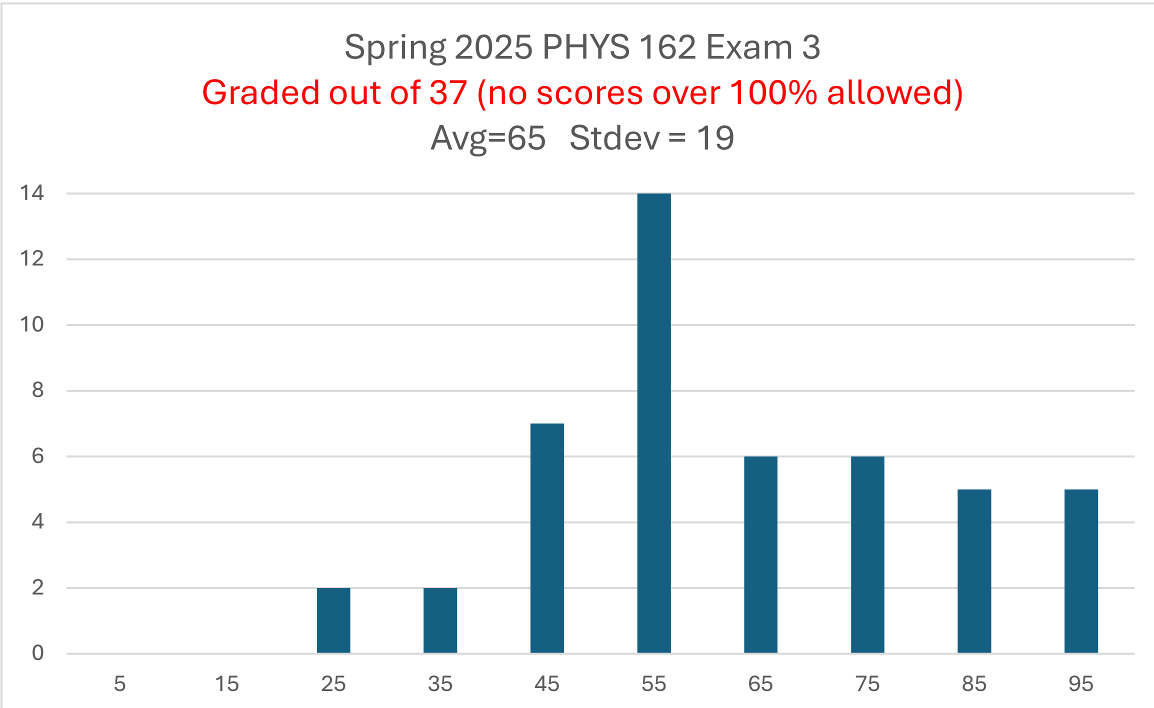
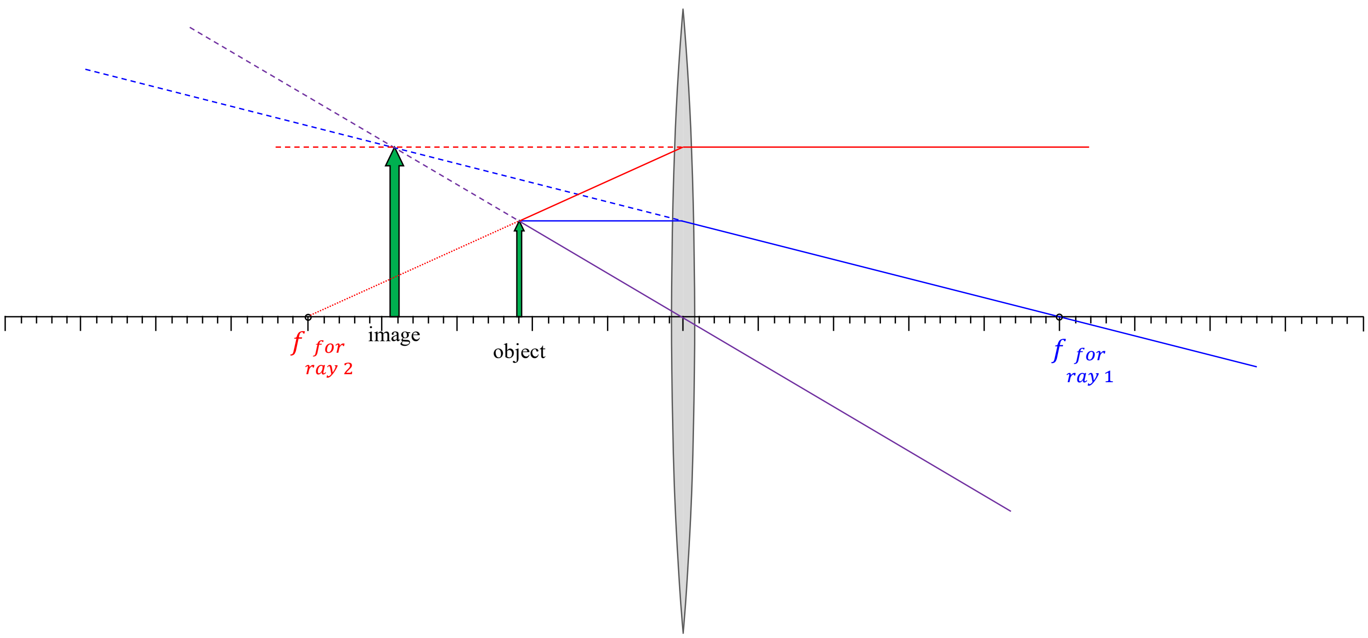
162sp25t3aSoln

