{sphere} = 4\pi R^2$



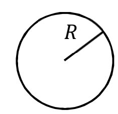
Volume of a cylinder: $V_{cyl} = \pi R^2 H$

Volume of a rectangular box: $V_{box} = LWH$

Arbitrary shape volume: often (but not always) $V = (Area\ of\ base) \times (height)$

Area of rectangle: $A_{rect} = LW$

Area of circle: $A_{circle} = \pi R^2$
Circumference: $C = 2\pi R$



1.16 Use the above information to re-write the volume of a sphere in terms of diameter (D) instead of radius.

1.17 Use the above information to re-write the volume of a cylinder in terms of height (H) & diameter (D) instead of height & radius.

Common Conversions:
Tip: Generally we assume conversion factors are perfect numbers (infinite sig figs) unless otherwise noted.

$160\underline{9}\text{ m} = 1\text{ mi}$	$12\text{ in} = 1\text{ ft}$	$60\text{ s} = 1\text{ min}$	$1000\text{ g} = 1\text{ kg}$
$2.54\text{ cm} = 1\text{ in}$	$1\text{ cc} = 1\text{ cm}^3 = 1\text{ mL}$	$60\text{ min} = 1\text{ hr}$	$100\text{ cm} = 1\text{ m}$
$1\text{ cm} = 10\text{ mm}$	$1\text{ yard} = 3\text{ ft}$	$3600\text{ s} = 1\text{ hr}$	$1\text{ km} = 1000\text{ m}$
$1\text{ furlong} = 220\text{ yards}$	$528\underline{0}\text{ ft} = 1\text{ mi}$	$24\text{ hrs} = 1\text{ day}$	$180° = \pi\text{ rad}$
		$1\text{ fortnight} = 14\text{ days}$	

Example 1:
Convert 23 in to miles and write your answer *with appropriate prefix* and correct sig figs. Note: we typically assume all of the above conversion factors have infinite sig figs (with the exception of $160\underline{9}\text{ m} = 1\text{ mi}$).

$$\frac{2\underline{3}\text{ in}}{} \times \frac{2.54\text{ cm}}{1\text{ in}} \times \frac{1\text{ m}}{100\text{ cm}} \times \frac{1\text{ mi}}{160\underline{9}\text{ m}} = 0.0003\underline{6}3\text{ mi} = 3\underline{6}3\ \times 10^{-6}\text{mi} = 360\ \mu\text{mi}$$

Example 2:
As part of the problem statement in an exam question, your instructor tells you to assume $1\text{ lbs} = 2.\underline{2}\text{ kg}$. You are asked to use this (and your conversions listed above) to convert $234\frac{\text{kg}}{\text{m}^2}$ to PSI or $\frac{\text{lbs}}{\text{in}^2}$. Write the final *answer in scientific notation* with correct sig figs.

$$\frac{23\underline{4}\text{ kg}}{\text{m}^2} \times \frac{1\text{ lbs}}{2.\underline{2}\text{ kg}} \times \frac{1^2\text{ m}^2}{100^2\text{ cm}^2} \times \frac{2.54^2\text{ cm}^2}{1^2\text{ in}^2} = 0.06\underline{8}6\ \frac{\text{lbs}}{\text{in}^2} = 6.\underline{8}6\ \times 10^{-2}\text{ PSI} = 6.\underline{9}\ \times 10^{-2}\text{ PSI}$$

Density in physics given by $\rho = \frac{m}{V}$ and typically has units $\frac{\text{kg}}{\text{m}^3}$ or $\frac{\text{g}}{\text{cm}^3}$. Density can effectively be used to convert mass to volume and vice versa.

1.18 Two accelerations are given as $a_1 = 0.060\frac{\text{mm}}{\text{s}^2}$ and $a_2 = 49500\frac{\text{ft}}{\text{hr}^2}$

a) Determine the number of sig figs in each acceleration.
b) Convert a_1 to $\frac{\text{ft}}{\text{hr}^2}$ units. Write your final answer with correct sig figs and *scientific* notation.
c) Convert a_2 to SI units (m and s). Use correct sig figs & *engineering* notation with appropriate prefix.

1.19 Convert $6\underline{0}$ mph to m/s.

1.20 A space rock composed of aluminum and iron has a density of $1.010 \times 10^2\,\frac{\text{g}}{\text{in}^3}$. Convert to $\frac{\text{kg}}{\text{m}^3}$. Write your answer with correct sig figs and proper scientific notation. *Challenge:* What % of the rock (by volume) is aluminum?

