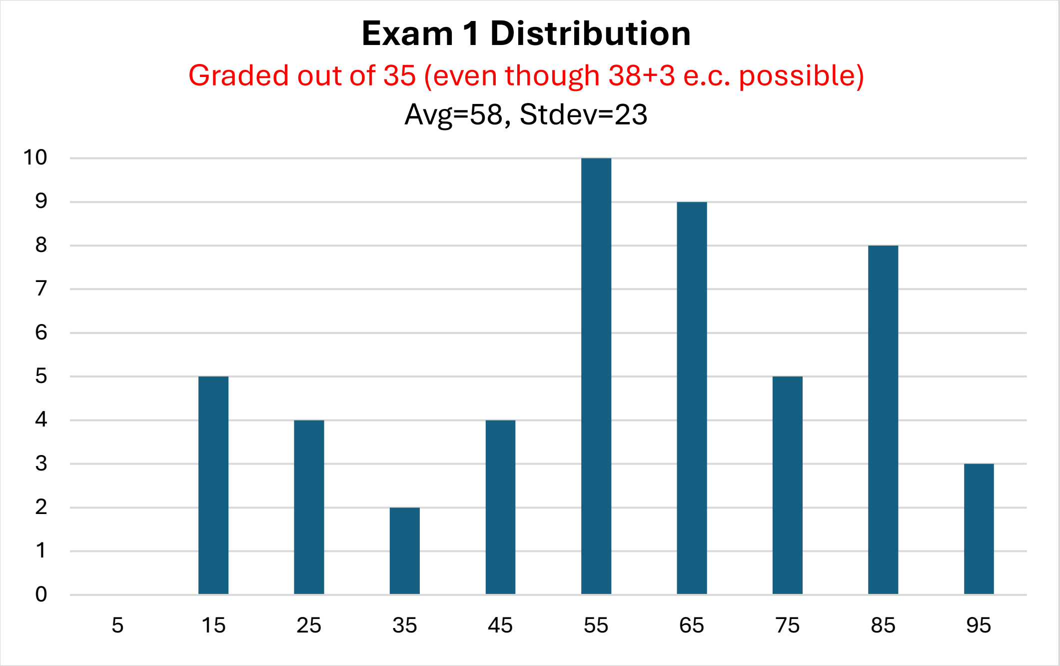
Version B, YELLOW

162sp26t1bSoln

Distribution on this page.
Solutions begin on the next page…



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1a) The specific heat of ethyl alcohol is smaller than the specific heat of water.
As such, less mass of water is required (compared to ethyl alcohol) to absorb the same amount of heat.

1b) In an isolated system, we expect:

$$Q_{alcohol\ warming} + Q_{metal\ cooling} = 0$$

$$Q_{alcohol\ warming} = -Q_{metal\ cooling}$$

$$mc_{alcohol}\Delta T_{alcohol} = -mc_{metal}\Delta T_{metal}$$

The water and the metal have the same mass.

$$\frac{c_{alcohol}}{c_{metal}}\Delta T_{alcohol} = -\Delta T_{metal}$$

The specific heat of all metals given in the table is significantly smaller than the specific heat of water.
Notice this implies the temperature change of the metal is larger ($|\Delta T_{metal}| > \Delta T_{alcohol}$).
The halfway point between the two initial temperatures is 60 °C.
The equilibrium temperature should be *less than* 60 °C.

2a) When describing temperature in physics, we should be using the Kelvin scale.
Both temperatures are within 1% of each other on the Kelvin scale (275.15 K versus 277.15 K).

2b) The rate of cooling for both conduction and radiation increases with temperature difference.
In this case, the warmer object has a larger temperature difference with the room.
The warmer object should cool more rapidly.

2c) Entropy change is given by

$$\Delta S = \int_{T_i}^{T_f} \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc\ln\frac{T_f}{T_i}$$

For both bricks, $T_f < T_i$. For both bricks, entropy change is negative.