Distribution on this page.
Solutions begin on the next page.



1a) From the figure we know object distance is $d_0 = +11.0$ cm.
From the problem statement we knew focal length was $f = +25.0$ cm.
We know the focal length is positive because it was a bi-convex lens (fat in the middle, positive focal length).
Using the $\frac{1}{f}$ equation I found $d_i = -19.6$ cm and $M = +1.79$ which both match well with the diagram.

The final image is upright, enlarged, and virtual.



2a) Image is 2.22 times smaller and upright. Magnification is between 0 & +1.

$$M = +\frac{1}{2.22} = 0.45\underline{0}45$$

2b) Use the $\frac{1}{f}$ equation to solve for d_i.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{d_o - f}{d_o f}$$

$$d_i = \frac{d_o f}{d_o - f}$$

Use this expression to eliminate d_i in the equation

$$M = -\frac{d_i}{d_o}$$

$$M = -\frac{\frac{d_o f}{d_o - f}}{d_o}$$

$$M = -\frac{f}{d_o - f}$$

Now solve for d_0:

$$d_o - f = -\frac{f}{M}$$

$$d_0 = f - \frac{f}{M}$$

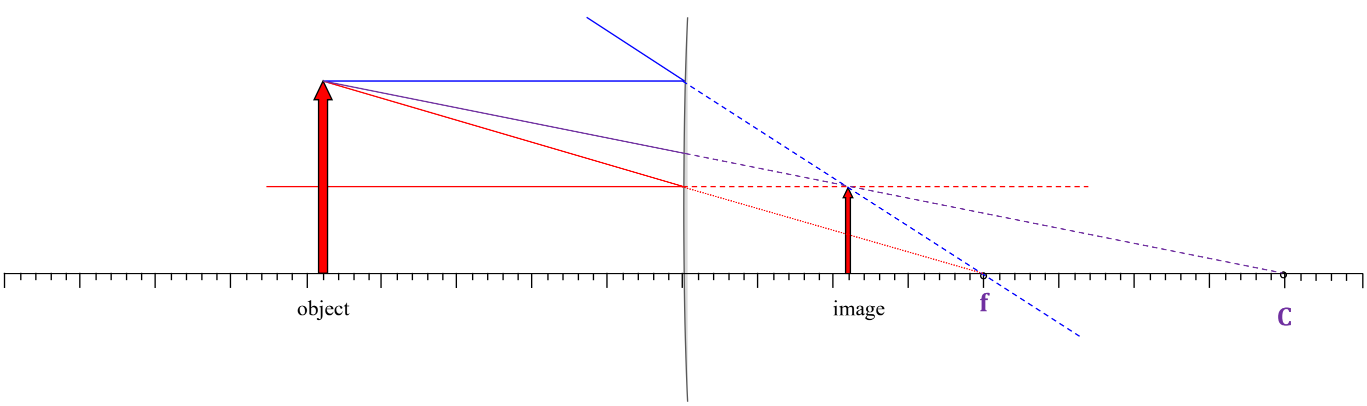
$$d_o = f\left(1 - \frac{1}{M}\right)$$

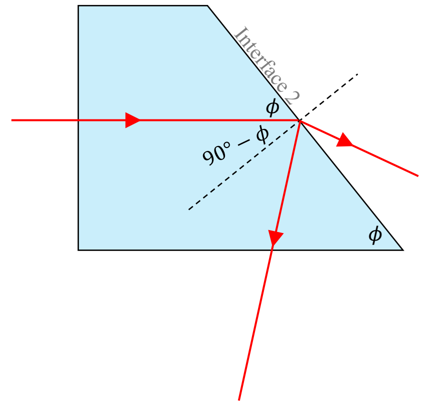
Finally, for a *convex* mirror we know $f = \frac{R}{2} = -20.0$ cm.

Note: a convex mirror has a *negative* radius (and thus *negative* focal length) according to sign convention.

$$d_0 = (-20.0 \text{ cm})(1 - 2.22) = 24.4 \text{ cm}$$

While not required, I did a quick ray diagram to check my work & caught an initial typo! This final diagram is in good agreement with the math.





3a) We are told TIR does NOT occur at interface 2.
Some light is reflected and some light is transmitted.
