

**1D MOTION**

**Position Vector** =  $\vec{x}$  or  $\vec{r}$  = location (magnitude & direction) relative to the origin

**Displacement Vector** =  $\Delta\vec{x}$  or  $\Delta\vec{r}$  = CHANGE in position (magnitude & direction)

**Distance Scalar** =  $\Delta x$  or  $\Delta r$  **WATCH OUT!** If the object changes direction  $\Delta x \neq \|\Delta\vec{x}\|$ !

**Instantaneous Velocity Vector** =  $\vec{v} = \frac{d\vec{x}}{dt}$  or  $\frac{d\vec{r}}{dt}$  includes magnitude & direction

**Average Velocity Vector** =  $\vec{v}_{avg} = \frac{\Delta\vec{x}}{\Delta t}$  or  $\frac{\Delta\vec{r}}{\Delta t}$  includes magnitude & direction

**Instantaneous Speed** =  $v = \|\vec{v}\|$  = the magnitude only of  $\vec{v}$

**Average Speed** =  $v_{avg} = \frac{\text{Total distance}}{\text{Total time}}$  **WATCH OUT!** If the object changes direction  $v_{avg} \neq \|\vec{v}_{avg}\|$ !

**Instantaneous Acceleration Vector** =  $\vec{a} = \frac{d\vec{v}}{dt}$  includes magnitude & direction

**Average Acceleration Vector** =  $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$  includes magnitude & direction

**WATCH OUT!** Even though the above definitions all have arrows, almost no physics book includes the arrows for 1D motion *equations*. For example, under constant acceleration displacement is given by

$$\Delta x \hat{i} = (v_{ix}t)\hat{i} + \left(\frac{1}{2}a_x t^2\right)\hat{i}$$

$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$$

After cancelling the  $\hat{i}$ 's you are supposed to know, in this instance,  $\Delta x$  implies *displacement*, not *distance*!!!

Usually we just assume right is the positive direction unless lots of things in the problem are going left.

Moving forward implies positive velocity. Moving backward implies negative velocity.

**If velocity and acceleration have the same sign the object is speeding up (opposite signs then slowing down).**

$v > 0$ AND $a > 0$	moving forward and speeding up
$v > 0$ AND $a < 0$	moving forward and slowing down
$v < 0$ AND $a < 0$	moving backward and speeding up
$v < 0$ AND $a > 0$	moving backward and slowing down

**Be careful when an object reverses direction!** At turnaround points the velocity is instantaneously zero but not necessarily the acceleration. Furthermore, when an object goes left then right (or up then down) the displacement will partially cancel while the two distances will not. Average velocity and average speed will not be equal in magnitude anymore!!! See below for an example.

**The following equations are only valid for CONSTANT acceleration. For problems in which the acceleration changes you need to split the problem into separate parts such that each part has constant acceleration.**

$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2 \quad v_{fx}^2 = v_{ix}^2 + 2a_x \Delta x \quad v_{fx} = v_{ix} + a_x t \quad \Delta x = \frac{1}{2}(v_{fx} + v_{ix})t$$

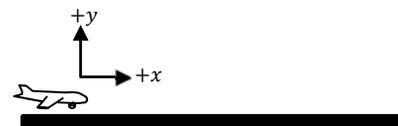
People often write the kinematics equations the following way instead:

$$x_f = x_i + v_{0x}t + \frac{1}{2}a_x t^2 \quad v_x^2 = v_{0x}^2 + 2a_x \Delta x \quad v_x = v_{0x} + a_x t \quad \Delta x = \frac{1}{2}(v_x + v_{0x})t$$

**Notice that these equations are exactly the same if we make the identifications below:**

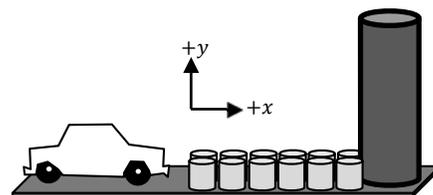
$$\Delta x = x_f - x_i \quad v_{fx} = v_x \quad v_{ix} = v_{0x}$$

**2.1<sup>1</sup>/<sub>4</sub>** Suppose the landing speed for a huge jet airplane is about 170 miles per hour. Assume the plane is to slow to a stop at a rate of  $0.175g$ . Assume this rate is constant from the instant the plane touches down until the instant it comes to a full stop. Here  $g$  is the magnitude of freefall acceleration near earth's surface. Note: the numerical value is always  $g = +9.8 \frac{\text{m}}{\text{s}^2}$  (plus sign added for emphasis... $g$  is always positive).



- Is  $g$  gravity?
- Is  $g$  the *acceleration* due to gravity?
- Under what circumstances does  $g = -9.8 \frac{\text{m}}{\text{s}^2}$ ?
- How far will the plane travel as it slows from touchdown to full stop? Answer in both meters and miles.
- How long does it take to stop? Assume this is elapsed time between touchdown and stop.