1.21 Near earth's surface we assume $1\text{ lbs} = 453.\underline{6}\text{ g}$. Convert $13.5\frac{\text{lbs}}{\text{day}^2}$ to SI units (kg and s). Write your final result with correct sig figs in *scientific* notation.

1.22 Convert $12.3\frac{\text{in}}{\text{hr}^2}$ to SI units (m and s). Use correct sig figs & *engineering* notation with appropriate prefix. Note: the units of your final answer should look something like $\frac{\text{km}}{\text{s}^2}, \frac{\mu\text{m}}{\text{s}^2}$, etc.

Using density to relate mass and volume

In most physics classes we use the symbol ρ (pronounced "rho") to represent density. The density equation is thus

$$\rho = \frac{m}{V}$$

Obviously this can be rearranged as needed to determine mass or volume. Another way to think of it is density can be used as a conversion factor between mass and volume.

1.23 I have $4.0\ \text{cm}^3$ of aluminum with density $2.7\ \frac{\text{g}}{\text{cm}^3}$. What is the equivalent mass?

1.24 I have 4.2×10^2 kg of aluminum with density $2700\ \frac{\text{kg}}{\text{m}^3}$. What is the equivalent volume?

Using volume & mass flow rates

"Volume Flow Rate" means $R_{vol} = \frac{Volume}{time} = \frac{V}{t}$ and has units of $\frac{\text{m}^3}{\text{s}}$ (another common unit is $\frac{\text{Gallons}}{\text{minute}}$)

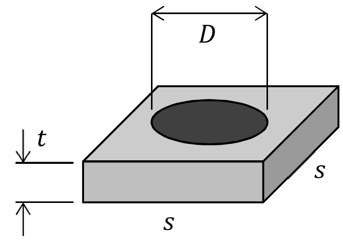
"Mass Flow Rate" means $R_{mass} = \frac{mass}{time} = \frac{m}{t}$ and has units of $\frac{\text{kg}}{\text{s}}$

Notice we can convert from mass flow rate to volume flow rate using density as above.

To convert a mass to a volume we divide by density (as in **Example 4** above). To convert R_{mass} to R_{vol} we similarly divide by ρ! Cool.

1.25 At one point during a flood, the peak water flow rate at the Oroville Dam spillway was $100{,}000\ \frac{\text{cubic feet}}{\text{second}}$. Determine the amount of $\frac{\text{kg}}{\text{day}}$ flowing over the dam. You may assume the density of water is $1000\ \frac{\text{kg}}{\text{m}^3}$. **Round your final answer to three sig figs and answer with correct <u>scientific notation</u>.**

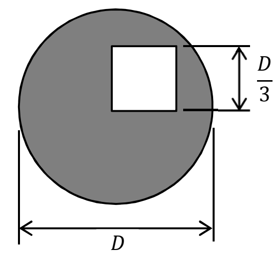
1.26 Putting it all together: A nut is modeled as a rectangular solid with a cylindrical hole drilled out of it. This assumes we are ignoring the threads. A sketch of such a nut is shown at right. Note: figure not to scale.



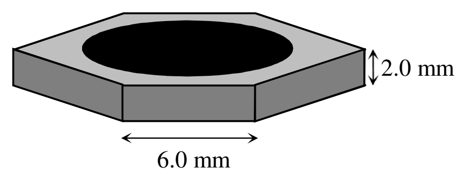
a) Determine the volume of metal in the nut in terms D, s, and t.
b) You are told the nut is made of gold with density 19.3 $\frac{\text{g}}{\text{cm}^3}$. The nut is $4.50\ \times 10^{-3}$ m thick with side $s = 2.00$ in and hole diameter 50.3 mm. Determine the mass of the nut in kg. Write your answer with correct sig figs and scientific notation.

1.27 Assume a nucleus is a sphere of diameter 12 μÅ. You are told $1\ \text{Å} = 10^{-10}$ m. Determine the surface area of the nucleus in fm^2. Write your answer with correct sig figs.

1.28 A plate has a square hole cut from it. The circle has diameter D and the square hole has side length $\frac{D}{3}$. The mass of the plate, *after the square piece is removed*, is m. The thickness of the plate is t. Figure not to scale. Determine an algebraic expression for the density. Write the answer as a number with three sig figs times $\frac{m}{D^2t}$.