The light transmitted goes into a lower index medium.
Light is bent *away* from the normal as it enters a lower index medium.
The light ray entering air is deflected downwards.

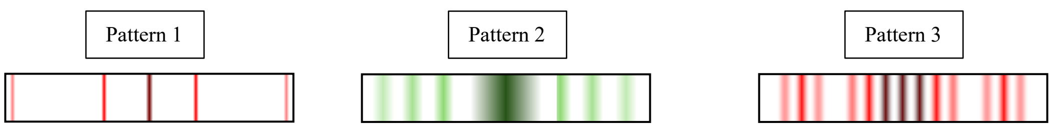
3b) As seen in lab, **blue light is refracted more than red light.**

3c) To experience TIR, we require two things:

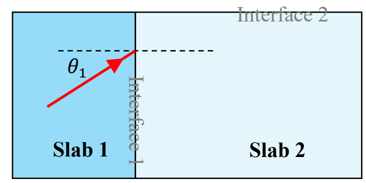
1) Light must be travelling in a higher index medium and hit an interface with a lower index medium.
2) The incidence angel must be *larger* than the critical angle given by $\theta_c = \sin^{-1}\left(\frac{n_{lo}}{n_{hi}}\right)$.

From the geometry of this scenario, we see *decreasing* ϕ causes a larger angle of incidence.
We require $\phi' < \phi$ to cause TIR at interface 2 of the new prism.

4a) Pattern 1. A diffraction grating has so many grooves we typically only see a few very narrow bright bands in the interference pattern it produces. Pattern 2 looks like a single slit. Pattern 3 looks like a double slit. You notice the single slit pattern has a central max which is twice as wide as the bright fringes.



4b) We see the central portion of pattern 3 has equally spaced bright fringes.
This looks like a double slit pattern.
We know each slit of the double slit pattern has some finite width.
Almost certainly the gaps in the pattern are caused by single slit diffraction effects are most likely extinguishing the fourth bright fringe.



5a) Slab 1 is made from cubic zirconia with index of refraction 2.14.
Slab 2 is made from an unknown material with index 1.450.