**2.1<sup>1</sup>/<sub>2</sub>** Perhaps you have seen those yellow barrels near a highway underpass designed to prevent death in the event of a car crash? It is my understanding they are called sand barrel arrays. To get a feeling for how these might work, let's assume the barrels cause a constant acceleration even though the acceleration actually changes quite a bit during the collisions. Assume a car comes in at  $35.0 \frac{\text{m}}{\text{s}} \approx 80 \text{ mph}$  and stops in a distance of 9.00 m.



- Determine the time to stop.
- Determine the acceleration in  $\frac{\text{m}}{\text{s}^2}$  using the coordinate system shown.
- For comparison, assume no yellow barrels were present. If you hit the concrete you would come to a full stop in perhaps 1.5 m...approximately the distance between your face and the concrete structure at the moment your bumper impacts the concrete. Compute the acceleration for this case.
- People like to use  $g$ 's when discussing acceleration. Convert your accelerations from the previous parts to  $g$ 's by dividing each term by  $g$ . Example:  $a = 8.25 \frac{\text{m}}{\text{s}^2}$  implies  $\frac{a}{g} = 0.842$  implies  $a = 0.842g$ . Notice you don't have to include units when describing accelerations in terms of  $g$ 's (because the units are hidden inside...  $g = 9.8 \frac{\text{m}}{\text{s}^2}$ ).

Note: the acceleration a human can survive without death varies depending on several factors including the duration of acceleration and each particular human's body. Try a web search to learn more if you are interested.

Side note: In practice the barrels are not all filled with the same amount of sand. The first barrels to be impacted might be about half full while the barrels adjacent to the concrete structure are completely full. In real life we would not expect acceleration to be constant.

**2.1<sup>3</sup>/<sub>4</sub>** *Instructor tip:* use blue marker for first stage symbols and red marker for second stage symbols. A track star runs the 100-meter dash. She accelerated from rest to her top speed at a rate of  $4.75 \frac{\text{m}}{\text{s}^2}$  for the first 15.0 m of the race (figure not to scale). The runner finished the race at her top speed.



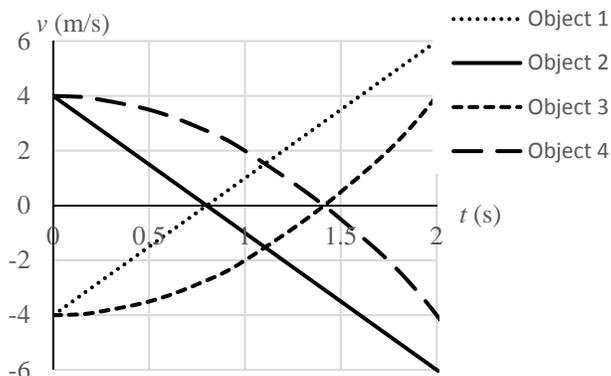
- What is the top speed of the runner?
- What was her time for the race?

**Going Further:** Create a spreadsheet to do the calculations for you. Then mess around with the value used for  $a$  until you get the current women's world record time of about 10.5 seconds.

**Going Further:** In real life, as you get going faster and faster your acceleration should decrease. A slightly better model might be accelerating at  $5.5 \frac{\text{m}}{\text{s}^2}$  for the first 7.5 m, then accelerating at  $4.0 \frac{\text{m}}{\text{s}^2}$  for the next 7.5 m. Try making a spreadsheet to compute the time of the race. See if the time is faster or slower than the initial conditions.

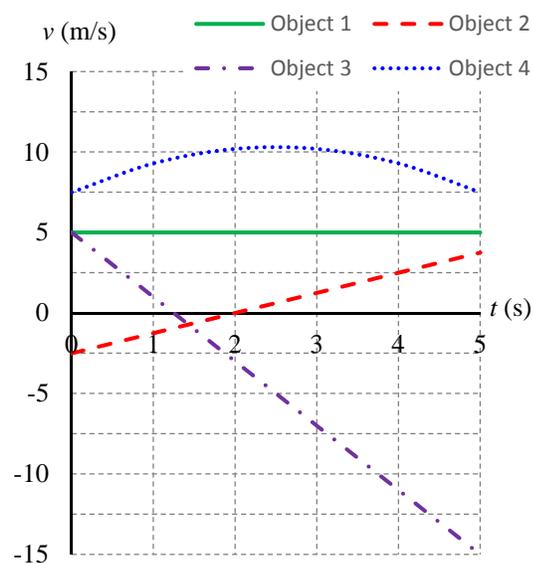
2.29¼ The graphs of velocity versus time are shown for objects moving in one dimension.

- Which object or objects have *constant* acceleration? If no object has *constant* acceleration, write “None”.
- Which object or objects have *positive* acceleration? If no object has *positive* acceleration, write “None”. To be clear, for this part the acceleration could *changing* but it should always be *positive*.
- At what time (or over what time interval) is **object 4 at rest**? Assume we are restricting our discussion to times between 0 and 2.00 s. If no such time or interval occurs write “Never happens”.
- At what time (or over what time interval) is **object 4 speeding up**? Assume we are restricting our discussion to times between 0 and 2.00 s. If no such time or interval occurs write “Never happens”.
- Rank the speeds of the objects at  $t = 2.00$  s from largest to smallest clearly indicating any ties.
- For instance, if you think object 1 is the fastest, objects 2 & 3 are tied for second, and object 4 is slowest you should write  $v_1 > v_2 = v_3 > v_4$ .