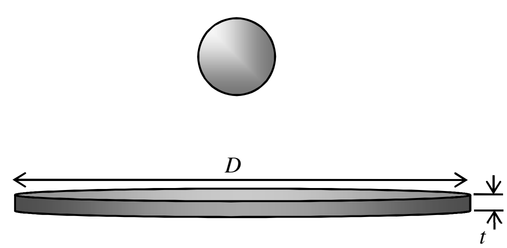


1.29 Many feel this is tricky: Consider the hexagonal nut shown in the figure. The nut is a regular hexagon with a circular hole drilled in it. For this problem ignore the threads inside the hole. Suppose each side of the hexagon is s = 6.0 mm long. The hole has diameter of D = 10.7 mm. The nut is t = 2.0 mm thick. The density of aluminum is 2.7 g/cm³.



a) Determine the volume of the nut *algebraically* (in terms of the variables before you plug in any numbers).
b) Determine mass of the nut. Get the sig figs correct & answer in engineering notation. Note: this is not a realistically sized nut…try sketching it to size to see what I mean.

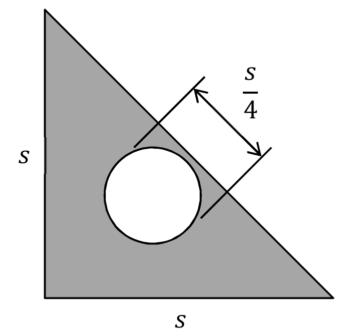
1.30 A spherical drop of oil is dropped onto the surface of a smooth pond. Within a matter of seconds after touching the surface, the oil rapidly spreads out into the shape of a pancake (very short cylinder). The oil slick formed on the surface eventually ends up having a thickness roughly equal to a single molecule of oil! Let's use a demo to figure out thick the oil molecule is.



a) Determine the diameter of the initial drop.
b) Determine an expression for t in terms of the other variables.
c) Use values provided by your instructor during the demo to determine the thickness. As a back-up, I found a video where a 0.5 μL (yes, that's micro!) drop created an oil slick with diameter 60 cm.

One can learn more about this by doing a web search for "Benjamin Franklin oil drop". I learned of a scientist named Agnes Pockles (1862-1935). She was denied higher education. In her kitchen she created a device for performing this type of experiment with much greater precision. Her device is the precursor to the Langmuir trough which is still used in biophysics labs today. Her work was published in the prestigious journal *Nature* with the help of Nobel Laureate Lord Rayleigh. She had numerous other publications/contributions and was a pioneer of the field of surface science. At age 70 she was awarded an honorary doctorate.

1.31 A 45-45-90 triangular plate has a circular hole cut from it. The two sides of the right triangle are length s while the hole has diameter $\frac{s}{4}$. The mass of the plate, *after the circular piece is removed*, is M. The thickness of the plate is x. Figure not to scale. Determine an algebraic expression for the density. Write the answer as a number with three sig figs times $\frac{M}{s^2 x}$.

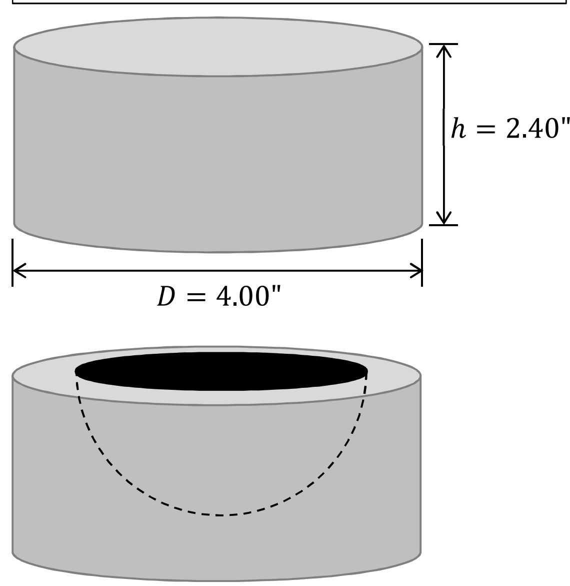


1.32 This problem was inspired by spherical ice ball makers which seem to be all the rage these days. We will consider a simplified model of such a device.

A cylinder of brass has diameter 4.00 inches, height 2.40 inches and density $8.73\,\frac{\text{g}}{\text{cm}^3}$. A hemisphere of radius R must be removed from the cylinder to allow space for the ice. *After the hemispherical hole is made*, the desired weight is 5.55 lbs. Near earth's surface humans typically assume $1 \text{ lbs} = 0.453\underline{6}$ kg.

Determine the radius of the hemispherical hole which gives the desired weight of 5.55 lbs. **Answer in units of meters with correct sig figs and scientific notation.** I will also accept engineering notation with appropriate prefix in lieu of scientific notation.

I want to cut a hemispherical hole. *After the hole is cut*, the weight is 5.55 lbs. Figure out the radius of the hemispherical hole.