When light enters a medium with a lower index of refraction, it is possible is could experience total internal reflection. Unfortunately, without knowing angle θ_1, we cannot know if the input ray will TIR.

The best answer is "Impossible to determine without knowing θ_1."

Note: if we did know $\theta_1 < \theta_c$, then we would know some light reflects at interface 1 and some refracts into slab 2. Refracted light is bent *away* from the normal when entering a medium with *lower* index of refraction.

5b) **<u>At interface 2</u>**, we can determine the critical angle for TIR using

$$n_{slab\ 2} \sin\theta_c = n_{surrounding} \sin 90°$$

$$\theta_c = \sin^{-1}\left(\frac{n_{surrouding}}{n_{slab\ 2}}\right)$$

$$\theta_c = \sin^{-1}\left(\frac{1.234}{1.450}\right)$$

$$\boldsymbol{\theta_c = 58.\underline{32}°}$$

5c) I drew the picture at right to help me determine the angles involved.
I started at interface 2.
I assumed the angle to the normal was the critical angle from part b.
This helped me determine

$$\theta_2 = 90.00° - \theta_c = 31.\underline{68}°$$

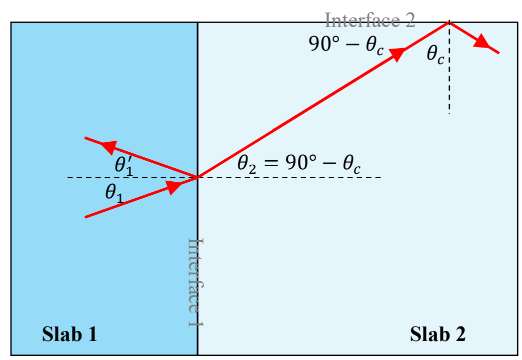
Next I used Snell's law to determine θ_1.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\sin\theta_2\right)$$

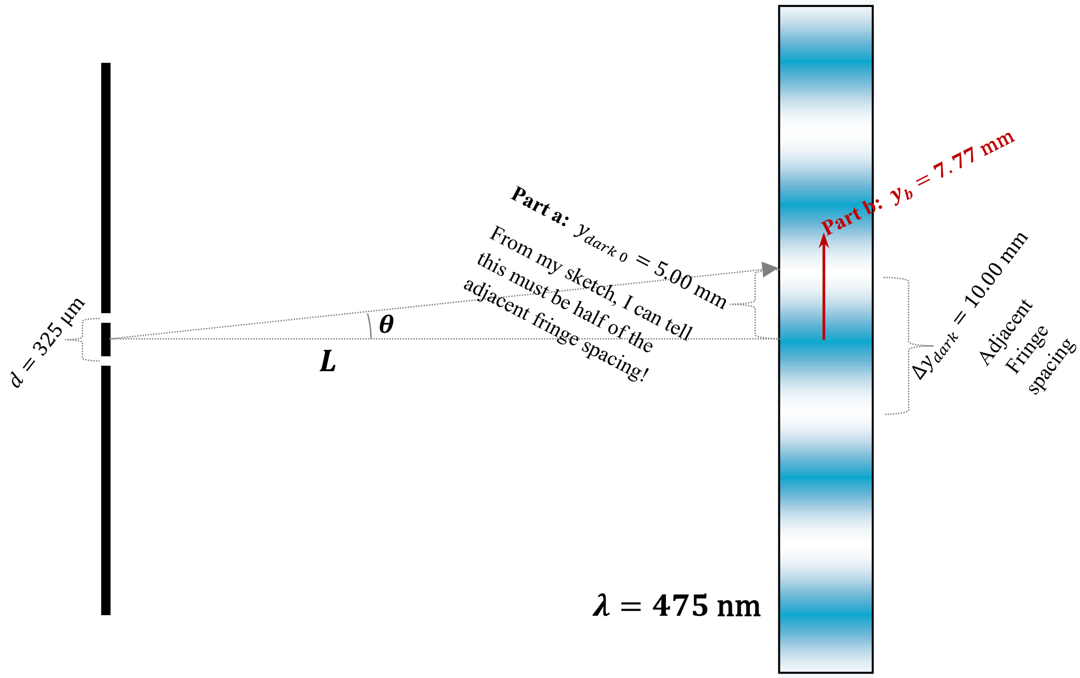
$$\theta_1 = \sin^{-1}\left(\frac{1.450}{2.14}\sin 31.\underline{68}°\right)$$