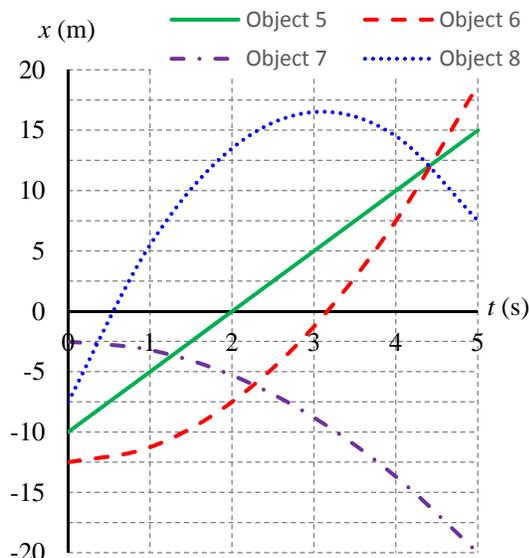
2.29½ Four objects move in one dimension. A plot showing velocity versus time is shown at right for all four objects.

- Rank the initial speeds from smallest to largest clearly indicating any ties.
- Which objects, if any, have non-zero *constant* acceleration?
- Which objects, if any, are not *moving*?
- Which objects, if any, have  $a_x = 0$ ?
- Which objects, if any, reverse direction?
- Rank the final speeds from smallest to largest clearly indicating any ties.
- Monotonically increasing velocity* implies velocity is either constant or increasing. Said another way,  $a_x \geq 0$  for all time. Which objects have monotonically increasing velocity?
- Do any of the objects show monotonically changing *speed*?
- Rank the *displacements* of each object (most positive to most negative).
- Rank the *distance traveled* by each object.



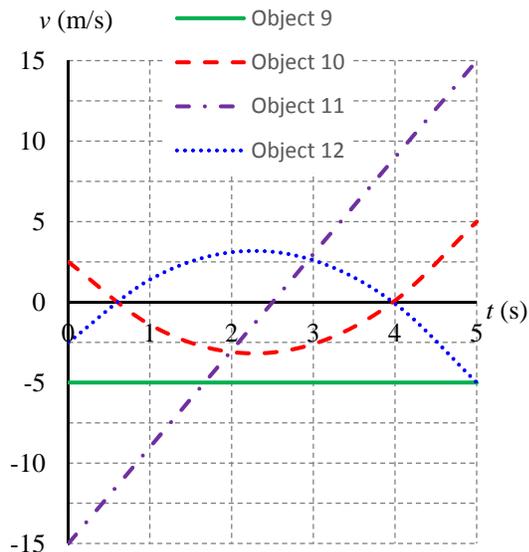
2.29<sup>3/4</sup> Four objects move in one dimension. A plot showing position versus time is shown at right for all four objects.

- Rank the initial speeds from smallest to largest clearly indicating any ties.
- Rank the final velocities from least negative to most negative.
- Which objects, if any, have non-zero *constant* acceleration?
- Which objects, if any, have  $a_x \leq 0$  at all points in time (monotonically *decreasing velocity*)?
- Do any of the objects show monotonically changing *speed*?
- Which objects, if any, have  $a_x = 0$ ?
- Which objects, if any, reverse direction?
- What would the plot of  $x$  vs.  $t$  look like for an object at rest?
- Rank the *displacements* of each object (most positive to most negative).
- Rank the *distance traveled* by each object.



2.29<sup>7/8</sup> Four objects move in one dimension. A plot showing velocity versus time is shown at right for all four objects.

- Rank the initial speeds from smallest to largest clearly indicating any ties.
- Which objects, if any, have non-zero *constant* acceleration?
- Which objects, if any, are not *moving*?
- Which objects, if any, have  $a_x = 0$ ?
- Which objects, if any, reverse direction?
- Rank the final speeds from smallest to largest clearly indicating any ties.
- Which objects, if any, have  $a_x \geq 0$  at all points in time (monotonically *increasing velocity*)?
- Which objects, if any, have  $v_x \geq 0$  at all points in time (monotonically *increasing position*)?
- Do any of the objects show monotonically changing *speed*?



2.29<sup>15/16</sup> Four objects move in one dimension. A plot showing position versus time is shown at right for all four objects.

- Rank the initial speeds from smallest to largest clearly indicating any ties.
- Rank the final velocities from least negative to most negative.
- Which objects, if any, have non-zero *constant* acceleration?
- Which objects, if any, have  $a_x \leq 0$  at all points in time (monotonically *decreasing velocity*)?
- Which objects, if any, have  $v_x \leq 0$  at all points in time (monotonically *decreasing position*)?
- Which objects, if any, have  $a_x = 0$ ?
- Which objects, if any, reverse direction?
- Rank the *displacements* of each object.
- Rank the *distance traveled* by each object.

