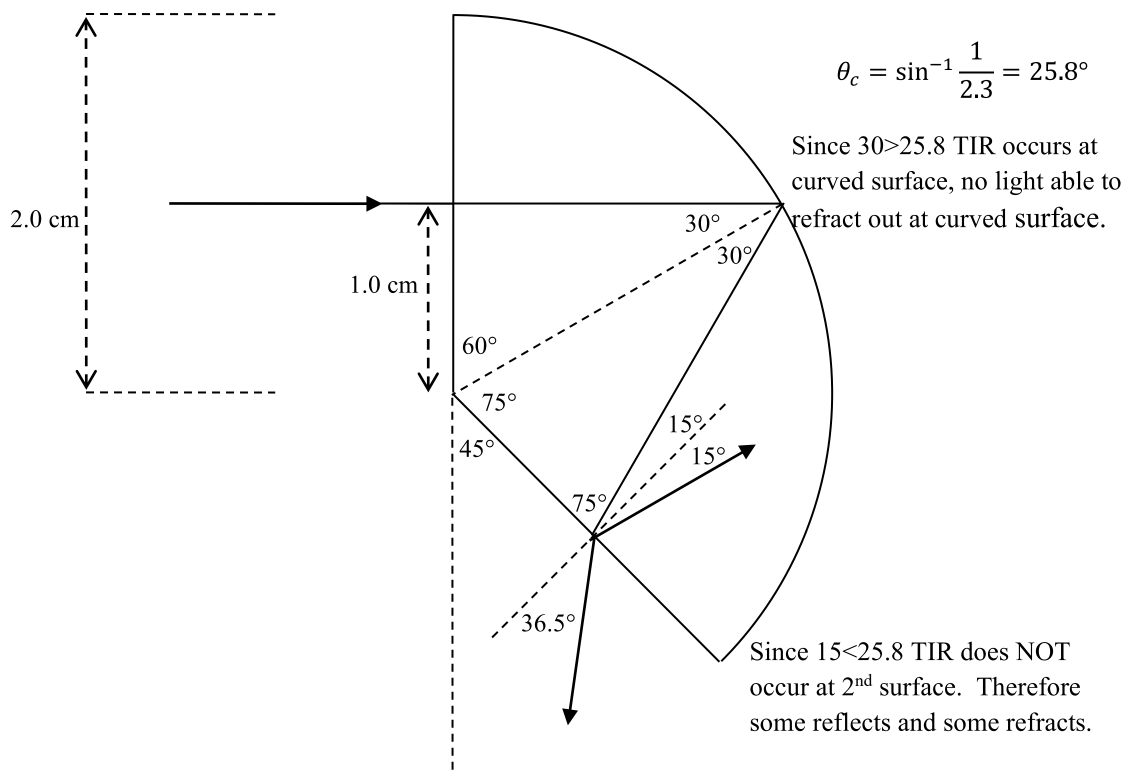
Disclaimer: Also practice single slit, double slit, diffraction grating, and polarizers (and anything else I forgot). Just putting this much together has fried my brain…

1) A crystal with index of refraction 2.3 is cut into the unusual shape shown in the figure. The shape is essentially a semi-circle with a 45° slice removed. The radius is 2.0 cm. Light is normally incident on the vertical face at the midpoint of that face.

a) Sketch any reflected and refracted rays for the first time light reaches the curved surface. Clearly label the figure with all angles labeled in a clearly legible fashion. Show all work for credit.
b) Sketch any reflected and refracted rays for the first time the light reaches the slanted surface.
c) Lastly, would things differ significantly for the curved surface if the crystal was submerged in water with index 1.33?



Note: if the crystal is immersed in water the TIR critical angle would change to

$$\theta_c = \sin^{-1}\frac{1.33}{2.3} = 35.3°$$

That means light would refract at the curved surface as well as the 45 degree surface.