$$\boldsymbol{\theta_1 = 20.8°}$$



5d) I'm going to imagine I used a slightly *larger* angle for θ_1 and think through the consequences.
A slightly *larger* value of θ_1 would cause a slightly *larger* value of θ_2.
This would cause the light ray to impact interface 2 at a slightly *smaller* angle to the normal.
TIR would no longer occur at this slightly *smaller* angle to the normal at interface 2!
Conversely, a *smaller* value for θ_1 causes a *larger* impact angle to the normal at interface 2 (TIR still occurs).
The value of θ_1 determined in part 5c must be a *maximum* angle to cause TIR at interface 2.

6a) The first thing I would do is draw a sketch and label it with the parameters given in the question. The key insight I gained from making the sketch: the zeroth dark fringe must be located at $y = 5.00$ mm above the central max! I know this because the spacing between the two zeroth order dark fringes is 10.00 mm. This is exactly how we measured y in the lab!



$$\Delta r_{dark} = (odd\ \#)\frac{\lambda}{2}$$

$$\Delta r_{dark} = \left(m + \frac{1}{2}\right)\lambda \quad \text{for } m = 0, 1, 2, \ldots$$

$$d \sin\theta = \left(0 + \frac{1}{2}\right)\lambda \quad \text{use } m = 0 \text{ for } 0^{th} \text{ dark fringe}$$

Use the small angle approximation then verify it is small after the fact. If not small, recompute using $\sin\theta = \frac{y}{\sqrt{y^2+L^2}}$.

$$\frac{d\, y_{dark\ 0}}{L} = \frac{\lambda}{2} \quad \text{for } 0^{th} \text{ dark fringe}$$

$$L = \frac{2d\, y_{dark\ 0}}{\lambda}$$

WATCH OUT for all those different prefixes when you are plugging in numbers!

$$L = \frac{2(325 \times 10^{-6}\text{ m})\,(5.00 \times 10^{-3}\text{ m})}{475 \times 10^{-9}\text{ m}} = 6.8\underline{4}2\text{ m}$$

Notice $y = 5.00$ mm $\ll L = 6.84$ m. Using the small angle approximation is valid. No need to recompute.

6b) Use the intensity equation for double slits with $y = 7.77$ mm.

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d y}{\lambda L} = \frac{2\pi(325 \times 10^{-6}\text{ m})\,(7.77 \times 10^{-3}\text{ m})}{(475 \times 10^{-9}\text{ m})(6.8\underline{4}2\text{ m})} = 4.8\underline{8}2\text{ rad} = 27\underline{9}.7°$$

WATCH OUT! At the central max $\phi = 0$. We are told $I = 0.888\frac{\text{W}}{\text{m}^2}$. Notice this implies $4I_0 = 0.888\frac{\text{W}}{\text{m}^2}$ (*not* $I_0 = 0.888\frac{\text{W}}{\text{m}^2}$).

$$I = 4I_0\cos^2\frac{\phi}{2} = \left(0.888\frac{\text{W}}{\text{m}^2}\right)\cos^2\left(\frac{27\underline{9}.7°}{2}\right) = \mathbf{0.519\frac{W}{m^2} = 519\frac{mW}{m^2}}$$

6c) Using a *smaller slit spacing* (smaller d) with the same λ & L causes the fringe spacing to *increase*!

7a) **Almost no reflection at $y = 0$.**

The problem wording indicates the sapphire wedge is a thin film.
I use the equation sheet to look up the indices of refraction involved.
While we do not know the exact index of refraction of crown glass, we do know it is smaller than sapphire.