$[\mathrm{M}] = \mathrm{mass} = \mathrm{kg}$ $\quad [\mathrm{L}^2] = \mathrm{area} = \mathrm{m}^2$ $\quad [\mathrm{T}] = \mathrm{time} = \mathrm{s}$ $\quad \left[\frac{\mathrm{L}}{\mathrm{T}^2}\right] = \mathrm{acceleration} = \frac{\mathrm{m}}{\mathrm{s}^2}$

$[\mathrm{L}] = \mathrm{length} = \mathrm{m}$ $\quad [\mathrm{L}^3] = \mathrm{volume} = \mathrm{m}^3$ $\quad \left[\frac{\mathrm{L}}{\mathrm{T}}\right] = \mathrm{velocity} = \frac{\mathrm{m}}{\mathrm{s}}$ $\quad \left[\frac{\mathrm{L}\cdot\mathrm{M}}{\mathrm{T}^2}\right] = \mathrm{force} = \frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2} = \mathrm{N}$

Be careful. When you write v it doesn't mean the same thing as $[v]$.

- Here v is the variable speed which includes both a number and units when you substitute in a value.
- Here $[v]$ implies you should write only *the units of speed*.
- After you write the units of speed, the square brackets disappear.
- Notice the following tricky points:
  - The variable m (italicized) implies mass which has *units* of kg
  - The unit m (not italicized) stands for meters, the SI unit of length
  - $[m] = \mathrm{kg}$ while $[x] = \mathrm{m}$

*****A handout with explanations & worked examples can be found online.*****
The handout is linked next to the Chapter 1 Solutions link online (in the comments sections).

1.33 Use this first problem as the example. Blatantly follow the solution if desired. In the following equations L is radius, v is velocity, V is volume, m is mass, and g is the magnitude of acceleration due to gravity. Consider the following equations and determine the units of the variable specified.

a) $K = \frac{1}{2}mv^2$. Find the units of K.
b) $\rho = \frac{m}{V}$. Find the units of ρ.
c) $P_D = \frac{1}{2}\rho v^2$. Find the units of P_D.
d) $\omega = \sqrt{\frac{g}{L}\tan\theta}$. Find the units of ω.

1.34 Consider the following set of measurements shown in the table. Notice the units are given for each measurement in parentheses in the column headings.

t (s)	*m* (kg)	*M* (kg)	*x* (m)	*r* (m)	*s* (m)	*v* (m/s)	*a* (m/s^2)	*F* (N)
1.00	2.50	8.1	4.200	1.00	6.28	5.39	6.4	16.5

a) A force equation involving drag is given by
$$ma = -bv^2$$
Determine the appropriate units for the constant b. Answer only in SI units (s, kg, & m in this case).

b) An equation for the force due to gravity is given by
$$F = \frac{GmM}{r^2}$$
Determine the appropriate units for "big G". Answer only in SI units (s, kg, & m in this case).

c) Explain why the following formula cannot be correct. Hint: first determine the units of each term on the right side and compare to the units of x.
$$x = s\frac{at}{v} - t\sqrt{\frac{m}{F}}$$

d) A common formula in oscillations is given by
$$x = A\cos\omega t$$
where ω is angular frequency, A is amplitude, t is time, and x is distance from equilibrium. Determine the units of A and ω. Hint: In general, functions and their arguments have no units. We say the arguments of trig functions are radians, but radians are simply a place holder unit.

e) A common formula in circuits is given by
$$Q = Q_{max}e^{-\alpha t}$$
What are the units of α? Think: do the units of Q and Q_{max} matter?

1.35 An equation for speed is determined as $v = \sqrt{\frac{rg}{\mu_s}}$ where g is the magnitude of the acceleration due to gravity and r is a radius. Determine the units of μ_s.

1.36 Suppose you have a force equation given by

$$F = ax - bx^3$$

where force (F) is measured in Newtons and distance (x) is in m. Recall, $1\ \mathrm{N} = 1\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}$. Determine the units of the constants a and b.

1.37 Suppose you have a force equation given by

$$F = cv + dv^2$$

where force (F) is measured in Newtons and speed (v) is in $\frac{\mathrm{m}}{\mathrm{s}}$. Recall, $1\ \mathrm{N} = 1\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}$. Determine the units of the constants c and d.