3) When dealing with mirrors, only a concave mirror can create a real image. Note: it can also create a virtual image as well depending on object distance. This information clarifies that the 2nd sentence can only be possible when the object is in front of the concave side of the bowl. For a concave mirror we know that $f=+R/2$ where R is the radius of curvature of the bowl. Also, for a real image we know that q is a positive # by the sign conventions. Combining this information into equation form we get

$$\frac{1}{p}+\frac{1}{0.80\text{m}}=\frac{1}{f}=+\frac{2}{R}$$

Turning the bowl around the curvature will flip sign; it is now convex so $f=-R/2$. A convex mirror makes only virtual images so this part checks with the wording of the problem. Furthermore, we will know that q is now a negative number by sign convention. This gives the equation

$$\frac{1}{p}+\frac{1}{-0.40\text{m}}=\frac{1}{f}=-\frac{2}{R}$$

Use algebra to show $R=16/15\approx1.07$m and $p=1.6$ m. Before moving on, consider when a convex mirror forms a virtual image instead of a real image. We know that for virtual images q must be negative. Use the $1/f$ eqt'n and verify this can only occur if $p<f$. Notice that, since $f=R/2\approx53.3$cm we have not yet entered that domain. This makes checks out with our problem as we were told the concave mirror formed a real image (corresponding to $p>f$).

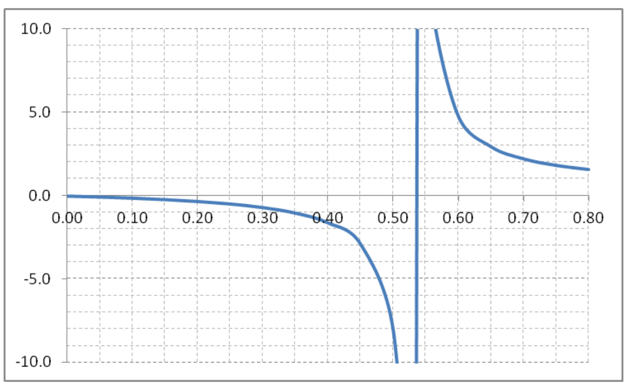
For the ray diagrams, first check with your neighbors. See your instructor if there are any unusual discrepancies. For extra practice, make sure you can do an example for each type of mirror when $p<f$! Then repeat all of those cases for both types of lenses.

Lastly, show that using the $1/f$ equation and solving for q you obtain

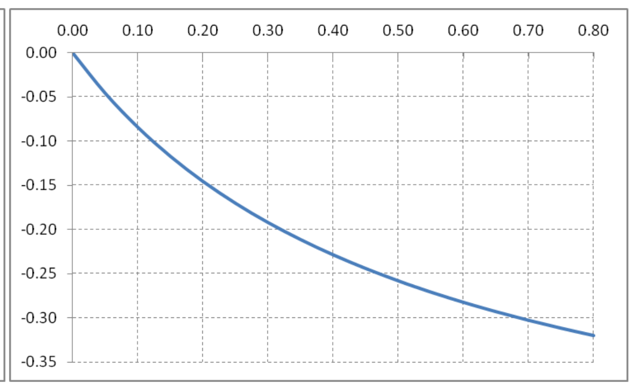
$$q=\frac{fp}{p-f}$$

Plugging in numbers gives:

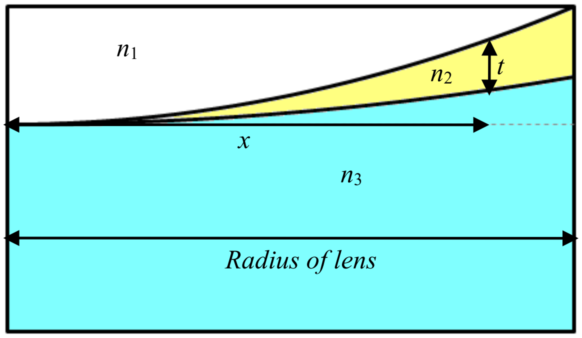
$$q=\frac{\frac{8}{15}p}{p-\frac{8}{15}}=\frac{8p}{15p-8}\text{ for the concave side and }q=\frac{-\frac{8}{15}p}{p+\frac{8}{15}}=\frac{-8p}{15p+8}\text{ for the convex side}$$



Here we see the vertical asymptote at about $p=0.533$ m; the asymptote occurs when $p=f$ for a concave mirror.



For the convex mirror no such asymptote exists.