2d) We were told m & c are the same for both bricks. Entropy change is given by

$$\Delta S = \int_{T_i}^{T_f} \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc\ln\frac{T_f}{T_i}$$

The first 1.00 °C of cooling is a slightly larger fractional temperature change for brick B.
Brick B has a slightly larger entropy change.
Said another way, consider the fractions inside the ln term.

$$\frac{T_{fB}}{T_{iB}} = \frac{274.15\text{ K}}{275.15\text{ K}} \approx 0.99636 \quad \text{while} \quad \frac{T_{fA}}{T_{iA}} = \frac{276.15\text{ K}}{277.15\text{ K}} \approx 0.99639$$

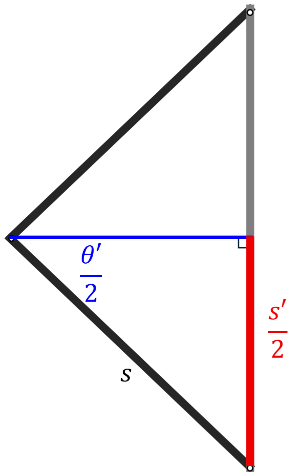
Notice this implies brick B experiences a slightly larger entropy change.
Recall block B is also cooling for slightly a longer time; it is closer to room temp and thus requires more time to cool by 1.00 °C.

3a) Heat flows left to right. Warmer reservoir is on the left.

3b) **Answer:** $Q_{12} = Q_{23}$. In steady state the power delivered though each point in the rod is the same.
Since $\mathcal{P} = \frac{Q}{\Delta t}$, the heat flow past one point in the rod is the same as heat flow past any other point!

3c) **Answer:** $\Delta T_{12} > \Delta T_{23}$. The average cross-sectional area between points **2** & **3** is larger than the average cross-sectional area between points **1** & **2**. Because $\mathcal{P} = kA\Delta T/x$, the smaller area requires a larger ΔT.

4a) The grey rod increases in length while the black rods don't.
The triangle gets taller but narrower.
In the real world this change might be so small it isn't obvious to the naked eye.
The figure below shows an exaggerated version of what happens.



4b) Assume the temperature has increased to final temperature T.
At this point the aluminum rod has increased in size to length s'.
Notice my triangle uses half of the *new* angle θ' instead of θ.

$$\sin\frac{\theta'}{2} = \frac{\frac{s'}{2}}{s}$$

$$\sin\frac{\theta'}{2} = \frac{s'}{2s}$$

$$\sin\frac{\theta'}{2} = \frac{s(1+\alpha\Delta T)}{2s}$$

$$\sin\frac{\theta'}{2} = \frac{1+\alpha(T-20.0\text{ °C})}{2}$$

$$\frac{\theta'}{2} = \sin^{-1}\left[\frac{1+\alpha(T-20.0\text{ °C})}{2}\right]$$

$$\boldsymbol{\theta'(T) = 2\sin^{-1}\left[\frac{1+\alpha(T-20.0\text{ °C})}{2}\right]}$$

Many tried to use cosine or tangent. Those seem like reasonable choices at first…but they are bad ideas.
Think: using a sine function involves the side of the triangle which is constant. If you involve the height of the triangle, you have TWO terms which change size.

Before going on, it is good practice to verify this result does two things:

1) At temperature $T = 20.0$ °C this result should produce the original angle $\theta' = \theta = 60.0°$.
2) As temperature increases, this result should produce $\theta' > 60.0°$.

Verifying condition 1 is met:

$$\theta'(20.0\text{ °C}) = 2\sin^{-1}\left[\frac{1+\alpha(20.0\text{ °C}-20.0\text{ °C})}{2}\right]$$

$$\theta'(20.0\text{ °C}) = 2\sin^{-1}\left[\frac{1}{2}\right]$$

$$\theta'(20.0\text{ °C}) = 2(30.0°)$$

$$\theta'(20.0\text{ °C}) = 60.0°$$

Verifying condition 2 is met:

$$\theta'(T) = 2\sin^{-1}\left[\frac{1+\alpha(T-20.0\text{ °C})}{2}\right]$$

For $T > 20.0$ °C, the term inside the inverse sine function is larger than ½.
The inverse sine function produces angles larger than 30.0° for arguments greater than ½.
This gives $\theta'(T) > 60.0°$ for $T > 20.0$ °C.

4c) If the expansion of the black rods was non-negligible, the angle will not change as much.
Think: if all rods were aluminum, the angle wouldn't change at all!
Our function *overestimates* the change in angle.

