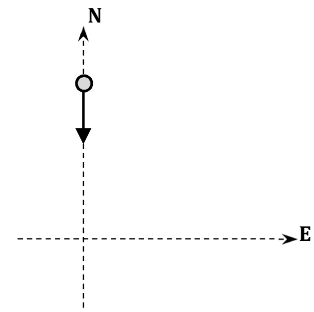
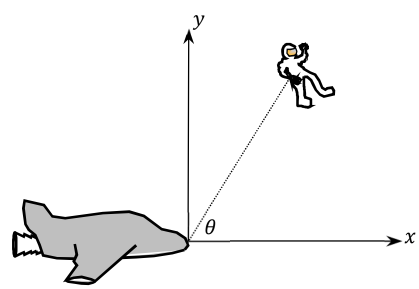
Solutions follow on the next two pages…

A hovercraft travels over a level surface.
As viewed from above, we see the hovercraft (grey dot) moving in two dimensions.
Initially the hovercraft is 2.22 m *north* of the origin, moving *south* at $4.44\,\frac{\text{m}}{\text{s}}$.
The hovercraft accelerates to the *east* at $6.66\,\frac{\text{m}}{\text{s}^2}$.
1a) How much time elapses before the hovercraft crosses the x-axis?
1b) How far is the hovercraft from the origin as it crosses the x-axis?
**1c) How fast is the hovercraft moving as it crosses the x-axis?
1d) What direction is the hovercraft moving as it crosses the x-axis?
Sketch the answer and label an angle in your sketch.

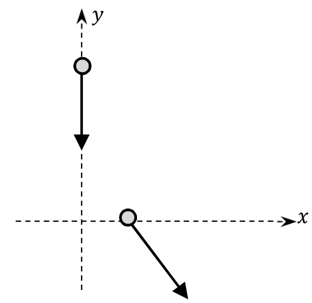


An astronaut is initially 3.33 m from the nose of her ship at angle $\theta = 66.6°$.
She initially moves with speed $1.11\,\frac{\text{m}}{\text{s}}$ to the *left*.
She accelerates *upwards* at $4.44\,\frac{\text{m}}{\text{s}^2}$.
2a) How much time elapses before she crosses the y-axis?
2b) How far is she from the origin as she crosses the y-axis?
**2c) How fast is she moving as she crosses the y-axis?
2d) What direction is she moving as she crosses the y-axis?
Sketch the answer and label an angle in your sketch.



Going further: plot the distance of the astronaut from the nose of the ship as a function of time for the first two seconds of motion (using a 0.10 s time increment).
Hint: first use Excel to generate data for horizontal position & vertical position as functions of time.

1a) We are asked about the instant in time hovercraft passes the horizontal axis.



Δx	?	Δy	-2.22 m
v_{ix}	0	v_{iy}	$-4.44\frac{\text{m}}{\text{s}}$
v_{fx}	?	v_{fy}	$-4.44\frac{\text{m}}{\text{s}}$
a_x	$6.66\frac{\text{m}}{\text{s}^2}$	a_y	0
t	?		

A list of knowns is shown at right.
Notice, in this case, $a_y = 0$.
This implies $v_{fy} = v_{iy}$!

Time can be found using

$$\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$$

$$\Delta y = v_{iy}t$$

$$\boldsymbol{t = \frac{\Delta y}{v_{iy}} = \frac{-2.22\text{ m}}{-4.44\frac{\text{m}}{\text{s}}} = 0.500\text{ s}}$$

1b) When the hovercraft crosses the horizontal axis, horizontal position equals distance from the origin.

$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$$

$$x_f - x_i = \frac{1}{2}a_x t^2$$

Since initial *horizontal* position is zero we know $x_i = 0$.

$$x_f = \frac{1}{2}a_x t^2$$

$$x_f = \frac{1}{2}a_x\left(\frac{\Delta y}{v_{iy}}\right)^2$$

$$\boldsymbol{x_f = 0.83\underline{2}5\text{ m}}$$

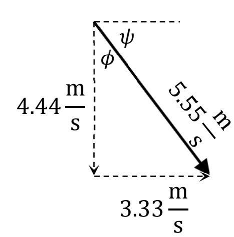
I'm keeping the extra sig fig in case I use this value for the next part…

1c) & 1d) The final two parts are essentially asking for the final velocity vector in polar form.
In particular, the question asking "how fast" is essentially asking for *speed* (not the velocity vector).
One finds

$$v_{fx} = v_{ix} + a_x t = 3.33\frac{\text{m}}{\text{s}}$$

A sketch of the final velocity vector is shown at right.

One finds *speed* $\boldsymbol{v_f = 5.55\frac{\text{m}}{\text{s}}}$ with *heading* $\boldsymbol{\psi = 53.1°}$ S of E or $\boldsymbol{\phi = 36.9°}$ E of S.



x_i	$3.33\cos 66.6°$ $\mathbf{1.323\ m}$	y_i	$3.33\sin 66.6°$ $\mathbf{3.056\ m}$
x_f	0	y_f	?
Δx	$-1.323\ \mathrm{m}$	Δy	?
v_{ix}	$-1.11\,\frac{\mathrm{m}}{\mathrm{s}}$	v_{iy}	0
v_{fx}	$-1.11\,\frac{\mathrm{m}}{\mathrm{s}}$	v_{fy}	?
a_x	0	a_y	$4.44\,\frac{\mathrm{m}}{\mathrm{s}^2}$
t	?		

2a) The list of knowns and unknowns is shown at right.
We are asked to find time to reach the vertical axis.
Reaching the vertical axis corresponds to reaching $x_f = 0$.
Because $a_x = 0$ we know $v_{fx} = v_{ix}$ and can also write

$$\Delta x = v_{ix} t$$

$$t = \frac{\Delta x}{v_{ix}}$$

$$\boldsymbol{t = 1.192\ \mathrm{s}}$$

2b) When the astronaut crosses the vertical axis, the distance from the origin corresponds to the astronaut's vertical position.

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

$$\Delta y = \frac{1}{2} a_y t^2$$

$$y_f - y_i = \frac{1}{2} a_y t^2$$

$$y_f = y_i + \frac{1}{2} a_y t^2$$

$$y_f = y_i + \frac{1}{2} a_y \left(\frac{\Delta x}{v_{ix}}\right)^2$$

$$y_f = 3.056\ \mathrm{m} + \frac{1}{2}\left(4.44\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(\frac{-1.323\ \mathrm{m}}{-1.11\,\frac{\mathrm{m}}{\mathrm{s}}}\right)^2$$

$$\boldsymbol{y_f = 6.21\ \mathrm{m}}$$

2c) & 2d) We are essentially asked to find the final velocity in polar form (speed and direction).
In particular, the question asking "how fast" is essentially asking for *speed* (not the velocity vector).

$$v_{fy} = v_{iy} + a_y t$$

$$v_{fy} = a_y t$$

$$v_{fy} = a_y \left(\frac{\Delta x}{v_{ix}}\right)$$

$$v_{fy} = 5.2\underline{9}2\,\frac{\mathrm{m}}{\mathrm{s}}$$

A sketch of the astronaut's *velocity* (as she crosses the vertical axis) is shown at right.

Notice one finds *speed* $v_f = 5.41\,\frac{\mathrm{m}}{\mathrm{s}}$ with *heading* $\phi = 78.2°$ or $\psi = 11.8°$.