1.38 For now, assume pressure (P) is measured in Pascals (Pa). Notice that the variable P is italicized but the units Pa are not. Note: you will learn in chapter 14 $1\ \mathrm{Pa} = 1\frac{\mathrm{N}}{\mathrm{m}^2}$. Recall also $1\ \mathrm{N} = 1\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}$. Assume volume ($V$) is measured in m^3. Temperature is measured in kelvin (K). Number of moles (n) has units of moles (mol).

a) Perhaps you have heard of the ideal gas law ($PV = nRT$). Determine the units on the constant R.
b) There is another gas law which provides a better model for certain situations called "the van der Waals equation of state". The equation of state is

$$\left[P + a\left(\frac{n}{V}\right)^2\right]\left(\left(\frac{V}{n}\right) - b\right) = RT$$

Determine the units of the new constants a and b.

Note: I read somewhere once that before 1968 *degrees* kelvin was the norm but I guess you shouldn't say the degrees anymore. Many capitalize Kelvin as it comes from a name. I don't really care if you capitalize or not but in formal writing perhaps you would look up appropriate usage in the NIST style guide?

Note[2]: if you use chemistry units for pressure and volume (atm and L respectively) books sometimes use r (not R).

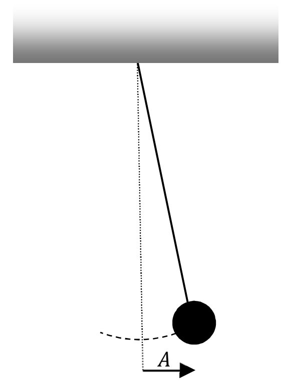
Note[3]: If you set the constants equal to zero in the van der Waals state equation to zero you get back the ideal gas law.

1.39 Physics & Aerospace Majors: Force is measured in Newtons (N) where 1 N = 1kg·m/s^2. From experience we expect the force keeping an object in circular motion should increase with increasing speed and mass but also with decreasing radius. If we model the force as

$$F = \frac{m^k v^n}{r^q}$$

where k, n, and q are positive exponents we know our model will fit our expectations. If only we could figure out the exponents. Use unit analysis to determine the correct exponents of mass, speed, and radius.

1.40 Physics & Aerospace Majors: Consider the simple pendulum shown at right. A simple pendulum consists of a small mass tied to the end of a string that oscillates back and forth along the curved path shown. If the diameter of the mass is much smaller than the length of the string, we may treat the mass as a point mass (mass with no size at all). We assume the string has negligible mass compared to the point mass. We also assume the string doesn't stretch (inextensible string).



The time for the mass to swing back and forth one time is called the period (T) of oscillation. Without knowing anything more, we might suspect the T to depend on the length of the string L, the mass of the sphere m, the amplitude of the oscillation A, and the magnitude of the acceleration due to gravity (g). Said another way, we expect

$$T = kL^a m^b g^c A^d$$

where a, b, c, & d are exponents (positive or negative) and k is a *dimensionless* (aka unitless) constant. Note: amplitude is the distance the ball travels. Doing a brief experiment in front of the class reveals that, for small angles, the amplitude doesn't affect the period! Removing this term, by setting $d = 0$, gives

$$T = kL^a m^b g^c$$

Use dimensional analysis to determine the other exponents. You might be surprised at the results.

1.41 Physics & Aerospace Majors: In fluid dynamics it is of interest to study a dimensionless parameter known as Reynolds number (Re). For a sphere moving in a fluid, the following parameters might be of interest:

1) ρ = density of the fluid
2) D = diameter of the sphere
3) μ = Greek letter "mu" = dynamic viscosity of the fluid $\left(\text{units of } \frac{\text{kg}}{\text{m}\cdot\text{s}}\right)$
4) v = speed of the sphere

To incorporate the above elements, let us assume

$$Re = \rho^1 D^a \mu^b v^c$$

where a, b, and c are exponents (could be positive or negative). The exponent of ρ is set to 1 by convention. Use dimensional analysis to determine the exponents of each term. It turns out Re is extremely useful in fluid mechanics. Do a web search for "video Reynolds number" to see some amazing stuff.

1.42 Late in Chapter 6 will derive a formula $\psi = \sqrt{g \tan\theta(R + L\sin\theta)}$ where R is a radius, L is the length of a string, θ is the angle made by the string, and g is the magnitude of the acceleration due to gravity. Determine the units of ψ. This symbol is the Greek letter psi, pronounced like "sigh".

1.43 You are told an equation exists such that

$$\frac{x}{v^3} = k\sqrt{\frac{a}{rt^2}}$$

where x is position (same units as distance), v is velocity, t is time, a is acceleration, r is a distance, and k is an unknown constant. Determine the units of the unknown constant k. The answer may look pretty unusual as I made up the craziest equation I could think of to give you some hard practice.

