9.33 1D Calculus Suppose a rod with length L has linearly increasing density. **This is an example of non-uniform density.** Based on the coordinate system shown, density is given by $\lambda = \alpha x$.

a) Based on the density function, where do you expect the center of mass to be: in the middle, towards the right end, or towards the left end? Hint: which end of the rod should be heavier based on λ?
b) What are the units of α?
c) Determine the total mass of the object in terms of α and L. Total mass is given by $M = \int dm$. Remember to check the units. The answer to part b) is used to help you check your units to your answer for part c).
d) Determine the horizontal location of center of mass of the object in terms of L (is it $L/2$, $L/3$, etc).
e) Think: compare your result to a right triangle with the point at the origin. <u>*Why*</u> are the results the same?

9.33$\frac{1}{3}$ Suppose the previous questions was modified. You may still assume the total length of the rod is L. This time, however, the rod is centered on the origin. Furthermore, the density is changed to $\lambda = \alpha x^2$.

a) What must change in your previous work? The density equation changes…but does anything else change?
b) Why is it physically unreasonable to use the as the density equation $\lambda = \alpha x$ for this scenario?

9.33$\frac{2}{3}$ A non-uniform, thin rod has a mass distribution that is given by $\mu(x) = ax^n$. Here n is unknown number (greater than 1) and a is an unknown constant. This density assumes the coordinate system is aligned with the left end of the rod as shown in the figure. The rod has total length $L = 1.00$ m and mass $m = 0.100$ kg. By balancing the object, you determine the center of mass is 14.29 cm from the right end of the rod (shown by the black **x** in the figure). Figure not to scale. Determine the exponent n. Express your answer as a decimal with 3 sig figs.