4) A cross-sectional view (right half only) of an unusual camera lens is shown. The lens is designed such that the surface between materials 1 and 2 is described by the equation $y=ax^2$ where y is the height above the horizontal axis shown. The surface between materials 2 and 3 is the top of a plano-concave lens with radius of curvature 2.00 m. The radius of the lens is 30.0 cm.

a) a has units of 1/m

b) $\lambda/2$ shift at the 1-2 interface, no shift at the 2-3 interface

c) Assume that y_2 is the vertical distance above the center of lens for the 1-2 interface. Likewise y_1 is the vertical distance above the center of the lens for the 2-3 interface. Here we find that

$$t = y_2 - y_1 = ax^2 - R + \sqrt{R^2 - x^2} = ax^2 - R + R\sqrt{1 - \frac{x^2}{R^2}}$$

Note: since for almost all x in this problem $\frac{x^2}{R^2} \ll 1$ the above equation simplifies to

$$t = ax^2 - R + R\left(1 - \frac{x^2}{2R^2} + order\left(\frac{x^2}{R^2}\right)^2\right) \approx x^2\left(a - \frac{1}{2R}\right)$$

One sees immediately that a must be greater than $1/2R$ for this problem to make sense physically. Since R=2.00m we find the restriction on values of a to be a>0.25m^{-1}.

d) We expect (with one phase reversal) that the equation for bright fringes is

$$2n_2t = \left(m + \frac{1}{2}\right)\lambda$$

In this problem we want the same thickness to be valid for both red and blue. This gives rise to

$$\left(m_r + \frac{1}{2}\right)\lambda_r = \left(m_b + \frac{1}{2}\right)\lambda_b$$

$$\left(m_r + \frac{1}{2}\right)\frac{\lambda_r}{\lambda_b} = m_b + \frac{1}{2}$$

The left side simplifies to $\left(m_r + \frac{1}{2}\right)\frac{750\text{nm}}{450\text{nm}} = \left(m_r + \frac{1}{2}\right)\frac{5}{3} = \frac{5}{3}m_r + \frac{5}{6}$

Giving us $\frac{5}{3}m_r + \frac{5}{6} = m_b + \frac{1}{2}$

Using a common denominator & $some\ algebra\ we\ find$ $\frac{10m_r + 2}{6} = m_b$

For this to work one needs the quantity $10m_r + 2 = (a\ multiple\ of\ 6)$. Checking starting with the lowest numbers for m_r and working upward one quickly discovers (through trial and error) that m_r=1 and m_b=2. This corresponds to a thickness of $t \approx 352$nm. Plugging this into our approximate result for $t(x)$ we find

$$352\text{nm} = x^2(a - 0.25)$$

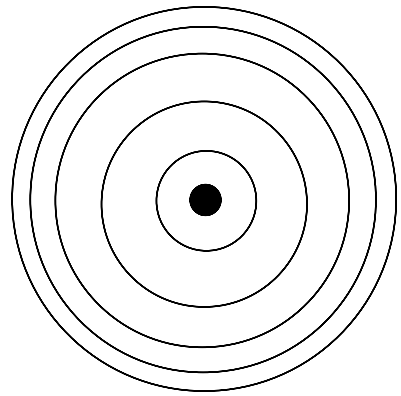
$$x = \sqrt{\frac{351.6 \times 10^{-9}\text{m}}{(a - 0.25\text{m}^{-1})}}$$

Suppose a = 0.26 m^{-1}. Then one finds that

$$x = \sqrt{\frac{351.6 \times 10^{-9}\text{m}}{(0.01\text{m}^{-1})}} = 5.93\text{mm from the center, clearly } \frac{x^2}{R^2} \ll 1 \text{ is valid}$$

Note, we expect the red pattern to be more spaced out than the blue pattern (it has a larger wavelength). That implies that at whatever thickness that red and blue both experience a strong reflection we expect m_b>m_r which gives a reality check on this result. This is also why I choose to solve for m_b instead of m_r.

e) Fringes most closely spaced at left side where thickness of the film changes least rapidly. From the top, the pattern of fringes will look exactly like a topographical map of the film. Center will be black for a 1 phase reversal thin film interference pattern.



f) That thickness of the upper film is given by

$$t = t_{max} - ax^2 = ar^2 - ax^2 = a(0.30\text{m})^2 - ax^2 = a(0.09\text{m}^2 - x^2)$$

Going from air into n_1 and from n_1 into n_2 will both have a phase reversal equivalent to $\lambda/2$. The bright and dark conditions will be opposite that of a single phase reversal problem.

g) Apply the liquid then spin the lens at a constant rate until it dries. A faster spin will cause the film to pile up more on the outside (effectively making for a larger value of *a*). Unusual to note: a type of mirror is made from liquid mercury in this manner for a special type of telescope that only looks straight up (called a zenith telescope). This technique can be used to make a large high quality mirror on the cheap; don't drink the mirror!

BONUS PUZZLE: To understand the bouncing laser beam demonstration google "Harvard bouncing laser", click the first link, then scroll down the page approximately halfway.