1.44 Most of the following terms are actually used in physics problems at some point during your first semester. Which of the following terms have matching units? There may, or may not be, more than one match. Assume that m is mass in kg, h & r are distances in m, v is speed in m/s, a & g are accelerations in m/s^2, and t is time in seconds.

ma	mgh	mvt	mvr	$\frac{1}{2}mv^2$	$m\frac{v^2}{r}$	mht

1.45 In the solutions I made a set of instructions with screen captures which should help you make plots. Open the solutions and follow the instructions before attempting the next two problems.

1.46 Do after completing **1.45**. Later in the semester we encounter some tricky equations. Determine θ.

$$2\cos\theta - 2.232 + \sin\theta = 0$$

1.47 Do after completing **1.45**. In a problem relating to diffraction one must solve the transcendental equation

$$\frac{\phi}{\sqrt{2}} = \sin\phi$$

where ϕ is in radians. This equation determines the angle for full width at half maximum (FWHM) of the central bright fringe of a single slit diffraction pattern. Solve the equation for ϕ.

Absolute, Fractional and Percent Errors

Suppose you measure a distance as $x = 2.0$ cm and you expect to be off by no more than half of the rightmost digit.

Absolute error in x is $\delta x = \frac{1}{2}(rightmost\ digit) = \frac{1}{2}(0.1) = 0.05$ cm.
The symbol δ is the lowercase delta from the Greek alphabet.
While conventions in other books vary, many assume δ indicates absolute error.

Fractional error is given by $\frac{\delta x}{x} = \frac{0.05\text{ cm}}{2.00\text{ cm}} = 0.025 \approx 0.03$.
Notice fractional error has no units.
Common convention is to round errors to 1 sig fig.
Exception: if the first digit is 1, round to 2 sig figs.

Percent error is simply expressing the fractional error as a percent.
Percent error is thus $\frac{\delta x}{x} \times 100\% = \frac{0.05\text{ cm}}{2.00\text{ cm}} \times 100\% = 0.025 \times 100\% \approx 3\%$
Percent error is a good way to compare the quality of measurements with different units.

1.48 Determine the *percent* error in each of the following measurements. To be clear, the $\pm$ number in each measurement is the *absolute* error.

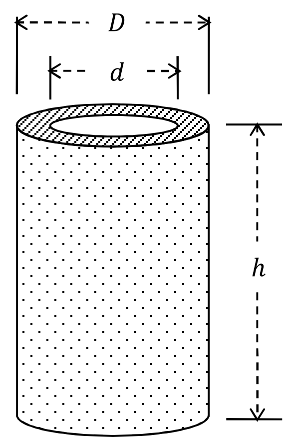
a) $\rho = 50.1 \pm 0.1 \frac{\text{g}}{\text{cm}^3}$
b) $x = 3.0 \pm 0.1$ m
c) $d = 1.0 \pm 0.3$ m
d) $T = 0.060 \pm 0.001$ s
e) $t = 6\underline{0} \pm 1$ s
f) $f = 9.0 \pm 0.5 \frac{\text{m}}{\text{s}^2}$
g) $g = 9.5 \pm 0.5 \frac{\text{m}}{\text{s}^2}$
h) $h = 10.5 \pm 0.5 \frac{\text{m}}{\text{s}^2}$
i) $i = 19.5 \pm 0.5 \frac{\text{m}}{\text{s}^2}$
j) $j = 1\underline{0} \pm 1 \frac{\text{m}}{\text{s}^2}$

1.49 The diameter of regulation table tennis balls must be $D = 40.0 \pm 0.5$ mm. The mass of regulation balls must be $m = 2.70 \pm 0.03$ g.

a) Determine the volume of a 40.0 mm ball. Express with correct sig figs, scientific notation, and units of m³.
b) Determine the surface area of 40.0 mm ball. Express with correct sig figs, scientific notation, and units of m².
c) The ball is a celluloid spherical shell with thickness t. The celluloid has density ρ. Determine an expression for the thickness of the shell in terms of ρ, m and D.
d) Assume the celluloid used to make the balls has density $1375\,\frac{\text{kg}}{\text{m}^3}$. Determine the <u>range</u> of thicknesses of a regulation ball. Express your answer as $t = \# \pm \#$ and as $t = \# \pm \%$ in units of mm similar to the diameter specification above. Assume we may ignore the mass of the air inside the ball.
e) Assume the air inside the ball has density $1.29\,\frac{\text{kg}}{\text{m}^3}$. Determine the mass of the air inside the ball. Express your answer as a percentage of the ball's mass ($m = 2.70$ g). Notice this mass is roughly equal to the uncertainty in the ball's mass. It was probably reasonable to ignore it in our previous calculation.
f) **Challenge**: In part c it is easiest to use the volume of plastic used in the ball is
$$V = (Surface\ Area) \times thickness = (S.A.) \times t$$
A more precise method uses $V = V_{outer} - V_{inner}$ where V_{outer} uses diameter D and V_{inner} uses diameter $D - 2t$. Determine the difference in calculated volume between the methods in terms of D and t.
g) **Challenge:** for what thickness does the approximation $V = (S.A.) \times t$ cause a 1% error in the volume calculation. Answer as a fraction times D.