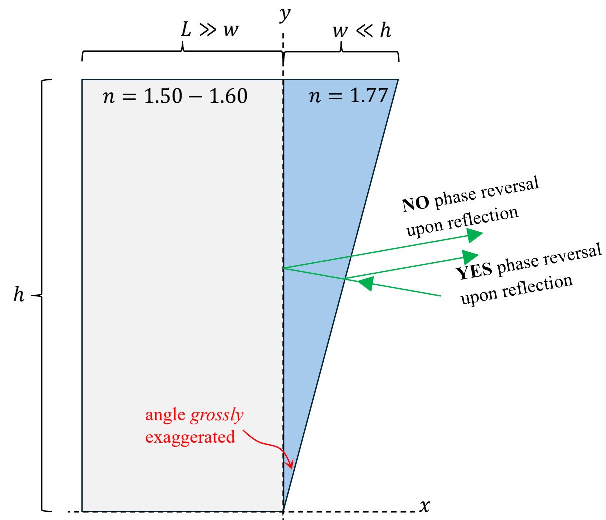


Figure not to scale.

We expect one phase reversal (one $\frac{\lambda}{2}$ flip).

Constructive	Destructive
$2n_{film}t = \left(m + \frac{1}{2}\right)\lambda$	$2n_{film}t = m\lambda$

Both equations use $m = 0, 1, 2, \ldots$

At $y = 0$, thickness is zero.
The destructive condition matches $t = 0$ for $m = 0$.

7b) **Expect a pattern of fringes with uniform spacing as y increases.**

Thickness is changing. We expect fringes.
Thickness changes at a constant rate. We expect fringe spacing is the same everywhere.

7c) Before plugging in numbers, I can tell I need to determine thickness as a function of vertical position y.
I notice thickness varies linearly. I write

$$t = slope \cdot y + intercept$$

I'm going to try the following then confirm it produces the correct results at $y = 0$ & $y = h$.

$$t = \frac{w}{h}y + 0$$

CHECK: When $y = 0$ I get $t = \frac{w}{h}0 = 0$. When $y = h$ I get $t = \frac{w}{h}h = w$. Looks good.

NOTE: you could also say the equation of the line is $y = slope \cdot x + intercept = \frac{h}{w} \cdot x$ then solve for thickness $x = \frac{w}{h} \cdot y$.

NOTICE! The parameter L has nothing to do with the problem. The thickness of the glass is unimportant.

Using the bright fringe (constructive) equation gives

$$2n_{film}t = \left(m + \frac{1}{2}\right)\lambda$$

$$2n_{film}\frac{w}{h}y = \left(m + \frac{1}{2}\right)\lambda$$

$$w = \frac{\left(m + \frac{1}{2}\right)\lambda h}{2n_{film}y}$$

WATCH OUT! Because the equation starts with integer $m = 0$, the 44th bright fringe corresponds to $m = 43$!
This is similar to a list in Python where the index of a list starts with zero!
I'm changing all length to meters so I don't screw up the prefixes.

$$w = \frac{\left(43 + \frac{1}{2}\right)(555 \times 10^{-9}\ \text{m})(0.200\ \text{m})}{2(1.77)(0.0325\ \text{m})}$$

$$\boldsymbol{w = 41.\underline{9}6\ \mu\text{m}}$$

8) The initial intensity is I_0 polarized vertically.

$$I_1 = I_0 \cos^2\theta$$

Notice I_1 is angled $90° - \theta$ from the 2nd polarizer.

$$I_2 = I_1 \cos^2(90° - \theta)$$

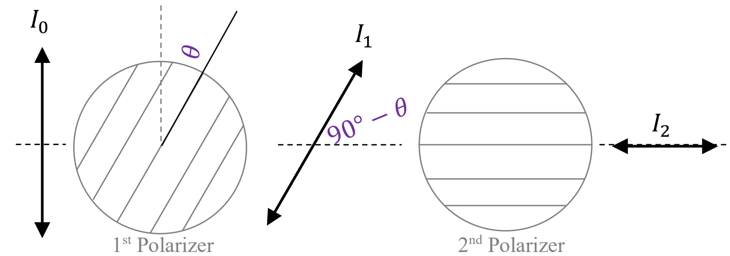
Now plug in I_1.

$$I_2 = (I_0 \cos^2\theta)\cos^2(90° - \theta)$$

Notice this calculation implies we should get identical results for both θ & $(90° - \theta)$.