5) This is trickier than it seems at first.
In particular, at first glance it seems like it is impossible to solve this problem without knowing final temperature!
THINK!
If only *some* of the ice melts, you are supposed to recognize the final state has metal in thermal equilibrium with an ice-water mixture. This only occurs if the temperature of all materials is the melting temperature of ice!

$$Q_{ice\ warms\ to\ 0\ ^\circ\mathrm{C}} + Q_{some\ ice\ melts} + Q_{metal\ cools} = 0$$

$$m_{ice}c_{ice}\Delta T_{ice} + (some\ \%)m_{ice}L_{f\ ice} + m_{metal}c_{metal}\Delta T_{metal} = 0$$

$$m_{ice}\left[c_{ice}\Delta T_{ice} + (some\ \%)L_{f\ ice}\right] = -m_{metal}c_{metal}\Delta T_{metal}$$

$$m_{ice} = -\frac{m_{metal}c_{metal}\Delta T_{metal}}{c_{ice}\Delta T_{ice} + (some\ \%)L_{f\ ice}}$$

$$m_{ice} = -\frac{(875\ \mathrm{g})\left(235\ \frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}\right)(-99.9^\circ\mathrm{C})}{\left(2200\ \frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}\right)(+3.00\ ^\circ\mathrm{C}) + (4.00\%)\left(334\ \frac{\mathrm{kJ}}{\mathrm{kg}}\right)}$$

At this point, I notice the minus sign out front cancels the minus sign in the numerator.
I notice I need to change the units of grams to kilograms in the numerator.
I also notice I should get rid of the prefix kilo in the last term of the denominator.
I should also rewrite the % as a decimal.

$$m_{ice} = \frac{(0.875\ \mathrm{kg})\left(235\ \frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}\right)(99.9^\circ\mathrm{C})}{\left(2200\ \frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}\right)(+3.00\ ^\circ\mathrm{C}) + (0.0400)\left(334\times 10^3\ \frac{\mathrm{J}}{\mathrm{kg}}\right)}$$

Recall $1\ ^\circ\mathrm{C} = 1\ \mathrm{K}$…those units do cancel! We do eventually get units of kg.

$$m_{ice} = 1.0292\ \mathrm{kg}$$

Unless otherwise specified, we typically round any final result to three sig figs
Exception: many engineers use 4 sig figs if the first digit of the answer is 1.

$$\boldsymbol{m_{ice} = 1.029\ \mathrm{kg} = 1.03\ \mathrm{kg}}$$

Either of these last two forms is acceptable.

A final note: because we were told thermal equilibrium is reached, we assumed all un-melted ice, all melted ice, and the metal must all be at the same final temperature. If the block of ice is big enough, one could argue it is possible for some of the ice to remain at sub-zero temperatures. Think: ice is not a great conductor of heat. In the real world, some of the ice would melt and all of the ice around the melted portion would be at 0 °C. We might expect a small fraction of the ice to still be subzero if the block was huge compared to the chunk of metal. That said, it would be impossible for us to calculate that scenario without complex numerical models and significant computation. As a result, for test questions we assume all ice warms to 0 °C and realize this calculation is only an estimate which approximately models the real world.

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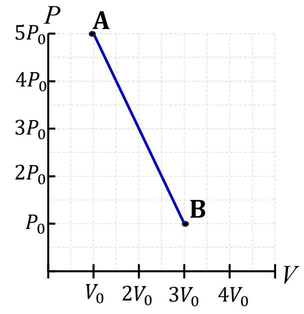

6a) In going from point A to point B, the volume increases.
Work done BY the gas (on the piston) is positive.

$$W_{done\ ON\ gas} = -W_{done\ BY\ gas}$$

Therefore, work done ON the gas (by the piston) is negative.

6b) For ANY process, we know

$$\Delta E_{int} = \frac{f}{2} nR\Delta T$$

$$\Delta E_{int} = \frac{f}{2}\left(P_f\, V_f - P_i V_i\right)$$

$$\Delta E_{int} = \frac{f}{2}\left[(P_0)(3V_0) - (5P_0)(V_0)\right]$$

$$\boldsymbol{\Delta E_{int} = -f P_0 V_0 < 0}$$

6c) While I didn't ask, what about the sign of Q_{in}? Can you figure it out or not?
I should have put this on there to make you think but I was worried the test was getting too long.
Try it for fun, I'll put the answer on the next page…
This part would be good to do as extra practice after you do the timed practice test.