Note: perhaps you are wondering if the air inside the ball affects the scale reading of the ball when it is placed on a mass balance. We will learn about this when we discuss buoyant forces much later…

1.50 Disclaimer: A did a quick search for specs on PVC. The two main types appear to be Schedule 40 and Schedule 80. Depending on which source you use you might find slightly different numbers. That said, any numbers in the ballpark should serve our purpose just fine here. A Schedule 40 PVC pipe has density ρ, outer diameter D, inner diameter d and height h.



a) Determine an expression for the surface area of the sidewall of the pipe. The sidewall is indicated by sparse dots in the figure.
b) Determine an expression for the surface area of the end. The end is shaded with diagonal lines in the figure.
c) Determine an expression for the mass of the pipe.
d) For 1-inch nominal pipe, one website stated $D = 1.315$ in, $d = 1.029$ in, and $\rho = 1295\,\frac{\text{kg}}{\text{m}^3}$. Nominal means "size in name only"; a 1-inch nominal pipe is approximately but not exactly 1 inch in diameter. Determine the mass *per unit length* of such a pipe. Answer with correct sig figs in scientific notation using $\frac{\text{g}}{\text{cm}}$.
e) In most calculations we would probably ignore the area of the two ends of the pipe compared to the sidewall. If the pipe is short, this will introduce errors in our calculations. What length of 1-inch pipe has the combined area of both end caps equal to a 1% correction to the sidewall area?

1.51 Read about order of magnitude calculations (Fermi calculations). Do the following estimations without the internet.

a) Estimate the number of heartbeats in your lifetime.
b) Estimate the maximum speed of a garden variety snail in furlongs per fortnight. Note: one furlong is an eighth of a mile. A fortnight is a unit of time equal to two weeks.
c) Estimate how many gallons of gasoline are required to drive a car across the country. I'm assuming it is not electric…
d) Estimate how much water you use per day.
e) Estimate the volume of air in a room then use that to estimate the number of air molecules in that room.
f) Estimate the time in your life spent at stop lights.
g) Estimate the time in your life you will waste on paperwork…or perhaps some things are better left unknown.
h) Estimate the number of stars you can see on a clear night.
i) Estimate the area of your college campus…or the number of bathrooms on campus.

Obscure comment: On occasion the sig fig rules give answers that vary depending on the order of operation! This serves as a reminder to us that sig fig rules are merely an estimate of precision. By using the sig fig rules we will be in the right ballpark but might not always have the best estimate of precision. In lab you will learn more about error analysis. This is a skill that takes many years to master…

Example: Suppose you are given two spheres with radii 9.6 cm and 5.2 cm respectively. You, Rogelio, and Vega are asked to find the combined volume of the two spheres. You are each told to keep track of the sig figs.

You decide to the problem as follows:

$$\begin{aligned} V_{tot} &= V_1 + V_2 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 \\ &= 4.18\underline{9} \times 9.\underline{6}^3 + 4.18\underline{9} \times 5.\underline{2}^3 \\ &= 3\underline{7}06 + 5\underline{8}9 \\ &= 4\underline{2}95 = 4.3 \times 10^3 \text{cm}^3 \end{aligned}$$

Rogelio decides to do the problem by first factoring out the constants like this:

$$\begin{aligned} V_{tot} &= V_1 + V_2 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi(r_1^3 + r_2^3) \\ &= 4.18\underline{9}\left(9.\underline{6}^3 + 5.\underline{2}^3\right) \\ &= 4.18\underline{9}\left(8\underline{8}5.7 + 1\underline{4}0.6\right) \\ &= 4.18\underline{9}\left(10\underline{2}5.3\right) \\ &= 42\underline{9}5 = 4.30 \times 10^3 \text{cm}^3 \end{aligned}$$