I rewrote this result using $\cos(90° - \theta) = \sin\theta$.

$$I_2 = I_0 \cos^2\theta \sin^2\theta$$



Before plugging in the trig identities, use the given information $I_2 = 0.1234 I_0$. Note: this is essentially a 3 sig fig number.

$$0.1234 I_0 = I_0 \cos^2\theta \sin^2\theta$$

$$0.1234 = \cos^2\theta \sin^2\theta$$

This is screaming out for a trig identity. Below are the solution styles I saw used during the test.

Use $2\sin\theta\cos\theta = \sin 2\theta$	**Use Reduction Formulas**	**Use $\sin^2\theta = 1 - \cos^2\theta$**
$0.1234 = \cos^2\theta\sin^2\theta$ $0.1234 = \frac{1}{4}\sin^2 2\theta$ $\sin^2 2\theta = 0.4936$ $\sin 2\theta = \pm 0.70257$ $2\theta = \pm 44.633°$ $\boldsymbol{\theta = \pm 22.3°}$ Recall this result is also valid for $90° - \theta$: $\boldsymbol{\theta = \pm 67.7°}$ This gives all possible correct results between $-90°$ & $+90°$.	$0.1234 = \cos^2\theta\sin^2\theta$ $0.1234 = \left(\frac{1+\cos 2\theta}{2}\right)\left(\frac{1-\cos 2\theta}{2}\right)$ $0.1234 = \frac{1}{4}(1 - \cos^2 2\theta)$ $0.4936 = 1 - \cos^2 2\theta$ $\cos^2 2\theta = 0.5064$ $\cos 2\theta = \pm 0.71162$ $2\theta = 44.63°$ **OR** $135.37°$ $\boldsymbol{\theta = 22.3°}$ **OR** $\boldsymbol{67.7°}$ Add 90° to each θ to get angles between 90° & 180°	$0.1234 = \cos^2\theta(1 - \cos^2\theta)$ Let $x = \cos^2\theta$. $0.1234 = x(1 - x)$ $x^2 - x + 0.1234 = 0$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(0.12324)}}{2(1)}$ $x = 0.85581$ **OR** 0.14419 $\cos\theta = 0.9251$ OR 0.3797 $\boldsymbol{\theta = 22.3°}$ **OR** $\boldsymbol{67.7°}$ Add 90° to each θ to get angles between 90° & 180°

Specifying the angles between 0° & 180°:

$$\boldsymbol{\theta = 22.3°,\ 67.7°,\ 112.3°,\ \&\ 157.7°}$$

Note: after turning the polarizer 180° you are essentially back to 0°.

Extra Credit: an inverted image requires magnification less than 1.
To get a feeling for this, solve for magnification algebraically.
As we've seen a million times by now, the $1/f$ equation gives

$$d_i = \frac{d_o f}{d_o - f}$$

$$M = -\frac{d_i}{d_o} = -\frac{f}{d_o - f}$$

For an inverted image we require $M < 0$.

$$-\frac{f}{d_o - f} < 0$$

Think: for a diverging lens we know $f < 0$.
This implies we require a negative denominator.

$$d_o - f < 0$$
$$d_o < f$$

Think: $f < 0$ for a diverging lens.
The object distance must be a negative number that is more negative than the focal length.
Essentially, we require a virtual object farther from the lens than the focal points.

This was actually the example given in the virtual object ray diagram in the lab handout!
Note: virtual objects will only appear in multi-lens systems (or mirror with lens systems).