6c) At first you would probably bust out the first law and solve for Q_{in}.

$$\Delta E_{int} = Q_{in} - W_{by}$$

$$Q_{in} = \Delta E_{int} + W_{by}$$

Recall from parts 6a) and 6b) we know $\Delta E_{int} < 0$ and $W_{by} > 0$.

At first glance it seems impossible to determine.

HOWEVER, hopefully you recall we can determine W_{by} using area under the curve!

The area is that of a trapezoid given by

$$W_{by} = Area_{trapezoid}$$

$$W_{by} = \frac{1}{2}(5P_0 + P_0)2V_0$$

$$W_{by} = 6P_0V_0$$

This gives

$$Q_{in} = -fP_0V_0 + (6P_0V_0)$$

Think: we don't know the number of degrees of freedom (f). For a diatomic gas at very high temperatures, $f = 7$. Notice this implies $Q_{in} < 0$. However, at lower temperatures we expect $f = 3$ or $f = 5$. Under those circumstances $Q_{in} > 0$. The best answer is "impossible to determine the sign of Q_{in}".

7) I'm hoping this was an easy one.

$$\mathcal{P}_{emitted} = \sigma A_{surface} e(T^4 - T_{env}^4)$$

We were told the temperature of the environment surrounding the star was negligible.

$$\mathcal{P}_{emitted} = \sigma A_{surface} e T^4$$

Every problem we've ever done using stars in physics typically assumes the star is a sphere.

$$\mathcal{P}_{emitted} = \sigma(4\pi r^2)eT^4 = \sigma\left[4\pi\left(\frac{d}{2}\right)^2\right]eT^4 = \pi\sigma d^2 e T^4$$

For a perfect blackbody, emissivity is $e = 1$.

$$\mathcal{P}_{emitted} = \pi\sigma d^2 T^4$$

$$d = \sqrt{\frac{\mathcal{P}_{emitted}}{\pi\sigma T^4}}$$

$$d = \sqrt{\frac{3.43 \times 10^{24}\ \mathrm{W}}{\pi\left(5.67 \times 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2 \cdot K^4}}\right)(8888\ \mathrm{K})^4}}$$

$$\boldsymbol{d = 5.55 \times 10^7\ \mathbf{m}}$$

8) We are asked to determine the average of v^3 for the given distribution.

$$\overline{v^3} = \int_0^\infty v^3\, f(v)\, dv$$

$$\overline{v^3} = \int_0^\infty v^3 \left[2\pi v \left(\frac{m}{2\pi k_B T}\right) \exp\left(-\frac{mv^2}{2k_B T}\right)\right] dv$$

Pull out the 2π, group the v^3 & v into a v^4, and let $\alpha = \frac{m}{2k_B T}$.

$$\overline{v^3} = 2\pi \int_0^\infty v^4 \left(\frac{\alpha}{\pi}\right) e^{-\alpha v^2}\, dv$$

Pull out the constant and cancel the $\pi's$.

$$\overline{v^3} = 2\alpha \int_0^\infty v^4\, e^{-\alpha v^2}\, dv$$

Use integral I_4 from the table.

$$\overline{v^3} = 2\alpha\left(\frac{3}{8}\sqrt{\frac{\pi}{\alpha^5}}\right)$$

$$\overline{v^3} = \frac{3\alpha}{4}\sqrt{\frac{\pi}{\alpha^5}}$$

Follow instructions and get all terms inside a single radical.

$$\overline{v^3} = \sqrt{\frac{9\pi}{16\alpha^3}}$$

Follow instructions and write the final answer in terms of $m, k_B, \& \, T$.

$$\overline{v^3} = \sqrt{\frac{9\pi}{16\left(\frac{m^3}{2^3 k_B^3 T^3}\right)}}$$

Follow instructions and simplify the result to a single fraction.

$$\boldsymbol{\overline{v^3} = \sqrt{\frac{9\pi k_B^3 T^3}{2m^3}}}$$

Since a lot of you are engineers, you might be in the habit of writing this as:

$$\boldsymbol{\overline{v^3} = 3.76\sqrt{\frac{k_B^3 T^3}{m^3}}}$$

I'll accept that result as well as it is somewhat standard in some circles.