Finally, Vega determines an estimate of precision based on percent errors. She first makes the assumption that when someone measures 9.6 cm they are implying 9.6 ± 0.05 cm. Similarly, she assumes that the other measurement was 5.2 ± 0.05 cm. This implies the percent error on each measurement is given by:

$$\%\text{err in } r_1 = \frac{0.05}{9.6} \times 100\% = 0.52\%$$

$$\%\text{err in } r_2 = \frac{0.05}{5.2} \times 100\% = 0.96\%$$

Since each radius was cubed, Vega expects the error for each radius to contribute three times. Then Vega further takes the cautious approach and assumes the worst case scenario; Vega assumes that the error in measuring one radius will not cancel out the error in measuring the other radius. For example, Vega is assuming that either both radii were measured too large or too small (not one of each). This gives a total error as follows:

$$\begin{aligned} \%\text{err} &= 3(\%\text{err in } r_1) + 3(\%\text{err in } r_2) \\ &= 3(0.52\%) + 3(0.96\%) \\ &= 1.56\% + 2.88\% \\ &= 4.44\% \approx 4\% \end{aligned}$$

Using this method, Vega decides that the total volume is given by

$$4295 \pm 4.44\% = 4295 \pm 190 \approx 4295 \pm 200$$

Vega infers the 3rd digit isn't significant and the result should be written as $4.3 \times 10^3 \text{cm}^3$.

1D MOTION

Position Vector = $\vec{x}$ or $\vec{r}$ =location (magnitude & direction) relative to the origin

Displacement Vector = $\Delta\vec{x}$ or $\Delta\vec{r}$ =CHANGE in position (magnitude & direction)

Distance Scalar = Δx or Δr **WATCH OUT!** If the object changes direction $\Delta x \neq \|\Delta\vec{x}\|$!

Instantaneous Velocity Vector = $\vec{v} = \frac{d\vec{x}}{dt}$ or $\frac{d\vec{r}}{dt}$ includes magnitude & direction

Average Velocity Vector = $\vec{v}_{avg} = \frac{\Delta\vec{x}}{\Delta t}$ or $\frac{\Delta\vec{r}}{\Delta t}$ includes magnitude & direction

Instantaneous Speed = $v = \|\vec{v}\|$ =the magnitude only of $\vec{v}$

Average Speed = $v_{avg} = \frac{Total\ distance}{Total\ time}$ **WATCH OUT!** If the object changes direction $v_{avg} \neq \|\vec{v}_{avg}\|$!

Instantaneous Acceleration Vector = $\vec{a} = \frac{d\vec{v}}{dt}$ includes magnitude & direction

Average Acceleration Vector = $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$ includes magnitude & direction

WATCH OUT! Even though the above definitions all have arrows, almost no physics book includes the arrows for 1D motion *equations*. For example, under constant acceleration displacement is given by

$$\Delta x\hat{\imath} = (v_{ix}t)\hat{\imath} + \left(\frac{1}{2}a_x t^2\right)\hat{\imath}$$
$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$$

After cancelling the $\hat{\imath}'s$ you are supposed to know, in this instance, Δx implies *displacement*, not *distance*!!!

Usually we just assume right is the positive direction unless lots of things in the problem are going left.
Moving forward implies positive velocity. Moving backward implies negative velocity.
If velocity and acceleration have the same sign the object is speeding up (opposite signs then slowing down).

$v > 0$ AND $a > 0$	moving forward and speeding up
$v > 0$ AND $a < 0$	moving forward and slowing down
$v < 0$ AND $a < 0$	moving backward and speeding up
$v < 0$ AND $a > 0$	moving backward and slowing down

Be careful when an object reverses direction! At turnaround points the velocity is instantaneously zero but not necessarily the acceleration. Furthermore, when an object goes left then right (or up then down) the displacement will partially cancel while the two distances will not. Average velocity and average speed will not be equal in magnitude anymore!!! See below for an example.

The following equations are only valid for CONSTANT acceleration. For problems in which the acceleration changes you need to split the problem into separate parts such that each part has constant acceleration.

$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2 \qquad v_{fx}^2 = v_{ix}^2 + 2a_x\Delta x \qquad v_{fx} = v_{ix} + a_x t \qquad \Delta x = \frac{1}{2}\left(v_{fx} + v_{ix}\right)t$$

People often write the kinematics equations the following way instead:

$$x_f = x_i + v_{0x}t + \frac{1}{2}a_x t^2 \qquad v_x^2 = v_{0x}^2 + 2a_x\Delta x \qquad v_x = v_{0x} + a_x t \qquad \Delta x = \frac{1}{2}(v_x + v_{0x})t$$

Notice that these equations are exactly the same if we make the identifications below:

$$\Delta x = x_f - x_i \qquad v_{fx} = v_x \qquad v_{ix} = v_{0x}$$