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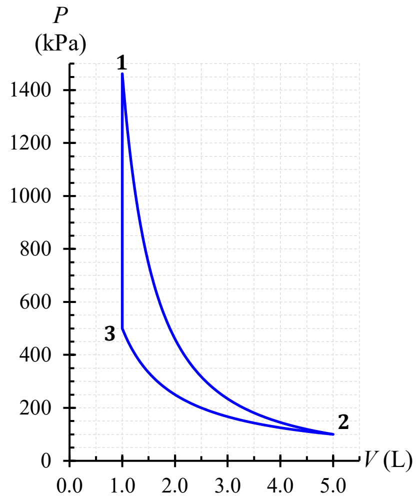
A PV-diagram for an engine using a *monatomic* gas is shown. The cycle runs clockwise. Temperature at point **3** is 1046 K. Assume work is done by the gas during every cycle (as in a 2-stroke engine). Note: $P_1 = 1461.5$ kPa. The other volumes & pressures line up on gridlines. To make things line up, one of the temperatures is unrealistically high!



9a) Determine n. Use $PV = nRT$ on point 3.

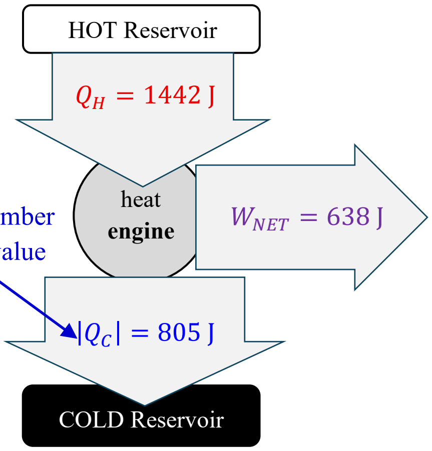
9b) Determine T_1 & T_2. Use $PV = nRT$ ratios.

9c) Determine the best value to use for the adiabatic constant γ based off discussions in class & homework.

You'll know your results are good enough if your rounding digit matches my rounding digit (assuming we both actually rounded)

9a	$n = 0.057\underline{4}9$ mol
9b	$T_1 = 30\underline{5}7$ K $T_2 = 1046$ K
9c	$f = 3$ for montoamic $\gamma = \frac{f+2}{f} = \frac{5}{3} = 1.667$

	Process Name	Q_{in} (J)	$W_{\boldsymbol{by\ gas}}$ (J)	ΔE_{int} (J)
1 → 2	Notice $P_2V_2 \neq P_1V_1$ $P_2V_2^\gamma = P_1V_1^\gamma = 10.08$ **Adiabatic**	0	$\frac{1}{1-\gamma}(P_fV_f - P_iV_i)$ $+14\underline{4}2$	$nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}(P_2V_2 - P_1V_1)$ $-14\underline{4}2$ Verify with 1st law
2 → 3	Notice $P_2V_2 = P_3V_3$ **Isothermal**	Use 1st law after getting W_{by} & ΔE_{int} $-80\underline{4}.7$	$nRT\ln\frac{V_3}{V_2} = P_2V_2\ln\frac{V_3}{V_2} = P_3V_3\ln\frac{V_3}{V_2}$ $-80\underline{4}.7$	0
3 → 1	**Isochoric**	$nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}(P_1V_1 - P_3V_3)$ $+14\underline{4}2$	0	$nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}(P_1V_1 - P_3V_3)$ $+14\underline{4}2$ Verify with 1st law
For the Entire Cycle		Sum the rows above $6\underline{3}7.3$	Sum the rows above $6\underline{3}7.3$	Sum the rows above 0 Verify with 1st law



Did you remember the absolute value on Q_c?

Efficiency

$$\boldsymbol{\eta = \frac{W_{NET}}{Q_H} = 44.2\%}$$

%'s are not always error calcs. A common error was writing η with fewer than 3 sig figs…

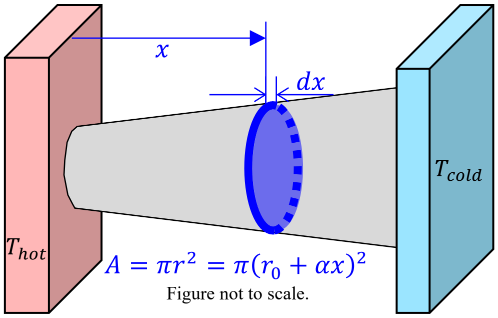


Figure not to scale.

Extra Credit: Consider the figure at right.
Notice the cross-sectional area varies with horizontal position x!

$$\mathcal{P} = kA\frac{dT}{dx}$$

Think: to get the known parameter ΔT involved we must integrate!
In general $\frac{dT}{dx} \neq \frac{\Delta T}{\Delta x}$. If you simply subbed in $\frac{\Delta T}{L}$ expect zero points.
Also, because the area is a function of x, *separate* before you *integrate*!

$$\frac{\mathcal{P}}{A}dx = k\,dT$$

Note: I know in the end we are solving for k. I left it on the left side to make solving for k easy at the end.

$$\frac{\mathcal{P}}{\pi r^2}dx = k\,dT$$

Note: I can tell I need a substitution…why not try $r = r_0 + \alpha x$ giving $dr = \alpha\,dx \ \rightarrow dx = \frac{1}{\alpha}dr$.
Also, the $x = 0$ limit gives $r(0) = r_0$ while the $x = L$ limit gives $r(L) = r_0 + \alpha L$.

$$\frac{\mathcal{P}}{\pi r^2}\left(\frac{1}{\alpha}dr\right) = k\,dT$$

$$\frac{\mathcal{P}}{\pi\alpha}\int_{r_0}^{r_0+\alpha L}\frac{dr}{r^2} = k\int_{T_{hot}}^{T_{cold}} dT$$

$$\frac{\mathcal{P}}{\pi\alpha}\left[-\frac{1}{r}\right]_{r_0}^{r_0+\alpha L} = k(T_{cold} - T_{hot})$$

$$-\frac{\mathcal{P}}{\pi\alpha}\left(\frac{1}{r_0+\alpha L} - \frac{1}{r_0}\right) = -k\Delta T$$

I wanted to put this on the regular test but it got too long. Be sure to actually practice this question after you complete the timed practice test.

Without experience, you might solve for k at this point and be *mostly* correct. However, with experience, when you see something like that left hand side you know you can simplify further using a common denominator.

$$-\frac{\mathcal{P}}{\pi\alpha}\left[\frac{r_0 - (r_0 + \alpha L)}{(r_0 + \alpha L)r_0}\right] = -k\Delta T$$

$$-\frac{\mathcal{P}}{\pi\alpha}\left[\frac{-\alpha L}{(r_0 + \alpha L)r_0}\right] = -k\Delta T$$

$$\frac{\mathcal{P}L}{\pi(r_0 + \alpha L)r_0} = -k\Delta T$$

$$k = -\frac{\mathcal{P}L}{\pi(r_0 + \alpha L)r_0\Delta T}$$

WATCH OUT! At this point, it looks like we found a negative value for k.
THINK! Energy is being transferred from the high temperature reservoir to the low temperate reservoir at rate $\mathcal{P}$.
This energy is *leaving* the high temperature reservoir. We should have said $-\mathcal{P} = kA\frac{dT}{dx}$ to start this problem!
Fixing up this minus sign error gives

$$\boldsymbol{k = \frac{\mathcal{P}L}{\pi(r_0 + \alpha L)r_0\Delta T}}$$

For reference, compare this to simply using the average radius to compute area then plugging in $\frac{\Delta T}{L}$:

$$r_{avg} = r_0 + \frac{\alpha L}{2} \quad \rightarrow \quad A = \pi\left(r_0 + \frac{\alpha L}{2}\right)^2 \quad \rightarrow \quad -\mathcal{P} \approx k\pi\left(r_0 + \frac{\alpha L}{2}\right)^2\frac{-\Delta T}{L} \quad \rightarrow \quad k \approx \frac{\mathcal{P}L}{\pi\left(r_0 + \frac{\alpha L}{2}\right)^2\Delta T} = \frac{\mathcal{P}L}{\pi\left(r_0^2 + r_0\alpha L + \frac{\alpha^2L^2}{4}\right)\Delta T}$$

This result is identical except for the $\frac{\alpha^2L^2}{4}$ term! This approximation is a decent approximation whenever $\frac{\alpha^2L^2}{4} \ll r_0\alpha L$.